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Equipotential Energy Exchange Depends on Density of Matter

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Abstract: Potential energy exchanged for work done by radial/center-bound gravitational force field along equipotential surfaces (where the radial potential remains unchanged by definition), depends also on density of matter of the source mass that is responsible for generating the locally dominant field. To the extent that equipotential (hence nonradial) parts of trajectory paths (of the objects that move within the field) are exposed to the (nonrotating) field's influence, the moving objects' energy is lost (regardless of direction of the nonradial part of their motion) and transferred to the field. Mathematically derived, with the use of (new) synthetic mathematics, a new law governing nonradial exchange of potential energy generalizes the previous law that has been derived from physical considerations and had already been confirmed by several experiments and observations.

Key words: Potential energy; Work done; Nonradial effects; Radial/center-bound gravitational field

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1. INTRODUCTION

While Poisson equation contains density of mass, which is the dynamic/energetic—hence rather procedural feature of the main source mass that generates the locally dominant gravitational field, its free space counterpart (i.e. Laplace equation) does not. None of primary field equations of both pre-relativistic and relativistic theories of gravitation, however, contain density of matter, which is rather static/structural (i.e. definitely constitutional) feature of massive bodies. One can mistakenly argue that density of matter (also known as specific gravity) is maybe even more truly physical an attribute than that of mass, and therefore has no place in free space, which is empty by definition.

Since Laplace equation is of abstract geometric nature, however, and all geometries are about structures, it should be possible to obtain some structural relationships by connecting procedural and structural features of fields. This is because every structure corresponds to a certain (even if unknown) procedure and vice versa. This line of thought is mathematically unavoidable; and so if our mathematics was yet unable to make such necessary connection then maybe it is incomplete.

Also the fact that annihilating particle-antiparticle pairs dissolve into energy by unfolding the pair into radiation, strongly suggests that density of matter should affect work done and potential energy of gravitational fields if energy is to be conserved, because mass is a form of energy too.

Moreover, since frequency shifts appear near collapsing black hole (Hawking & Ellis, 1973, p.309), then less dense bodies should exhibit similar effect even if much smaller. Such effects do not suddenly start appearing when the black hole begins collapsing, but are likely inherent to all fields. During the collapse the shifts presumably become more pronounced and thus are more easily recognizable than before.

Finally, after Galilei demonstrated that all bodies fall towards center of the Earth's gravity field with the same radial acceleration regardless of their composition, physicists declared that density of matter does not count. But even if density of matter does not influence purely radial interactions within fields, it would be a mathematical miracle if the same were true for other than radial (hence nonradial) interactions, because equipotential surface contains volume whose internal change must somewhat affect happenings within the field that is generated by the mass source enclosed therein.

As a matter of fact, formerly unanticipated experimental data (to be briefly explained below) hinted at the possibility that density of matter matters for phenomena happening on equipotential surfaces. The experiments have been reconciled only after density of matter of the mass sources that generated their respective fields was taken into account in a new nonradial formula (Czajko, 2000). In the present note more general nonradial formula with matter density will be derived mathematically. It suggests that even constant/average density of matter of the massive source (body) that generates the locally dominant gravitational force field affects nonradial phenomena happening within the field just as specific heat affects work being done by flow of the heat—this was pointed out by Kellog (1929, p. 77f).

2. MATHEMATICAL INCOMPLETENESS INADVERTENTLY CONCEALED SOME ASPECTS OF PHYSICAL REALITY

When Lagrange made his phenomenal discovery in 1773 AD linking Newtonian gravitational force **F** to gradient $\nabla V = -\mathbf{F}$ of the generic scalar potential function $V = 1/r$ that solves the Laplace equation, he launched new approach to force fields' interactions which culminated in what is now called potential theory by establishing its mathematical foundation for any center-bound fields (Birkhoff, 1973, p.360).

But when rate of work done by the force field over a distance r has subsequently been defined as just $dW = \mathbf{F}\bullet d\mathbf{r}$ (by analogy to formula admissible for a single standalone force which does not vary when distance r is fixed) then the notion of "total potential energy" (as the energy that is spent on work done by the given force field) became effectively meaningless for center-bound force fields (Czajko, 2013). The definition is incomplete because it disregards product differentiation rule. In multisourced fields both magnitudes and directions of these vectors can change even on equipotential surfaces.

