# Analysis of Traffic Flow at Signalized Junctions in Uyo Metropolis 

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#### Abstract

Traffic flows at three signalized junctions in Uyo metropolis were considered and the data used for the analysis were collected during peak periods (morning and evening) for three consecutive days. The performance measures of congestion such as the average queue length, arrival and saturation flow rates as well as the average waiting time of vehicle per cycle have been calculated for a better understanding of the traffic situation in the city by motorists and proper vehicular logistics. With the random nature in which vehicles arrive and depart, we assumed a Poisson arrival process and deterministic (constant) service time with the service rate $\mu=1 /$ s which incorporate both the stochastic and deterministic components of delay estimation. Using the concept of the Canadian delay model in Hellinga and Liping (2001), we obtained the overall delay given an observation interval. An estimate of the mean overall delay which is the average time a vehicle could be delayed at any signalized junction in Uyo metropolis is approximately 63s.


Key words: Traffic lights; Queue length; Cycle length; Congestion; Phase; Performance measures

## 1. INTRODUCTION

Traffic congestion leads to delays, decreasing flow rate, higher fuel consumption and thus has negative environmental effect. It is caused by irregular occurrences such as traffic accidents, vehicle disablements, and so on. Traffic congestion and flows can be modeled in various ways under different assumptions. According to Woensel and Vandaele (2007), it can be subdivided into interrupted and uninterrupted flows. The interrupted flow is regulated by an external means such as traffic lights or traffic police, while the uninterrupted flow is defined as all flows regulated by vehiclevehicle interaction and interaction between vehicles traveling on a roadway.

For the purpose of this research, only the interrupted flow will be considered. The fixed-cycle traffic light control is a typical example of an interrupted traffic flow, where vehicles arrive at an intersection controlled by traffic light and the traffic light alternates between green and red periods of effective duration $\boldsymbol{g}$ and $\boldsymbol{r}$, and delayed vehicles are assumed to depart during the green periods at equal time intervals. Several assumptions have been made about the arrival and departure process of vehicles at traffic lights controlled junctions. Webster (1958) modeled vehicular traffic at signalized intersections assuming a Poisson arrival process and went on to derived performance characteristics such as the mean queue length and the vehicle total waiting time. He however presented the mean delay of a vehicle in closed form, an expression which was partially based on simulation. These assumptions were accepted and modified by several other authors; see for example, Miller (1968) and Rouphail, Tarko and Li (1996). Years later, Leeuwardeen (2000) agreed with Webster (1958) assumptions when working on the fixed-cycle traffic light queue. He assumed that vehicles arrive at the traffic light according to the Poisson arrival cases (like the compound Poisson and geometric cases) and made comparison to detect the more effective and efficient distribution. He made an assumption which he termed the fixed-cycle traffic light assumption for many cycles in which the queue clears before the green period terminates, all vehicles that arrive during the residual green period pass through the system and experienced no delay whatever.

Apart from the Poisson arrival cases, there are other assumptions such as the $D / D / 1, D / G / 1, M / G / 1$, and so on. Example of such cases is in Yusuf and Black (2000) who developed models governing traffic waiting time for time dependent vehicle arrival and departure rates at a busy road junction in central Bangkok, Thailand. The long suffering Bangkok commuters were subjected to what appears to be at times, arbitrary and irrational traffic control at road junctions. They considered the problem of controlling the traffic flow through optimization of the traffic light phases using two criteria. The first criterion for the optimization is the minimization of the total vehicle waiting time and second criterion is termed the "fairness criterion" which tries to prioritize equalization of the queue length at a junction rather than the minimization of the unimpeded total waiting time of all vehicles at a junction.

The "countdown time" is another aspect of traffic lights in which the time (in seconds) left for changes in either the red or green phases are displayed in decreasing order for necessary precaution to motorists. Ibrahim, Karim and Kidwal (2005) determined the ideal saturation flow at a countdown signalized junction under Malaysian road conditions. They measured the saturation flow by regressing the average flow values with lane widths to obtain a linear regression model:
$S=1020+265 w$, where $S$ is the Saturation flow rate of veh/hr and $w$ is the lane width in meters.

