CSCanada
Studies in Mathematical Sciences
Vol. 6, No. 2, 2013, pp. [18–27]
DOI: 10.3968/j.sms.1923845220130602.1988

ISSN 1923-8444 [Print]
ISSN 1923-8452 [Online]
www.cscanada.net
www.cscanada.org

A Multivariate Linear Regression on the Effect of Foreign Trade on Foreign Exchange Rates of Naira Using Bootstrap Approach

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Received: January 3, 2013/ Accepted: March 7, 2013/ Published: May 31, 2013

Abstract: This paper fits a multivariate linear Regression model for the relationship between four response variables; foreign trades (cost of oil imported, cost of non-oil imported, cost of oil exported and cost of non-oil exported) and two predictor variables; foreign exchange rate of US Dollar and Pounds Sterling between 1960–2010. The null hypothesis that no linear relationship exists between foreign trades and foreign exchange rate were proposed. The model indicates an adequate relationship between the foreign trade and exchange rate. We observed from model 1 to model 4 of the multivariate linear regression that the multivariate bootstrap method when compared with the analytic method yielded a reduced standard error. This shows that multivariate bootstrap is good methods in controlling variation among observations. We further observed that the exchange rate of US Dollar and Pounds Sterling contributed 68% effect on the Nigerian foreign trades while 32% may be attributed to other unexplained factors. This indicates that multivariate bootstrap method offer considerable potentials in the estimation of multivariate linear regression parameters. Thus, the results reveal that bootstrap resampling allows empirical assessment of the analytical model and it is a good approximation to the analytical model.

Key words: Multivariate; Bootstrap; Foreign trade; Foreign exchange rate; Linear regression

Obiora-Ilouno H. O., & Mbegbu J. I. (2013). A Multivariate Linear Regression on the Effect of Foreign Trade on Foreign Exchange Rates of Naira Using Bootstrap Approach. *Studies in Mathematical Sciences, 6*(2), 18–27. Available from http://www.cscanada.net/

index.php/sms/article/view/j.sms.1923845220130602.1988 DOI: 10.3968/j.sms.192384522 0130602.1988

1. INTRODUCTION

The evolution of the foreign exchange in Nigeria was influenced by a number of factors such as the changing pattern of international trade, institutional changes in the economy and structural shifts in production. The increased export of crude oil in the early 1970's, following the sharp rise in its prices, enhanced official foreign exchange receipts. The market witness a boom during this period and the management of foreign exchange resources finds it imperative to ensure that shortage did not arise [7]. Adubi and Okunmadewa [1] carried out a study on price exchange role volatility and Nigeria's Agricultural trade flows. They argued that changes in income earning of export crop producer income as a result of either increase/decrease in international world price of the exports of devaluation of the currency and the subsequent increase in producer prices. Adubi and Okunmadewa [1] said that price/exchange rate changes may lead to major decline in future output if they are unpredictable and erratic.

Johnson and Wichern [4] define multivariate linear regression as modeling the relationship between m responses and a single set of predictor variables, where each response variable is assumed to follow its own linear regression model. Bootstrapping is a resampling technique in which sample of size n are obtained with replacement from the original sample of size n. The basic idea behind bootstrapping is to analyze the population by replacing the unknown distribution function F by the empirical distribution function $\hat{F}$ obtained from the sample (see [2,3,5]).

2. MATERIAL AND METHOD

General Form of Multivariate Linear Regression Model

According to Johnson and Wichern [4], multivariate linear regression model defines the relationship between m responses $Y_1, Y_2, ..., Y_m$ and a single set of r predictors, $Z_1, Z_2, ..., Z_r$.

$$\left.\begin{array}{l} Y_1 = \beta_{01} + \beta_{11}Z_1 + \beta_{21}Z_2 + \cdots + \beta_{r1}Z_r + \varepsilon_1 \\ Y_2 = \beta_{02} + \beta_{12}Z_1 + \beta_{22}Z_2 + \cdots + \beta_{r2}Z_r + \varepsilon_2 \\ \vdots \; \vdots \; \vdots \; \vdots \; \vdots \\ Y_m = \beta_{0m} + \beta_{1m}Z_1 + \beta_{2m}Z_2 + \cdots + \beta_{rm}Z_r + \varepsilon_r \end{array}\right\} \tag{2.1}$$