One might ask why was the, operationally complete and thus mandatory, product differentiation rule/law $dW(\mathbf{F}\bullet\mathbf{r}) = \mathbf{F}\bullet d\mathbf{r}+\mathbf{r}\bullet d\mathbf{F}$ cut short, even though effective/resultant force field vectors could generally vary even when the distance r is fixed? Violation of the mathematical law could not be justified. The inconsiderate definition actually defines away the very part of reality it pertains to.

Notice that the right-hand side (RHS) term $\mathbf{r}\bullet d\mathbf{F}$ has nonradial character because r is fixed there. Consequences of former violations of the product differentiation rule have been discussed by Czajko (2013).

Although magnitude of force field due to single source mass remains constant at the same fixed distance, in some situations where the effective force field is due to more than one mass/force, the effective force acting along any equipotential surface surrounding just one of these source masses contributing to the effective/combined force field can vary. It was thus a conceptual mistake of the former physics to ignore the nonradial term, especially when multiple field's source masses cannot be simply amalgamated into as if single source mass, or a single center-bound force field vector.

Two experiments conducted by Sadeh, Knowles, & Yaplee (1968) and Sadeh, Knowles, & Au (1968) showed presence of (previously quite unanticipated) curious frequency decrease (hence extraneous energy loss) in rays/waves traversing gravitational fields of our Sun and Earth, respectively. None of then-accepted electromagnetic or gravitational theories of physics could explain the observed frequency decrease (Szekeres, 1968). However, new theory of nonradial effects of gravitational fields reconciled these two experiments and retrodicted their findings (Czajko, 2000). But although experimentally confirmed, the presence of nonradial contributions to potential energy in center-bound force fields virtually challenged the former (mathematically incomplete and thus operationally inadmissible) definition of work done (and the corresponding to it notion of potential energy). Moreover, it has also been shown that presence of other than radial (i.e. nonradial) effects of the usual radial force fields is mathematically unavoidable indeed (Czajko, 2011).

Even though force field of a distant star seems very small in our vicinity, the potential energy which the distant star imparted on rays coming towards us did not vanish (even though a part of it was lost on the radial relativistic gravitational frequency decrease, i.e. redshift) and the impact of our Sun on the energy of the rays whose path is close to the Sun is fairly easy to be measured as the rays' extraneous frequency decrease that was observed in the experiment (Sadeh, Knowles, & Yaplee, 1968). Also the impact of the Earth on radio waves resulted in similar frequency decrease (Sadeh, Knowles, & Au, 1968). The quite unanticipated results of these two experiments require taking into account also force vectors that are not radial.

By questioning the former (mathematically incomplete) definition of work done, the presence of definite nonradial effects of the usual radial gravitational forces acting in center-bound force fields challenged also the path-independence of work done theorem, which is not really applicable to any center-bound force fields (Czajko, 2013). When the theorem was eliminated from the context of center-bound fields, a new (operationally correct) geometric formula was developed for interactions within such fields (Czajko, 2013). Yet the geometric formula did not have any physical characteristics of the field itself.

In the present note I shall derive mathematically new expanded physical formula for the angle-dependent nonradial interactions alone, which is much more general and also more detailed than the previous (experiment-driven) formula (proposed by Czajko (2000)) for the nonradial interactions that do happen along equipotential surfaces within radial/center-bound force fields. The previous formula is not wrong for the local predicament of the aforesaid experiments, but the new, mathematically derived physical formula will take into account also the impact of perihelion position on nonradial effects/interactions, which was absent in the previously devised "physical" nonradial formula. The expanded formula to be derived here will include also density of matter of the field's source mass.