This work however, considers the fixed-cycle traffic light queue mostly used in major junctions in Uyo metropolis.

## 2. MODEL SPECIFICATION

According to Rouphail et al. (1996), the delay experienced by vehicles can be estimated given two components namely: the stochastic component and the deterministic component.

The stochastic component of delay estimation is founded on the steady-state probability of queuing theory which models traffic delay based on the statistical distribution of the arrival and departure process. Whereas, the deterministic component is formed on the fluid theory of traffic in which demand and service are treated as continuous variables described by flow rates which may vary over the time and space domain of the traffic system.

However, as traffic intensity increases, there is an increase likelihood of cycle failure, where, some cycles will begin to experience an overflow from previous cycles. The implication is that the stochastic component is specifically applied to undersaturated traffic situations, that is, for $x \leq 1$, while the time dependent process required to effectively discharge vehicles during over-saturated traffic situations can be explained by the deterministic component of delay. Hence, we model the process using the $(M / D / 1):(\infty / F C F S)$ queuing model with the following assumptions:
(1) The arrival of vehicles follows a Poisson distribution with arrival flow rate $(q)$, since vehicular arrival is random.
(2) The intersection has a fixed-cycle regulation.
(3) The interval between departures of vehicles is constant.
(4) There is only one server per road direction which occasionally takes a vacation to serve clients in another road direction.
(5) There is no limit in the service capacity.
(6) The service policy is non-gated constant time with clients served in a First-come-First-served regime.

However, the assumption of a single server is used to represent a single road direction, hence, for $\boldsymbol{n}$ road directions in a junction; we have $\boldsymbol{n}$ number of servers.

NOTATIONS
$c$ - Cycle length (Sec)
$g$ - Effective green time (Sec)
$r$ - Effective red time (Sec)
$A_{t}$ - Maximum queue length (number of arrivals during red)
$D_{t}$ - Maximum number of departure during green time, $g$
$q$ - Arrival flow rate of vehicle per second during red light
$q_{s}$ - Average arrival flow rate during the observation interval $\{0, T\}$
$S$ - Saturation flow rate of vehicle per second during green light
$S_{a}$ - Average Saturation flow rate given the observation interval $\{0, T\}$
$C_{a}$ - Capacity rate of vehicle per second
$Q_{0}$ - The overflow queue from previous cycle
$Y$ - Flow ratio
$X$ - Degree of saturation
$Y_{a}$ - Average flow ratio given $\{0, T\}$