The expectation and variance of error term ε are

$$E(\varepsilon) = E\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_m \end{bmatrix}$$

and

$$\text{Var}(\varepsilon) = \sigma^2 \quad \text{respectively.}$$

Let $[Z_{j0}, Z_{j1}, ..., Z_{jr}]$d enote the values of the predictor variables for the j^{th} trial. Let $Y_j = [Y_{j1}, Y_{j2}, ..., Y_{jm}]'$ be the responses, let $[\varepsilon_{j1}, \varepsilon_{j2}, ..., \varepsilon_{jm}]$ be the errors for j^{th} trial. Thus we have $n \times (r+1)$ design matrix

$$Z_{n\times(r+1)} = \begin{bmatrix} Z_{10} & Z_{11} & \cdots & Z_{1r} \\ Z_{20} & Z_{21} & \cdots & Z_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{n0} & Z_{n1} & \cdots & Z_{nr} \end{bmatrix} \tag{2.2}$$

If we set

$$Y_{n\times m} = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1m} \\ y_{21} & y_{22} & \cdots & y_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ y_{n1} & y_{n2} & \cdots & y_{nm} \end{bmatrix} = \left[Y_{(1)}|Y_{(2)}|\cdots|Y_{(m)}\right] \tag{2.3}$$

$$\beta_{(r+1)\times m} = \begin{bmatrix} \beta_{01} & \beta_{02} & \cdots & \beta_{0m} \\ \beta_{11} & \beta_{12} & \cdots & \beta_{1m} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{r1} & \beta_{r2} & \cdots & \beta_{rm} \end{bmatrix} = \left[\beta_{(1)}|\beta_{(2)}|\cdots|\beta_{(m)}\right] \tag{2.4}$$

$$\varepsilon_{n\times m} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \cdots & \varepsilon_{1m} \\ \varepsilon_{21} & \varepsilon_{22} & \cdots & \varepsilon_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \varepsilon_{n1} & \varepsilon_{n2} & \cdots & \varepsilon_{nm} \end{bmatrix} = \left[\varepsilon_{(1)}|\varepsilon_{(2)}|\cdots|\varepsilon_{(m)}\right] = \begin{bmatrix} \varepsilon'_{(1)} \\ \varepsilon'_{(2)} \\ \vdots \\ \varepsilon'_{(m)} \end{bmatrix} \tag{2.5}$$

where β is the $((r+1)\times m)$ matrix of parameters. Y is the $(n \times m)$ matrix of the response variables. ε is the $(n\times m)$ matrix of the errors or the residuals. Then, the multivariate linear regression model is

$$Y = Z\beta + \varepsilon \tag{2.6}$$

with $E\left(\varepsilon_{(i)}\right) = 0$ and $\text{cov}\left(\varepsilon_{(i)}, \varepsilon_{(k)}\right) = \sigma_{ik}I, \quad i, k = 1, 2, \cdots m$. Also, the m observed responses on the j^{th} trial have covariance matrix

$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1m} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{m1} & \sigma_{m2} & \cdots & \sigma_{mm} \end{bmatrix}$$

$$\hat{\beta}_{(i)} = (Z'Z)^{-1}Z'Y_{(i)}$$

$$\hat{\beta} = \left[\hat{\beta}_{(1)}|\hat{\beta}_{(2)}|\cdots|\hat{\beta}_{(m)}\right] = (Z'Z)^{-1}Z'\left[Y_{(1)}|Y_{(2)}|\cdots|Y_{(m)}\right] = (Z'Z)^{-1}Z'Y \tag{2.7}$$

Now for any choice of parameter $\beta = \left[b_{(1)}|b_{(2)}|\cdots|b_{(m)}\right]$, the resulting matrix of errors is $Y - Z\beta$. The resulting error sum of squares and cross-product matrix is

$$(Y-Z\beta)'(Y-Z\beta) = \begin{bmatrix} (Y_{(1)} - Zb_{(1)})'(Y_{(1)} - Zb_{(1)}) \cdots (Y_{(1)} - Zb_{(1)})'(Y_{(1)} - Zb_{(1)}) \\ \vdots \quad \vdots \\ (Y_{(m)} - Zb_{(m)})'(Y_{(m)} - Zb_{(m)}) \cdots (Y_{(m)} - Zb_{(m)})'(Y_{(m)} - Zb_{(m)}) \end{bmatrix} \tag{2.8}$$