3. TRADITIONAL PROOFS ENDORSED AXIOMS, NOT TRUTHS ABOUT REALITY

Given the obvious difficulty of the topic, which was lingering unrecognized for some 227 years (between 1773 and 2000 AD), I shall use examples from physics to illustrate the (new) synthetic mathematics that is being employed in the present note. By the same token, however, the proof of validity of the mathematical reasonings shall lie in the very experimental results that confirmed the former nonradial formula (Czajko, 2000) and indirectly thus also its more general equivalent to be derived in what follows. Although not incompatible with traditional mathematics, the synthetic mathematics is not rooted in presently accepted axiomatic systems of mathematical theories. Thus in order to prove the new results by regular derivation, one would have to append former axiomatic systems with several new concepts (such as abstract multispatial hyperspace

(Czajko, 2004), for example), which task is out of scope of the topic under investigation in this paper. Hence it shall be discussed elsewhere.

Traditional mathematical proof by derivation from axioms, previous theorems and few primitive notions validated only logical consistency of theorems leaving questions of their truth unanswered (Smart, 1967). For axioms need only be consistent, but they do not describe any known reality (Resnick, 1980, p.108). Some axioms used as premises in inductive inferences are arbitrary and often not quite understood (Delègue, 1908, p.21). With convenient axioms anything can be proved, but nothing established—warned Lorenzen (Lorenzen, 1951).

Axiomatization was supposed to remove mysticism from some abstract notions (Hilbert, Von Neumann, & Nordheim, 1927). Although Hilbert thought that concepts should reflect relations found in reality (Hilbert, 1992, p.17), he also believed that all mathematical reasoning can be somehow reduced to an elementary part of it (Wang, 1981, p.127). However, Kurt Gödel has demonstrated that any such abstract system is inherently incomplete if consistent, and any system holding set of all finitary methods cannot be proved consistent by such methods (Feferman, 1994).

Relying on axiomatization effectively narrowed the scope of mathematics so that its progress became synonymous with digging into deeper details of lesser relevance to reality, while leaving curious new aspects of reality untouched and the experiments which hinted at them unexplained.

Unanticipated phenomena revealed in troublesome experiments suggest that axiomatic systems are not always equipped to deal with the happenings observed in nature and thus need expansions. It has already been emphasized by Niels Bohr that any apparent disharmony in the description of experiences can be eliminated only by appropriate widening of the conceptual framework (Bohr, 1956).

Classical idea of proof as derivation from allegedly self-evident axioms and primitive notions, both of which were usually assumed by faith in human ability to perceive truths standing behind phenomena, served merely to uphold the lofty belief in infallibility of human minds, rather than validating truths about the reality the proofs pertained to. Therefore having experimental evidence of truths that escaped mathematical predictions we should try to improve the mathematics instead.

G. Choquet pointed out that the Euclid-Hilbert axiomatization of geometry (which was based on the notions of length, angle and triangle) so marvelously concealed the underlying vector space that the concept of vector remained unrecognized for ages (Choquet, 1969, p.14). Although the axiomatization was well-intended, what is good for devising proofs, could be detrimental to conceptualization of facts, which should be learned from the Nature and built from natural (not man-made) building blocks.

History of noneuclidean geometries is another example of futile efforts to prove that apparently self-evident axioms must be true, mainly because mathematicians could not imagine reality more sophisticated than their minds were able to fathom. With few exceptions, mathematics evolved as a science avoiding both asking and answering questions about character of observed phenomena.

Yet operationally sound mathematics developed very powerful tools and successful predictive methods, many of which are still untapped, often because they are incompatible with the present axiomatic systems that have been accepted as being sufficiently fundamental. In my choice of the presentation method I want to demonstrate that the (new) synthetic mathematics deployed in the present paper is well capable of predicting definitely new laws of nature and of mathematics itself. In either of the cases, truth of the new laws should be confirmed by experiments and observations at first. Then, after enhancing our axiomatic systems by incorporating into them the truths about reality that were learned from the newly synthesized laws, we could also prove their consistency. We cannot prove what we know is true/exists because our axiomatic systems are obsolete. On the other hand we did not update our old axiomatic systems upon newly found physical experiences.

For our mathematics to keep up with physics we should synthesize new laws from experimental hints. We have mathematics capable to do so, but despite knowing that proofs cannot ensure even consistency, not to mention truth, our minds are trapped in the unfounded old Euclid-Hilbert ideas.