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Q(0) - Vehicle queue length at the beginning of a cycle
Q(c) - Vehicle queue length at the end of a cycle
IR - Irrational Drivers
\DeltaD
AWT - Average waiting time of vehicle per cycle
D The overall delay given the observation period {0,T}
E(D) - Average overall delay
Var(D) - Variance of overall delay
```


## 3. METHODS OF ANALYSIS

We employed the "Approximate Expressions" by Miller (1963) for the identified $\{(M / D / 1):(\infty / F C F S)\}$ queuing model. Let

$$
\begin{equation*}
Q(c)=Q(0)+A_{t}-D_{t}+\Delta D_{t} \tag{3.1}
\end{equation*}
$$

where

$$
\Delta D_{t}=\left\{\begin{array}{lr}
D_{t}-Q(0)-A_{t}, & \text { if } Q(0)+A_{t}<D_{t}  \tag{3.2}\\
0, & \text { otherwise }
\end{array}\right.
$$

If the system is in equilibrium, then,

$$
\begin{equation*}
Q(0)=Q(c) \tag{3.3}
\end{equation*}
$$

Also, we have:

$$
\begin{align*}
& q=A_{t} / C  \tag{3.4}\\
& S=D_{t} / g  \tag{3.5}\\
& \mu=1 / S  \tag{3.6}\\
& Y=q / S  \tag{3.7}\\
& \lambda=g / c  \tag{3.8}\\
& \Rightarrow X=Y / \lambda \tag{3.9}
\end{align*}
$$

The queue is in equilibrium if

$$
\begin{equation*}
X<1 \tag{3.10}
\end{equation*}
$$

The delay (expected waiting time) of vehicles per cycle is given as:

$$
\begin{equation*}
A W T=\frac{(1-\lambda)}{2(1-Y)}[c(1-\lambda)]+\frac{Q_{0}}{q} \tag{3.11}
\end{equation*}
$$

### 3.1. The Overall Delay

Considering the cumulative arrival and departure of vehicles during the observation interval $\{0, T\}$, we define the total delay that vehicles experienced during this interval as the overall delay and express it as:

$$
\begin{equation*}
D=D_{1}+D_{2} \tag{3.12}
\end{equation*}
$$

Where $D_{1}$ is the portion of delay that will be incurred by a vehicle when the approach rate does not exceed signal capacity. It is the uniform delay.
$D_{2}$ is the portion of delay caused by the temporary overflows of queue resulting from the random nature of arrivals.

Using the Canadian delay estimation formula, Hellinga et al. (2001), we calculate the mean of the overall delay as follows:

$$
\begin{equation*}
E(D)=E\left(D_{1}\right)+E\left(D_{2}\right) \tag{3.13}
\end{equation*}
$$

where

$$
\begin{gather*}
E\left(D_{1}\right)=\frac{c(1-\lambda)^{2}}{2\left(1-\lambda x_{1}\right)}  \tag{3.14}\\
E\left(D_{2}\right)=0.25 c\left\{\left(X_{a}-1\right)+\sqrt{\left(X_{a}-1\right)^{2}+\frac{4 X_{a}}{C_{a}}}\right\} \tag{3.15}
\end{gather*}
$$

Therefore,

$$
\begin{equation*}
E(D)=\frac{c(1-\lambda)^{2}}{2\left(1-\lambda x_{1}\right)}+0.25 c\left\{\left(X_{a}-1\right)+\sqrt{\left(X_{a}-1\right)^{2}+\frac{4 X_{a}}{C_{a}}}\right\} \tag{3.16}
\end{equation*}
$$

The variance of the overall delay is given as;

$$
\begin{align*}
& V A R(D)=V A R\left(D_{1}\right)+V A R\left(D_{2}\right) \\
& =\frac{c(1-\lambda)^{3}\left(1+3 \lambda-4 \lambda X_{1}\right)}{12\left(1-\lambda X_{1}\right)^{2}}+\frac{X_{a}\left(4-X_{a}\right)}{12 c\left(1-X_{a}\right)^{2} C_{a}} \tag{3.17}
\end{align*}
$$

In order to compute the mean and variance of the overall delay, we compute

$$
\begin{gather*}
q_{a}=\frac{1}{n} \sum_{i=1}^{n} q_{i}  \tag{3.18}\\
S_{a}=\frac{1}{n} \sum_{i=1}^{n} S_{i}  \tag{3.19}\\
Y_{a}=q_{a} / S_{a}  \tag{3.20}\\
X_{a}=Y_{a} / \lambda  \tag{3.21}\\
C_{a}=\lambda S_{a} \tag{3.22}
\end{gather*}
$$