The selection $b_{(i)} = \hat{\beta}_{(i)}$ minimizes the i^{th} diagonal sum of squares $(Y_{(1)} - Zb_{(1)})'(Y_{(1)} - Zb_{(1)})$. Thus, $tr\left[(Y - Z\beta)'(Y - Z\beta)\right]$ is minimized by $\hat{\beta}$. Also, the $\mathrm{var}\left[(Y - Z\beta)'(Y - Z\beta)\right]$ is minimized by the least squares estimate $\hat{\beta}$. Using the least estimates $\hat{\beta}$, we can obtain the matrix of predicted values as,

$$\hat{Y} = Z\hat{\beta} = (Z'Z)^{-1}Z'Y \tag{2.9}$$

and the matrix of the residuals is (See Johnson and Wichern [4] pp. 314-316)

$$\hat{\varepsilon} = Y - \hat{Y} = \left[1 - Z(Z'Z)^{-1}Z'\right]Y \tag{2.10}$$

Adopting the least squares estimator $\hat{\beta} = [\hat{\beta}_{(1)}|\hat{\beta}_{(2)}|\cdots|\hat{\beta}_{(m)}]$, we obtain the multivariate multiple regression model

$$\underset{(n\times m)}{\hat{Y}} = \underset{(n\times(r+1))}{Z} \cdot \underset{((r+1)\times m)}{\hat{\beta}^{(b)}}$$

3. THE PROPOSED ALGORITHMS FOR ESTIMATING MULTIVARIATE LINEAR REGRESSION PARAMETERS

3.1. The Multivariate Bootstrap Algorithm on the Resampling Observations for Estimating the Multivariate Linear Regression Parameters

Let $W_i = (Y_{ji}, Z_{jr})'$ be the original sample of size n for the resampling, and assume that $W_i s$ are drawn independently and identically from a distribution of F. Let $Y_{ji} = (y_{j1}, y_{j2}, ..., y_{jm})'$ be the $(n \times m)$ matrix of the response variables, $Z_{jr} = (z_{j1}, z_{j2}, ..., z_{jk})'$ be the $(n\times m)$ is the matrix of dimension $n\times(r+1)$ for the predictor variables, where $j = 1, 2, \cdots, n; i = 1, 2, \cdots, m;$ and $r = 1, 2, \cdots, k$. Let the $(k \times 1) \times 1$ vector denote the $W_i = (Y_{ji}, Z_{jr})'$ values associated with i^{th} $(w_1, w_2, \cdots, w_n)$ observation sets.
Step 1:

Draw bootstrap sample $(w_1^{(b)}, w_2^{(b)}, \cdots, w_n^{(b)},)$ of size n with replacement from the observation giving n^{-1} probability of each Z_r value being sampled from the population and label the elements of each vector $W_i^{(b)} = \left(Y_{ji}^{(b)}, Z_{jr}^{(b)}\right)'$ where $j = 1, 2, \cdots, n; i = 1, 2, \cdots, m$ and $r = 1, 2, \cdots, k$. From the bootstrap Sample, form the vector for the response variable $Y_{ji}^{(b)} = \left(y_{j1}^{(b)}, y_{j2}^{(b)}, \cdots, y_{jm}^{(b)}\right)$ and the matrix of the predictor variables $Z_{jr}^{(b)} = \left(z_{j1}^{(b)}, z_{j2}^{(b)}, \cdots, z_{jk}^{(b)}\right)$.
Step 2:

Calculate the multivariate regression parameters from the bootstrap sample:

$$\hat{\beta} = \left[\hat{\beta}_{(1)}|\hat{\beta}_{(2)}|\cdots|\hat{\beta}_{(m)}\right] = (Z'Z)^{-1}Z'\left[Y_{(1)}|Y_{(2)}|\cdots|Y_{(m)}\right] = (Z'Z)^{-1}Z'Y$$

$\hat{B}_{(i)} = (Z'Z)^{-1}Z'Y_{(i)}$ using the multivariate regression method.
Step 3:

Repeat steps 1 and 2 for $v = 1, 2, \cdots, B$ say 1000 times, where B is the number of repetition.
Step 4:

Obtain the probability distributions $F\left(\hat{\beta}^{(b)}\right)$ of multivariate bootstrap estimates $\hat{\beta}^{(b1)}, \hat{\beta}^{(b2)}, \ldots, \hat{\beta}^{(bB)}$ and use $F\left(\hat{\beta}^{(b)}\right)$ to estimate the multivariate regression parameters, standard error and confidence interval. The multivariate bootstrap estimates are the means (see [6])

$$\bar{\beta}^{\left(multi_{(b)}\right)} = \frac{\sum_{v=1}^{B} \hat{\beta}^{(b_V)}}{B} \tag{3.1}$$