4. MATHEMATICALLY COMPLETE DEFINITION OF WORK DONE

When some additional contributions to the effective local field (such as engines of a spacecraft, laser pulses, or a distant star that ejected rays crossing our local field in not exactly radial direction with respect to the local gravity center) cannot be amalgamated into some sort of single-source force field, then the rate of work done function for energy interchange within radial/center-bound force fields should be defined more adequately as mathematically complete function of product:

$$\begin{aligned} dW(\mathbf{F},\mathbf{r}) &:= d(\mathbf{F}\cdot\mathbf{r}) = \mathbf{F}\cdot d\mathbf{r} + \mathbf{r}\cdot d\mathbf{F} \\ &= -F\cos 2\alpha dr + 2Fr\sin 2\alpha d\alpha - r\cos 2\alpha dF \end{aligned} \tag{1}$$

where the planar spherical angle α spreads between the vector connecting the local gravity center with perihelion of the trajectory path traveled by a mass m or rays/waves, and the pointing vector (Czajko, 2013). Since α is the angle of visibility of just half of the rays' path which extends from 0 to π/2, a double angle α is needed to see/cover the whole single-sided path extending from -∞ to +∞, with perihelion located at α = 0. The fact that magnitude of the work done function is measured by force times distance is uncontested. It is the inappropriate traditional handling of the work done function that is clearly quite inadmissible, because its former definition was operationally incomplete (Czajko, 2013).

Planar spherical angle corresponds to spherical distance which lies on a circle of the sphere. If a spherical distance is somewhat twisted, the angle is not planar spherical but spheroidal spherical, because the surface connecting curve of the twisted distance with center of the sphere is mangled.

To visualize the situation take rays coming from Taurus A, for example. The rays were passing just few solar radii from the surface of our Sun whose gravity field affected their energy/frequency in both radial and tangential directions (Czajko, 2000; Czajko, 2013; Czajko, 2011), just as it happened in the experiment (Sadeh, Knowles, & Yaplee, 1968). The equation (1) has been derived (Czajko, 2013) for trihedron comoving with the rays, not for the reference frame bound to the local gravitational force field of our Sun, however. It shows the changing rate of the work done function by the field due to its potential energy at each point along the path of the rays.

Although the moving trihedron shall not deprecate the local reference frame bound to our Sun, it gives us several conceptual advantages. First, instead of dealing with forces from different sources the trihedron allows us to investigate exchange of energy influenced by just one source (which is our Sun), whereas the impact of the other source (a distant star) should be represented by the path and energy of the rays coming from the star. We know the rays' regular frequency (which depends on the difference between gravity on that star and Earth (Beiser, 1973, p.67ff)) when our Sun was far away from their path and their diminished frequency when they pass near occultation. The trihedron allows us to estimate how our Sun affects the rays without knowing exactly all force fields involved therein.

One can see that only at the rays perihelion the angle-dependent tangential contribution to the angular part of the work done by our Sun's force field (and thus also to the corresponding to it potential energy) vanishes. Since the planar spherical angle α (of visibility of the path) from the gravity center of our Sun is given via the usual planar trigonometric relation $\cos\alpha = r_p/r$ hence the ratio of the radius of perihelion r_p to the varying radius r that is always pointing to the path/orbit (Czajko, 2013). Notice that perfectly circular orbit around the local gravity center of the Sun actually means unchanging/fixed perihelion, which is the same everywhere along such an idealized circular orbit.

5. ANGULAR FUNCTION OF WORK DONE BY GRAVITATIONAL FIELD

From the equation (1) we got the angle-dependent part of work done rate in the moving trihedron:

$$dW_r(\alpha) = 2Fr\sin 2\alpha d\alpha = 4Fr\sin\alpha\cos\alpha d\alpha = -4Fr\cos\alpha d(\cos\alpha)$$
$$= -4Fr\frac{r_p}{r}d\left(\frac{r_p}{r}\right) = -4r_p^2F\frac{dr}{r^2} = -4GMmr_p^2\frac{dr}{r^4} = dW_\alpha(r) \tag{2}$$

and so the amount (i.e. functional) of the angle-dependent expense of work done W_α is given by