The estimate of the overall delay, for $N=24$ is

$$
\begin{equation*}
E(\hat{D})=\frac{1}{N} \sum_{i=1}^{24} E\left(D_{i}\right) \tag{3.23}
\end{equation*}
$$

### 3.2. The Data

Data for this study were collected at three signalized traffic light controlled junctions in Uyo metropolis in Akwa Ibom State, Nigeria for three different days and each day at two peak periods - morning and evening. Only 5 cycles were considered in each case and results obtained are displayed in Table 1 of Appendix A. It contains the effective green time, red time and cycle length for the three intersections under study. These values are constant in each case irrespective of period of the day.

## 4. ANALYSIS OF PERFORMANCE MEASURES

### 4.1. Arrival Flow Rate

We employ equation 3.4, given as $q=A_{t} / c$.
For day $1\left(D_{1}\right)$, cycle $1\left(C_{1}\right)$ and Road direction 1- morning session $\left(\mathrm{RdM}_{1}\right)$, we obtained the arrival flow rate, $q$, as:

$$
q=\frac{7 v e h}{135 s}=0.052 v e h / s=3.11 v \mathrm{eh} / \mathrm{min} \approx 4 v \mathrm{eh} / \mathrm{min}
$$

The arrival flow rate for $D_{1}, C_{2}, R d M_{1}, \ldots D_{3}, C_{5} R d M_{4}$ are calculated in similar manner and the results are presented in Table 2 of Appendix B.

Similarly, for day $1\left(D_{1}\right)$, cycle $1\left(C_{1}\right)$ and Road direction 1- evening session $\left(\mathrm{RdE}_{1}\right)$, we obtained the arrival flow rate, $q$, as

$$
q=\frac{16 v e h}{135 s}=0.119 v e h / s=7.11 v e h / \mathrm{min} \approx 8 v e h / \mathrm{min}
$$

The arrival flow rate for $D_{1}, C_{2}, R d E_{1}, \ldots D_{3}, C_{5} R d E_{4}$ are calculated in similar manner and result displayed in Table 3, see also, Appendix B.

The rate at which vehicles arrive during the red phase and those that arrive during green phase and experienced no delay is measured by the mean arrival rate or the arrival flow rate. From Tables 2 and 3 , the $A_{t}$ column contains the number of vehicles (queue length) during red period.

### 4.2. Saturation Flow Rate

Recall (3.5) that: $S=D_{t} / g$.
Then, for day $1\left(D_{1}\right)$, cycle $1\left(C_{1}\right)$ and Road Direction1 morning session $\left(\mathrm{RdM}_{1}\right)$, we calculate the saturation flow rate thus:

$$
S=\frac{8 v e h}{30 s}=0.267 \mathrm{veh} / \mathrm{s}=16.0 \mathrm{veh} / \mathrm{min} \approx 16 v e \mathrm{eh} / \mathrm{min} .
$$

Then saturation flow rate for $D_{1}, C_{2}, R d M_{1}, \ldots D_{3}, C_{5} R d M_{4}$ and $D_{1}, C_{1}, R d E_{1}$, $\ldots D_{3}, C_{5} R d E_{4}$ are calculated in similar manner and results contain in Tables 4 and 5 respectively in Appendix C.

The saturation (or departure) flow rate is the rate at which vehicle depart during green period per unit time at traffic lights controlled intersections. The interval between departures of vehicles during the green time is assumed to be at constant interval. The green time is in seconds per vehicle and an increase in this time will results in a likely increased in the number of vehicles that will depart.

### 4.3. The Service Rate ( $\mu$ )

From (3.6), the service rate of vehicle per time length of green duration is given as $\mu=\frac{1}{S}=\frac{g}{D_{t}}$, where $S$ is the corresponding Saturation flow rate.

Then, for $D_{1}, C_{1}, R d M_{1}$, we obtained $\mu$ as:

$$
\mu=\frac{1}{0.267}=\frac{30 \mathrm{~s}}{8 v e h}=3.7 \mathrm{~s} / \mathrm{veh}
$$

For $D_{1}, C_{1}, R d E_{1}$, we have:

$$
\mu=\frac{1}{0.4 v e h / s}=\frac{30 s}{12 v e h} \approx 2.50 \mathrm{~s} / \mathrm{veh}
$$

The service rate in seconds per vehicle is calculated in similar manner and results presented in Tables 6 and 7 for $D_{1}, C_{2}, R d M_{1}, \ldots D_{3}, C_{5} R d M_{4}$ and $D_{1}, C_{2}, R d E_{1}$, ... $D_{3}, C_{5} R d E_{4}$ respectively; see Appendix D.

This is the rate at which vehicles are served by the traffic light per unit time. The server (the green light) takes a vacation as soon as its time length expired, thus, serving clients in another road direction. Those vehicles who do not obey the traffic lights rule are not considered to be served by the server; hence, they are lost to the system. The service rate can be hampered by 2 factors namely:

1) The presence of long vehicle (e.g., trucks, Lorries, etc.) in the queue, and
2) The presence of a large crowd of motorcyclists in the queue.

### 4.4. The Traffic Flow Rate Ratio (Traffic Intensity)

Recall from (3.7), $Y=\frac{q}{S}$.
Then, for $D_{1}, C_{1}, R d M_{1}$, we obtained the flow ratio thus:

$$
\begin{aligned}
& q=\frac{7 v e h}{135 s}=0.052 v e h / s=3.111 \mathrm{veh} / \mathrm{min} \\
& S=\frac{8 v e h}{30 s}=0.267 v e h / s=16 v e h / \mathrm{min}
\end{aligned}
$$

Therefore,

$$
Y=\frac{3.111 v e h / \min }{16 v e h / \min }=0.19
$$

Similarly, the traffic flow ratio for $D_{1}, C_{2}, R d M_{1}, \ldots D_{3}, C_{5} R d M_{4}$ and $D_{1}, C_{1}$, $R d E_{1}, \ldots D_{3}, C_{5} R d E_{4}$ are obtained and presented in Table 8 of Appendix E.

The traffic flow ratio is otherwise called the traffic intensity in an ordinary queuing concept. It measures the congestion level of the queuing system. The closer the traffic intensity to zero, the efficient the queuing system is. However, as if it increases, it implies that some cycles have experienced an overflow queue which cannot be discharged from previous cycle as a result of the random nature of the arrival and departure process.

### 4.5. The Degree of Saturation

Using (3.9), we have: $X=\frac{Y}{\lambda}=\frac{C q}{S g}$.
Then, for $D_{1} C_{1} R d M_{1}$, we obtained the degree of saturation using $\lambda=\frac{g}{c}$ in Table 4.3 a and 4.3 b as:

$$
\lambda=\frac{g}{c}=\frac{30 s}{135 s}=0.22
$$

Therefore, $X=\frac{0.19}{0.22}=0.86$ for $D_{1} C_{1} R d M_{1}$.

The degree of saturation for $D_{1} C_{2} R d M_{1}, \ldots, D_{3} C_{5} R d M_{4}$ and $D_{1} C_{2} R d E_{1}, \ldots$, $D_{3} C_{5} R d E_{4}$ are calculated in similar manner and the results are displayed in Table 9 of Appendix F.

The degree of saturation measures the saturation level of vehicles given the arrival and departure rate with regards to the green cycle ratio. The queuing system is assumed to be at equilibrium if $X<1$, that is, the number of arrivals can be discharge in a single green period. At $X=1$, the numbers of arrivals can also be discharged but at a uniform rate, that is, no vehicle has passed using the residuals green time. At $X>1$, we have the system to be over-saturated as the likely presence of an overflow queue is certain, except for the irrational drivers' behaviour.