Thus, the multivariate bootstrap regression equation is

$$\underset{(n \times m)}{\hat{Y}} = \underset{(n \times (r+1))}{Z} \cdot \underset{((r+1) \times m)}{\hat{\beta}^{(b)}} \tag{3.2}$$

3.2. Data Presentation

The data obtained from Central Bank of Nigeria Bulletin 2010 edition on exchange of US Dollar and Pounds to Nigeria Naira currency and Nigeria foreign trade from 1960-2010 and shown in the Table 1.

Table 1
Exchange of US Dollar and Pounds to Nigeria Naira Currency and Nigeria Foreign Trade

Years	US Dollar	Pounds	Oil (import)	Non-oil (import)	Oil (export)	Non-oil (export)
1960	0.7143	2	26.952	404.83	8.816	330.612
1961	0.7143	2	31.668	413.37	23.09	324.166
1962	0.7143	2	32.966	373.468	34.42	302.652
1963	0.7143	2	37.258	377.916	40.354	338.306
1964	0.7143	2	46.398	461.028	64.112	365.19
1965	0.7143	2	47.876	502.186	136.194	400.58
1966	0.7143	2	22.042	490.666	183.94	384.228
1967	0.7143	2	23.228	417.872	144.772	340.866
1968	0.7143	2	39.642	345.76	73.998	348.12
1969	0.7143	2	42.766	454.616	261.93	374.062
1970	0.7143	1.7114	38.702	717.718	509.622	376.046
1971	0.6955	1.7156	50.4	1028.5	953	340.4
1972	0.6579	1.6289	45.2	944.9	1176.2	258
1973	0.6579	1.6289	41	1183.8	1893.5	384.9
1974	0.6299	1.4795	52.4	1684.9	5365.7	429.1
1975	0.6159	1.3618	118	3603.5	4563.1	362.4
1976	0.6265	1.1317	95	5053.5	6321.6	429.5
1977	0.6466	1.1671	102.2	6991.5	7072.8	557.9
1978	0.606	1.2238	110	8101.7	5401.6	662.8
1979	0.5957	1.2628	230	7242.5	10166.8	670
1980	0.5464	1.2647	227.4	8868.2	13632.3	554.4

To be continued

Continued

Years	US Dollar	Pounds	Oil (import)	Non-oil (import)	Oil (export)	Non-oil (export)
1981	0.61	1.2495	119.8	12719.8	10680.5	342.8
1982	0.6729	1.1734	225.5	10545	8003.2	203.2
1983	0.7241	1.1216	171.6	8732.1	7201.2	301.3
1984	0.7649	1.0765	282.4	6895.9	8840.6	247.4
1985	0.8938	1.1999	51.8	7010.8	11223.7	497.1
1986	2.0206	2.5554	913.9	5069.7	8368.5	552.1
1987	4.0179	6.5929	3170.1	14691.6	28208.6	2152
1988	4.5367	8.0895	3803.1	17642.6	28435.4	2757.4
1989	7.3916	12.0695	4671.6	26188.6	55016.8	2954.4
1990	8.0378	16.2419	6073.1	39644.8	106626.5	3259.6
1991	9.9095	17.4955	7772.2	81716	116858.1	4677.3
1992	17.2984	27.8684	19561.5	123589.7	201383.9	4227.8
1993	22.0511	33.2522	41136.1	124493.3	213778.8	4991.3
1994	21.8861	33.4252	42349.6	120439.2	200710.2	5349
1995	21.8861	34.524	155825.9	599301.8	927565.3	23096.1
1996	21.8861	34.7698	162178.7	400447.9	1286215.9	23327.5
1997	21.886	36.2166	166902.5	678814.1	1212499.4	29163.3
1998	81.0228	128.1561	175854.2	661564.5	717786.5	34070.2
1999	81.2528	126.4165	211661.8	650853.9	1169476.9	19492.9
2000	109.55	163.0323	220817.69	764204.7	1920900.4	24822.9
2001	112.4864	161.7534	237106.83	1121073.5	1839945.25	28008.6
2002	126.4	203.7442	361710	1150985.33	1649445.828	94731.85
2003	135.40	234.4	398922.31	1681312.96	2993109.95	94776.44
2004	132.67	249.9925	318114.72	1668930.55	4489472.19	113309.4
2005	130.4	221.2385	797298.94	2003557.39	7140578.92	105955.9
2006	128.27	249.3899	710683	2397836.32	7191085.64	133595
2007	117.968	234.0205	768226.84	3143725.79	8110500.38	199257.9
2008	119.7925	216.4808	1386729.93	380372.68	9913651.13	247839
2009	146	231.6438	1063544.18	4038990.2	8067233	289152.6
2010	148.455	228.6553	2073579.03	5931795.19	10639417.37	396377.2

Source: Central Bank of Nigeria.