$$W_\alpha(r) := \int_r^\infty dW_\alpha(r) = -\frac{4GMmr_p^2}{3r^3} = W_\alpha \tag{3}$$

when the radius of perihelion is fixed. As outcome of the integration process the amount of work done becomes also the functional/value W_α that is fixed on any equipotential surface determined by certain fixed radius r. Note that the distance r grows naturally to infinity here and so it renders the negative amount. In order to achieve this in traditional presentations of the topic, boundaries of the integration were purposely (but arbitrarily) reversed (from infinity to r) so that the work done could be conventionally defined as vanishing at infinity, just as the corresponding to it potential.

Notice that the same expression may represent both function and its functional, but when treated as function the amount of angular work done $W_\alpha(r)$ depends on the (chosen as independent in this notation) variable r, whereas functional W_α is determined by constants and all variables acting as fixed parameters. The work done might also be considered as function $W_\alpha(r,r_p)$ of radius/distance and radius of perihelion, or of other varying magnitudes too. Active symbols (i.e. those chosen as independent variables) in functions do vary, whereas in functionals the same symbols that can be varying in functions are just fixed configuration parameters always having certain definite values.

Despite the notational similarity, the operational roles of functions and functionals are different. Functions are mappings of sets, whereas functionals are mappings of single values which behave like compound parameters during integration and unlike functions, pure functionals cannot really be differentiated. Because of this clear operational distinction, which was largely ignored in many traditional presentations of mathematical reasonings, some abstract mathematical concepts were sometimes inadvertently misapplied. In order to avoid such pitfalls, one could think of a functional as representing a single picture/frame while function could be compared to animated movie/film composed of film sequences, each of which comprises several variations of a single picture/frame. The distinction between functions and their functionals is absolutely essential for understanding mathematical operations as well as some operational features of mathematically defined concepts. For concise descriptions of properties of functionals , see (Griffel, 2002, p.16ff; Gelfand & Fomin, 2000, p.2; Zubov, 1964, p.10; Castillo, Iglesias, & Ruiz-Cobo, 2005, p.9; Ryder, 1996, p.172ff; Riesz & Nagy, 1990, p.61; Whitehead & Russell, 1968, p.40).

Now consider perfectly spherical source mass M with uniform distribution of matter, in which case the angle α varies along equipotential surface (surrounding center of gravity of the mass M) where density of matter Q is the same and therefore the parameter Q is independent of the varying angle α (of visibility of the path) even though it is indirectly determined by α, and directly by r on behalf of α. Hence the impact of density of matter, if any, should depend on a certain new angular variable varying along the equipotential surface, not on the angle α that determines the surface itself and therefore plays the role of parameter codetermining the sought-for function of the new variable. The integrand in the equation (3) was function of the radius/pointer r, but not of Q, and the domain under consideration here was covered by variability of the angle α of visibility of the path.

Since the ensuing functional W_α is not determined by density of matter yet, we may designate the functional W_α also as differential function of a certain prospective (i.e. yet to be discovered and then determined) compound work done function $w_\alpha(Q(?))$ of the density of matter Q, which in turn should itself be an enhanced function of a certain as yet undisclosed but quite independent variable tentatively denoted by question mark. Hence by chain rule we get the following relation:

$$\{dw_\alpha(Q(?))\}(\nexists\!:Q) := \frac{\partial w_\alpha(Q(?))}{\partial ?} = \frac{\partial w_\alpha(Q(?))}{\partial Q(?)}\frac{dQ(?)}{d?} = W_\alpha \tag{4}$$

which is presumed here to be the rate $dw_\alpha(Q(?))$ of exchange of the angular (with respect to α) and angle-dependent (with respect to yet to be determined angular variable designated by the question mark) part of potential energy from equation (1), characteristic of the density of matter enclosed inside equipotential surfaces surrounding the center of locally dominant center-bound gravitational field.