### 4.6. Average Waiting Time of Vehicles per Cycle

From (2.11), we obtained the average waiting time of vehicles per cycle for $D_{1} C_{1} R d M_{1}$ as:

$$
A W T=\frac{135(1-0.22)}{2(1-0.19)}[(1-0.22)+0]=50.7 s \approx 51 \mathrm{~s} / \mathrm{veh}
$$

Therefore, the average waiting time of vehicle per cycle for $D_{1} C_{2} R d M_{1}, \ldots$, $D_{3} C_{5} R d M_{4}$ and $D_{1} C_{5} R d E_{4}, \ldots, D_{1} C_{1} R d E_{1}$ are obtained in similar manner and results presented in Table 10 in Appendix G.

The waiting time of vehicles per cycle varies with the number of vehicles in queue and the cycle length. The presence of an overflow queue from previous cycles contribute to an increment in the waiting time of the overflow vehicles, hence, the presence of an additional delay which must be accounted for in the estimation of delay.

### 4.7. The Overall Delay

Using (3.18)-(3.22), we calculate the parameters $q_{a}, S_{a}, X_{a}$ and $C_{a}$ for $D_{1} C_{1} R d M_{1}$, $\ldots, D_{1} C_{5} R d M_{1}$ to be used in our estimation of the mean overall delay as follows:

$$
\begin{aligned}
q_{a}=\frac{1}{5} \sum_{i=1}^{5} q_{i} & =\frac{1}{5}\{0.052+0.052+0.052+0.044+0.037\} \\
& =0.0474 \mathrm{veh} / \mathrm{s} \approx 2.84 \mathrm{veh} / \mathrm{min} \\
S_{a}=\frac{1}{5} \sum_{i=1}^{5} S_{i} & =\frac{1}{5}\{0.267+0.233+0.367+0.300+0.233\} \\
& =0.28 v \mathrm{eh} / \mathrm{s} \approx 16.8 \mathrm{veh} / \mathrm{min} \\
Y_{a}=\frac{0.047}{0.28}=0.17 ; \quad X_{a}= & \frac{Y_{a}}{\lambda}=\frac{0.17}{0.22}=0.77 \text { and } C_{a}=S_{a} \lambda=0.28 \times 0.22=0.06
\end{aligned}
$$

Then, using (3.16), we then calculate the mean overall delay as:

$$
\begin{aligned}
E(D) & =\frac{135(1-0.22)^{2}}{2(1-0.17)}+0.25(135)\left[(-0.23)+\sqrt{(-0.23)^{2}+\frac{4(0.77)}{0.06(635)}}\right] \\
& =54.06 \approx 55 \mathrm{~s} / \mathrm{veh}
\end{aligned}
$$

Also, the variance of the overall delay is given using (3.17) as:

$$
\begin{aligned}
\operatorname{Var}(D) & =135 \frac{(0.78)^{3}[1+3(0.22)-4(0.22)]}{12(1-0.22)^{2}}+\frac{0.77(4-0.77)}{12 \times 135 \times 0.06^{2} \times 0.23} \\
& =14.89 \mathrm{~s} / \mathrm{veh} \approx 15 \mathrm{~s} / \text { veh }
\end{aligned}
$$

Similarly, the parameters $q_{a}, S_{a}, Y_{a}, X_{a}, C_{a}$, the mean and variance of the overall delay are calculated in the same manner and results contained in Table 11 in Appendix H .

### 4.8. Estimation of the Mean Delay of Vehicle

Considering the three days observations, we obtain the sample mean of the overall delay to estimate the expected time a vehicle could be delayed at any of the traffic light controlled junctions in Uyo metropolis. Thus; from (3.23), we have:

$$
E(\hat{D})=\frac{1}{N} \sum_{i=1}^{24} E\left(D_{i}\right)
$$

where $N=24$ is the observed sample size.
Hence,

$$
E(\hat{D})=\frac{1}{24}\{53.84+72.42+\ldots+62.35+57.90\}=62.69 \mathrm{~s} / v e h
$$

which is the mean delay time for vehicles in Uyo metropolis.

## 5. SUMMARY AND CONCLUSION

The signal control effect on both the arrival and departure process is at constant rate in all cycles for both the under-saturated and over-saturated traffic situations. This, however, brings about unused green periods in under-saturated situations resulting in an increased number of non-delayed vehicles or an overflow queue in over-saturated situations resulting in excessive delay for the affected motorists.