4. RESULTS AND DISCUSSION

The data obtained from Central Bank of Nigeria Bulletin 2011 edition is on the exchange of US Dollar and Pounds to Nigeria Naira currency and Nigeria foreign trade (Oil Import and Export, Non-Oil Import and Export) from 1960-2010.

We intend to obtain the multivariate linear model that describes the relationship between foreign trade and foreign exchange rate of Naira per US Dollar and Pounds Sterling. Let

Y_1 = Oil import
Y_2 = Non - oil import
Y_3 = Oil export
Y_4 = Non - oil export
X_1 = Exchange rate of US Dollar

X_2 = Exchange rate of Pounds Sterling
Model 1: $Y_1 = \beta_{01} + \beta_{11}X_1 + \beta_{21}X_2$
Model 2: $Y_2 = \beta_{02} + \beta_{12}X_1 + \beta_{22}X_2$
Model 3: $Y_3 = \beta_{03} + \beta_{13}X_1 + \beta_{23}X_2$
Model 4: $Y_4 = \beta_{04} + \beta_{14}X_1 + \beta_{24}X_2$
We shall look at the adequacy of the models at 5% level of significance.

$$H_0: \text{ The models are not adequate } (\beta_{ij} = 0)$$
$$H_1: \text{ The models are adequate } (\beta_{ij} \neq 0)$$

The Analytic results of parameter estimation for model 1-4 are shown in Table 2.

Table 2
Analytic Estimate of Parameters of Multivariate Regression

	Model 1 $\hat{Y}_1$	**Model 2** $\hat{Y}_2$	**Model 3** $\hat{Y}_3$	**Model 4** $\hat{Y}_4$
Intercept	-24141.1	-69236.9	-162002.1	-5647.6
X_1	6692.8	12400.8	-45664.2	600.7
X_2	-386.7	4459.8	53673.1	384.2

For the models 1-4, the values of multiple R-square are 0.6257, 0.7126, 0.7535 and 0.6431 respectively. Adjusted R-square are 0.6101, 0.7006, 0.7432, and 0.6282 respectively at various P values of 5.736×10^{-11}, 1.0101×10^{-13}, 2.541×10^{-15} and 1.823×10^{-11} respectively. Averagely the R-square is 0.683725; Adjusted R-square is 0.670525 and P-value 2.777×10^{-13}. Since the P-value is less than 0.05, there is enough evidence to reject the null hypothesis and conclude that the models are adequate. We observed that the exchange rates of US Dollar and Pounds Sterling contributed 68% effect on the Nigerian Foreign Trades.

Furthermore, implementing the proposed multivariate bootstrap (see Appendix) to estimation of the parameters of models 1-4 yields Table 3.

Table 3
Estimation of Parameters of Models 1-4 Using Proposed Multivariate Bootstrap Algorithm

	Model 1 $\hat{Y}_1^{(b)}$	**Model 2** $\hat{Y}_2^{(b)}$	**Model 3** $\hat{Y}_3^{(b)}$	**Model 4** $\hat{Y}_4^{(b)}$
Intercept	-22876.8	-64926	-155566	-5285.8
X_1	6292.2	11777.5	-47387	517.99
X_2	-178.49	4631.7	54485.7	420.37

The Multivariate regression models obtained from Analytic method (see Table 2) are
Model 1: $Y_1 = -24141.1 + 6692.8X_1 - 386.7X_2$
Model 2: $Y_2 = -69236.9 + 12400.8X_1 + 4459.8X_2$
Model 3: $Y_3 = -162002.1 - 45664.2X_1 + 53673.1X_2$

Model 4: $Y_4 = -5647.6 + 600.7X_1 + 384.2X_2$

The multivariate regression models obtained from the proposed Multivariate Bootstrap Algorithm (see Table 3) are

Model 1: $\hat{Y}_1^{(b)} = -22038.3 + 5732.2X_1 + 130.998X_2$

Model 2: $\hat{Y}_2^{(b)} = -62246.3 + 9960.02X_1 + 5677.7X_2$

Model 3: $\hat{Y}_3^{(b)} = -151059 - 50960.96X_1 + 56405.65X_2$

Model 4: $\hat{Y}_4^{(b)} = -5129.05 + 392.63X_1 + 492.61X_2$

The Standard errors of parameters estimation for the models 1-4 are shown in Table 4.