Although the prospective function $w_\alpha(Q(?))$ is presumed to depend on the density Q, which in turn depends on a certain independently varying variable '?', the differential $dw_\alpha(Q(?))$ that is the derivative function signifying the rate at which the primary function $w_\alpha(Q(?))$ changes (i.e. could be altered) is independent of the matter density Q (i.e. the function/variable Q is clearly absent in the differential). This fact is written as $\{dw_\alpha(Q(?))\}(\nexists:Q)$ where the symbol $\nexists$ means absence (or nonexistence) of the variable Q in this expression, for the functional term W_α does not contain Q.

Recall that an actual (as opposed to merely symbolic) expression for a derivative of a function is effectively equal identically to its differential function: $W'(Q) \equiv dW(Q)$. This is pretty common and often quite explicit assumption—compare Zeidler (1995, p.228). The term dQ in the denominator of the derivative $dW_\alpha(Q)/dQ$ in the equation (4) is just a symbol indicating independently varying/active variable therein.

Caveat: Density of matter can depend on several distinct independent variables. In reference to potential energy, which is structural aspect of the field, density of matter could depend of certain structural/constitutional variables characterizing the main massive body that generates the locally dominant force field. However, although work done corresponds to the field's potential energy, it is not really structural but mainly rather procedural aspect of the given force field. Hence we are interested in a strictly operational/procedural variable. As I mentioned above, the sought-for quite independently varying variable '?' pertains to the impact of the (assumed as average and therefore constant) density of matter. We are not looking thus for any variable on which evolution of density matter itself could depend. The dependence Q(?) investigated herein is procedural, not structural.

Since neither the matter density Q nor the sought-for independent variable '?' is present in the differential $\partial W_\alpha(Q(?))/\partial Q$ in the equation (4), the rate of the work done function $dw_\alpha\,(Q(?))$ can be equal to the outer differential $\partial W_\alpha(Q)/\partial Q$ when the density of matter changes. We can write thus

$$\{dw_\alpha(Q(?))\}(\nexists\!:Q) := \frac{\partial w_\alpha(Q(?))}{\partial Q(?)} \Longleftrightarrow \frac{\partial w_\alpha(Q)}{\partial Q} \tag{5}$$

where the function (i.e. mapping of whole sets of values) of potential energy $w_\alpha(Q)$ of a force field characterized by a certain average density of matter Q corresponds to the functional/amount W_α of work done regardless of density of matter, because the functional is just a single value of the very function to be calculated in absence of the (fixed for the time being) variable Q. Hence we obtain:

$$\int_0^Q \{dw_\alpha(Q(?))\}(\nexists{:}Q)dQ = \int_0^Q \partial W_\alpha(\nexists{:}Q) = W_\alpha(\nexists{:}Q) \Rightarrow w_\alpha(Q(?)) \cdot Q = W_\alpha(\nexists{:}Q) \quad (6)$$

and so from the equations (6) and (3) we get the prospective function of work done by the field which is characterized/parametrized by the density of matter of the body/mass M that generates the field:

$$w_\alpha(Q(?)) = -\frac{4GMmr_p^2}{3Qr^3} = W_{rQ} \Longleftrightarrow \{w_r(Q(?))\}(\nexists{:}?) := W_{rQ} \quad (7)$$

which indicates that constant/average density of matter (or specific gravity) of the massive body that generates the locally dominant gravitational force field affects happenings within the field in very similar manner as specific heat affects work being done by flow of the heat—compare Kellog (1929)p.77f.

The mathematical rationale behind the equation (6) can be explained as follows: Since functional is just a value to be calculated, it cannot be integrated, because integrand must represent differential (i.e. it must be function obtained by actual differentiation). Functionals are compound parameters. But we can always integrate functions regardless of how detailed/explicitly they are expressed, indeed.

If the matter enclosed by the equipotential surface (whose density is determined by the radius r in perfectly spheroidal bodies) is uniformly distributed, it has an impact on the external ray that traverses the field only to the extent to which the ray interacts with the matter of the mass/body that generates the locally dominant force field. Hence the unknown variable '?' should be angular, because it runs along equipotential surfaces. But unlike the planar spherical angle of visibility α, which was based on the (assumed as being originally straightlinear) trajectory path, the sought-for independent variable '?' should be a spheroidal spherical angle, however. It should correspond to any curvilinear angular distance measured along equipotential surfaces surrounding gravity center.