From Table 3.1, traffic tends to be more congested in the evening period of the day for day 1 and 111 than in the morning, while the saturation flow rate seems to be on the increase if there is an increment in the service time. That is, for a green time of 30 s and beyond, there is a likely increase of vehicles being served.

For the three intersections, we observed that unlike Oron road by Gibbs street junction and Abak road by Udobio street junction which has fixed time length for all the road directions, Ikot Ekpene road by Ikpa road junction varies in the time length for the four different road directions to accommodate the difference in the congestion level of these roads. These changes in time is also necessary especially for the Oron road by Gibbs street junction which often experienced high overflow queue length in the opposing lanes of Oron road when compared to the less congested Gibbs and Udo-Umana street even at peak periods of the day.

The effect of the irrational drivers' component is usually higher in the morning especially during rush hours. Their actions usually lead to unprecedented cases of accidents which cause traffic delay. This usually results in damages to vehicular
parts and at times loss of lives. The essence of traffic police with motorbikes to effect arrest of traffic offenders is strongly recommended. Finally, the obtained mean delay time of $63 \mathrm{~s} /$ veh for vehicles in the city is crucial for effective planning of vehicular logistics.

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## APPENDIX A

EFFECTIVE GREEN TIME, RED TIME AND CYCLE LENGTH FOR THE THREE INTERSECTIONS

Table 1
Fixed Cycle Periods at Traffic Junctions in Uyo

| Day | Road <br> direction | Description | $\boldsymbol{g}$ | $\boldsymbol{r}$ | $\boldsymbol{c}$ | $\boldsymbol{\lambda}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\operatorname{Rd} M_{1}$ | Ikpa road (morning) | 30 | 105 | 135 | 0.22 |
|  | $\operatorname{Rd} E_{1}$ | Ikpa road (evening) | 30 | 105 | 135 | 0.22 |
|  | $\operatorname{Rd} M_{2}$ | Ik. Ekpene Rd to Ibom plaza (morning) | 35 | 115 | 150 | 0.23 |
|  | $\operatorname{Rd} E_{2}$ | Ik. Ekpene Rd to Ibom plaza (evening) | 35 | 115 | 150 | 0.23 |
|  | $\operatorname{Rd} M_{3}$ | Ibiam street (morning) | 23 | 142 | 165 | 0.14 |
|  | $\operatorname{Rd} E_{3}$ | Ibiam street (evening) | 23 | 142 | 165 | 0.14 |
|  | $\operatorname{Rd} M_{4}$ | Ibom plaza to Ik. Ekpene Rd (morning) | 52 | 113 | 165 | 0.32 |
|  | $\operatorname{Rd} E_{4}$ | Ibom plaza to Ik. Ekpene Rd (evening) | 52 | 113 | 165 | 0.32 |
| 2 | $\operatorname{Rd} M_{1}$ | Oron Rd to Ibom plaza (morning) | 28 | 95 | 123 | 0.23 |
|  | $\operatorname{Rd} E_{1}$ | Oron Rd to Ibom plaza (evening) | 28 | 95 | 123 | 0.23 |
|  | $\operatorname{Rd} M_{2}$ | Gibbs street (morning) | 28 | 95 | 123 | 0.23 |
|  | $\operatorname{Rd} E_{2}$ | Gibbs street (evening) | 28 | 95 | 123 | 0.23 |
|  | $\operatorname{Rd} M_{3}$ | Ibom plaza to Oron Rd (morning) | 28 | 95 | 123 | 0.23 |
|  | $\operatorname{Rd} E_{3}$ | Ibom plaza to Oron Rd (evening) | 28 | 95 | 123 | 0.23 |
|  | $\operatorname{Rd} M_{4}$ | Udoumana street (morning) | 28 | 95 | 123 | 0.23 |
|  | $\operatorname{Rd} E_{4}$ | Udoumana street (evening) | 28 | 95 | 123 | 0.23 |