Table 4
Standard Error of Parameter Estimates for Model 1-4 Using Multivariate Bootstrap Algorthms ($n = 51, B = 1000$)

	Model 1: $\hat{Y}_1$	**Model 1: $\hat{Y}_2$**	**Model 1: $\hat{Y}_3$**	**Model 1: $\hat{Y}_4$**
Analytic (Standard err) Intercept	5880.1	15958.3	33385.6	1174.7
X_1	802.78	2178.8	4558.2	160.33
X_2	471.41	1279.4	2676.6	94.18
Multivariate Bootstrap (Standard err) Intercept	420.72	1197.6	2271.2	87.11
X_1	410.71	1111.9	2116.99	83.01
X_2	232.93	624.82	1225.5	46.39

5. DISCUSSION AND CONCLUSION

Table 4 reveals great reduction in standard error for the multivariate Bootstrap and delete-5 algorithms when compared with the analytic method. This shows that both algorithms can be used in controlling variation among observations.

Also, we observed from model 1 to model 4 of the multivariate linear regression that the multivariate bootstrap method when compared with the analytic method yielded a reduced standard error. This shows that multivariate bootstrap is good methods in controlling variation among observations. We further observed that the exchange rate of US Dollar and Pounds Sterling contributed 68% effect on the Nigerian foreign trades while 32% may be attributed to other unexplained factors. This indicates that multivariate bootstrap method offer considerable potentials in the estimation of multivariate linear regression parameters.

Thus, the results reveal that bootstrap resampling allows empirical assessment of the analytical model. From the results obtained, we observe that the models obtain using bootstrap resampling approximates the analytical approach. This indicates that bootstrap methods offers considerable potentials for modeling in complex problems because it does not depend on the model assumptions. With the bootstrap results we confidently conclude that the model is adequate, and there is significant effect of foreign trade on foreign exchange rate of Naira.

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APPENDIX: IMPLEMENTATION OF MULTIVARIATE DELETE-d ALGORITHM FOR ESTIMATION OF THE PARAMETERS OF MULTIVARIATE LINEAR MODEL

```
#part 1: To read data
data=read.table( ”data(hap).txt”, header=T, sep=“ ”)
#Part 2
#Run this to get the original estimates
data=data.frame(data)
reg=lm(cbind(y1, y2, y3, y4) ~ x1 + x2, data=data)
reg
summary(reg)

#Delete d jacknife algorithm begins here
#d=no of rows to be deleted
#p=no of cols in the data
#data is the data matrix with the first p1 columns
#for the dept vars and the remaining p-p1 cols for the indpt vars
jack.a=function(data, p1, d)
{
p=ncol(data)
n=nrow(data)
u=combn(n,d) #Assign the matrix of all possible combinations to u
output=matrix(0,ncol=p1*(p-p1)+p1,nrow=ncol(u))
y=data[,1:p1] #the responses
x=data[,(p1+1):p] #the covariates
for (i in 1:(ncol(u)))
{
dd=c(u[,i])
yn=y[-dd,] #delete d rows of the independent var
xn=x[-dd,] #delete d rows of the dependent var
reg=lm(formula=yn ~ xn)
coef=coef(reg)
output[i,]=c(as.vector(coef))
}
output
}
#Use this part to run
data=data.matrix(data)
run=jack.a(data, 2, 5)
est = c(mean(run[,1], na.rm=TRUE), mean(run[,2], na.rm=TRUE), mean(run[,3],
na.rm=TRUE), mean(run[,4], na.rm=TRUE), mean(run[,5], na.rm=TRUE), mean(run[,6],
na.rm=TRUE), mean(run[,7], na.rm=TRUE), mean(run[,8], na.rm=TRUE), mean(run[,9],
na.rm=TRUE), mean(run[,10], na.rm=TRUE))
> est1=matrix(est,3,4)
> rownames(est1)=c(“Intercept”,“x1”,“x2”)
> colnames(est1)=c(“y1”,“y2”,“y3”,“y4”)
> est
```