Now let us find the sought-for independent variable '?' (that varies along equipotential surfaces whereon both the radius r and matter density Q remain constant) on which the generic rate of work done $dw_\alpha(Q(?))$ depends via Q. Since the variable '?' was absent in the outer rate $dw_\alpha(Q)$ we get:

$$dw_\alpha(Q(?)) = \frac{\partial w_\alpha(Q(?))}{\partial ?} = W_{rQ} \Rightarrow w_\alpha(Q(?)) = \int dw_\alpha(Q(?))$$

$$= W_{rQ}\int d? = \frac{-4GMmr_p^2}{3Qr^3}\int_0^\theta d? = \frac{-4GMmr_p^2\theta}{3Qr^3} \quad (8)$$

where the question mark was replaced by the angle-dependent (i.e. varying with angle measured along an equipotential surface) variable θ. The work done function $w_\alpha()$ depends on the impact of the field's density of matter (assumed as constant and enclosed by the equipotential surface that is determined by r) to the extent determined by the angular distance θ traveled along the surface. For an entire lapse on a flat circular orbit around gravity center of the nonrotating force field we get:

$$w_{\alpha}(Q, r_p) = \frac{-4GMmr_p^2}{3Qr^3}\int_0^{2\pi} d\theta = \frac{-2\upsilon GMm}{Qr_p^4} \tag{9}$$

where υ denotes volume of the sphere determined by the radius of the orbit/perihelion i.e. $r = r_p$. I should emphasize that the phrase 'impact of density of matter' means that the average density of matter is given as function of exposure to the matter of the source mass that generates the field.

The angular variable θ can also be replaced by curvilinear distance variable $\lambda = r\theta$ that represents the spherical distance measured along the given equipotential surface, in which case we can write:

$$w_r(Q(\theta)) = \frac{-4GMmr_p^2\theta}{3Qr^3} \Longleftrightarrow w_r(Q(\lambda)) = \frac{-4GMmr_p^2\lambda}{3Qr^4}$$

$$\Rightarrow w_{rQ}(Q(\lambda)) = \frac{-4kGMmr_p^2\lambda}{3Qr^4} \tag{10}$$

which enhances and supersedes the previous formula for extraneous frequency decrease that had been derived from hints deduced from the experiments by Sadeh, Knowles, & Yaplee (1968) and Sadeh, Knowles, & Au (1968) and other observational data (Czajko, 2000). The practical function $w_{rQ}(Q(\lambda))$ on the RHS contains coefficient k yet to be determined from experiments until more general/detailed function is developed, which shall be proposed elsewhere. For the equation (10) was derived for nonrotating fields and masses and assumed constant Q, M, m, r_p.

Note that that the nonradial potential energy function $w_r(Q(\lambda))$ that is spent on work done along equipotential surfaces depends on density of matter Q and equipotential distance λ while being just determined by both the radial distance r (which determines the surface) and the perihelion radius r_p that determines the distance to the path. The two distances only parametrize the function $w_r(Q(\lambda))$.

The extraneous frequency decrease has nonradial character for it was acquired on equipotential parts of the rays' trajectory path (where the usual radial gravitational potential does not change), in addition to the (radial) gravitational frequency shifts that depend on changing radial potential (Czajko, 2000).

6. EXPERIMENTAL EVIDENCE FOR PRESENCE OF NONRADIAL EFFECTS OF GRAVITY

When written in the present paper's notation, the formula for the extraneous frequency decrease found in the experiments by Sadeh, Knowles, and Yaplee (1968) and Sadeh, Knowles, and Au (1968) had been given by the following equation proposed by Czajko (2000):

$$\frac{\Delta f}{f} = -\frac{KM\lambda}{Qr} \cong -\frac{KM\lambda}{Qr_p^2} \tag{11}$$

where the extra energy (corresponding to work done by the respective gravitational fields of the Sun and Earth on the ray from Taurus A and on the radio waves captured at a distance measured along practically equipotential surface of Earth) was $E' = h\Delta f' = mc^2$ where c is the speed of light in vacuum, h is Planck constant and $\Delta f = f - f'$ is the extraneous frequency decrease (Czajko, 2000). The apostrophe here means altered frequency/energy measured on Earth, not derivative. The similarity of the equations (10) and (11) is not incidental. It demonstrates two ways of deriving the same new law of nature.

For the sake of simplicity of calculations it was assumed that $r = r_p$ determined the equipotential surface, which was fairly close to our Sun's surface and very close to the surface of the Earth (Czajko, 2000). Sadeh proposed the coefficient K which was calculated from data obtained in their experiments. Despite all the simplifying assumptions, the difference between results of the two experiments was just $K_{Sun}/K_{Earth} \approx 0.4\%$ once respective densities of matter have explicitly been taken into account.

Since Sadeh considered only the usual radial impact of gravity, his proposed radial-only formula did not include density of matter and could not reconcile their experiments, but left discrepancy of 390% or 3.9 which is practically equal to the ratio $5.52/1.42 \cong 3.9$ of specific gravities of Earth and our Sun, respectively (Weast, 1970, p.f145). Density of the field's matter is indispensable for nonradial effects (Czajko, 2000).

These nonradial effects induced over 25% discrepancy observed in certain solar spectra (Czajko, 2000) and also retrodicted the (found in several observations by Merat in 1974 and Dyson in 1921) excess over the Einstein's "flagship" prediction of deflection of light, which originated from the general theory of relativity (GTR) (Czajko, 2000).

Recall that by Einstein's own admission the GTR has not been devised for any other than purely radial phenomena, because, as he wrote: some tangential deviations [from the usual radial gravity] would be too slight if measured on the Earth, which was very common assumption back then (Einstein, 1923, p.161). The theory of nonradial effects of the radial fields does not defy the GTR, but complements it for nonradial interactions (Czajko, 2000), which were ignored in gravitational theories devised prior to 2000 AD.

Dependency of the attractive pull of gravity on density of mass was also found by spacecrafts orbiting the Moon and that find was quite recently confirmed by very ingenious experiment (Melosh et al., 2013). But the impact of mass concentrations was considered only in terms of their radial effects.

The Sadeh experiments effectively confirmed thus the previous experiment-driven nonradial formula (11) (Czajko, 2000) and indirectly support also the generalized formula (10) for nonradial effects.

The equation (10) shows explicit dependence of work done on the distance to perihelion, which was not taken into account (Czajko, 2000) but was rendered there by a certain constant to be determined from experiments, which pertained to perihelion. In the sense the generalized equation (10) also retrodicts the (previously quite unanticipated) results of those Sadeh experiments by Sadeh, Knowles, and Yaplee (1968) and Sadeh, Knowles, and Au (1968), see Czajko

(2000). A very preliminary analysis of slowly varying density of matter (in nonrotating or perhaps slowly rotating massive bodies and/or their gravitational fields) has already been briefly discussed (Czajko, 2004a).

Although the angle α of visibility of the path corresponds to the angle θ for straightlinear paths, for fast rotating masses and/or deflected (hence curvilinear, in general) paths it may be necessary to split the spherical angle θ (or the corresponding to it spherical distance λ that is measured along equipotential surfaces surrounding the mass M which generates the locally dominant gravitational field) into tangential and binormal parts. The variables θ and λ could reveal dynamical features.

Since negative sign in the equation (10) means energy loss on equipotential surfaces (regardless of the direction of motion), topologically speaking the surfaces delimit the fields from outside and thus belong to the surrounding them space, not to the mass or the force field.

7. CONCLUSION

Work done by locally dominant radial/center-bound gravitational force fields and thus also the corresponding to it exchange of potential energy depends on the exposure angle/path, and the fields' density of matter when the work/energy exchange takes place on trajectory paths lying on equipotential surfaces surrounding the center of gravity of the field.

The double dependence on path/angle and average/constant density of matter of the mass source that generates the field is also codependent on (or codetermined by) two distances (to the path and to the path's perihelion) from the local center of gravity. Unlike density of mass, however, density of matter is inversely proportional to the force field.

The (new) synthetic mathematics deployed here demonstrated predictive capabilities surpassing former attempts at mathematical construction/invention of new laws of nature.

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