| 3 | $\operatorname{Rd} M_{1}$ | Abak Rd to Ibom plaza (morning) | 30 | 115 | 145 | 0.21 |
|  | $\operatorname{Rd} E_{1}$ | Abak Rd to Ibom plaza (evening) | 30 | 115 | 145 | 0.21 |
|  | $\operatorname{Rd} M_{2}$ | Udobio street (morning) | 30 | 115 | 145 | 0.21 |
|  | $\operatorname{Rd} E_{2}$ | Udobio street (evening) | 30 | 115 | 145 | 0.21 |
|  | $\operatorname{Rd} M_{3}$ | Ibom plaza to Abak Rd (morning) | 30 | 115 | 145 | 0.21 |
|  | $\operatorname{Rd} E_{3}$ | Ibom plaza to Abak Rd (evening) | 30 | 115 | 145 | 0.21 |
|  | $\operatorname{Rd} M_{4}$ | Udo-Edwok street (morning) | 30 | 115 | 145 | 0.21 |
|  | $\operatorname{Rd} E_{4}$ | Udo-Edwok street (evening) | 30 | 115 | 145 | 0.21 |

## APPENDIX B

## ARRIVAL FLOW RATE

Table 2
Arrival Flow Rate for Morning Session


Table 3
Arrival Flow Rate for Evening Session

| C | Day 1 |  |  |  |  |  | Day 11 |  |  |  |  |  |  |  |  | Day 111 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\underset{\sim}{7}$ |  |  |  | $\sum_{\substack{\pi \\ ~ N}}^{\text {N }}$ |  |  |  | ${\underset{\sim}{\sim}}_{\text {N }}^{N}$ |  | ${\underset{\sim}{\sim}}_{\substack{\infty}}^{\infty}$ |  | 涊 |  | $\underset{\sim}{\underset{\sim}{x}}$ |  | $\underset{\sim}{\sim}$ |  | $\underbrace{\infty}_{\underset{\sim}{\sim}}$ | $\stackrel{\sim}{*}$ | $\sum_{2}^{8}$ |
| 1 | 16 | 8 | 17 | 873 | 23 | 9 | 16 | 8 | 9 | 5 | 10 | 5 | 8 | 4 | 16 | 7 | 11 | 5 | 1 | 8 | 12 | 5 |
| 2 | 18 | 8 | 22 | $\begin{array}{lll}9 & 7\end{array}$ | 17 | 7 | 11 | 6 | 8 | 4 | 14 | 7 | 4 | 2 | 19 | 8 | 13 | 6 | 14 | 6 | 10 | 5 |
| 3 | 15 | 7 | 25 | 1183 | 13 | 5 | 8 | 4 | 7 | 4 | 11 | 6 | 6 | 3 | 13 | 6 | 9 |  | 12 | 5 | 6 | 3 |
| 4 | 17 | 8 | 20 | 8146 | 20 | 8 | 17 | 9 | 5 | 3 | 9 | 5 | 8 | 4 | 8 | 4 | 6 | 3 | 18 | 8 | 11 | 5 |
| 5 | 16 | 8 | 13 | 642 | 30 | 12 | 17 | 9 | 6 | 3 | 10 | 5 | 8 | 4 | 10 | 5 | 8 | 4 | 23 |  | 8 | 4 |

## APPENDIX C

SATURATION FLOW RATE

Table 4
Saturation Flow Rate for Morning Session

| $C$. | Day 1 <br> Day 11 <br> Day 111 |
| :---: | :---: |
| $C_{i}$ |  |
| 1 | 8168145141417143071514304911236121020 |
| 2 | 7141933181214173792081881812247141428816 |
| 3 | 11221221411911163561315331022153010201020816 |
| 4 | 91815264111518163525153351181771417321122 |
| 5 |  |

Table 5
Saturation Flow Rate for Evening Session


## APPENDIX D

SERVICE RATE

Table 6
Morning Service Rate

| $C_{i}$ | Day 1 |  |  |  |  |  |  |  | Day 11 |  |  |  |  |  |  |  |  | Day 111 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\underset{\substack{10}}{\substack{1 \\ \hline}}$ |  | N | $\underset{\pi}{*}$ | $\sum_{i}^{\infty}$ |  |  |  | $\underset{\substack{10}}{\substack{10}}$ |  | $\underset{\sim}{N}$ |  | $\sum_{\substack{n}}^{\infty}$ |  |  |  | - |  | $\underset{\text { N }}{\substack{N \\ ~ N}}$ | $\pi$ | $\sum_{\substack{\infty \\ \hline}}^{\infty}$ |  | ¢ |
| 1 | 8 | 3.75 | 5 | . 12 | 2 | 54.60 | 14 | 3.71 | 14 | 2.00 |  | 4.00 | 14 | 42.00 |  | 7.0 | 11 | 2.72 | 6 | 5.00 | 10 | 3.00 | 8 | 3.75 |
| 2 |  | 4.29 | 19 | 1.84 | 1 | 23.0 | 12 | 4.33 | 17 | 1.65 | 9 | 3.11 | 8 | 3.50 | 8 | 2.50 | 12 | 2.50 | 7 | 4.29 | 14 | 2.14 | 8 | 3.75 |
| 3 |  | 2.73 | 12 | 2.92 | 4 | 5.75 | 9 | 5.78 | 16 | 1.75 | 6 | 4.67 | 15 | 1.87 | 10 | 2.80 | 15 | 2.00 | 10 | 3.00 | 10 | 3.00 | 8 | 3.75 |
| 4 |  | 3.33 | 15 | 2.33 |  | 5.75 | 15 | 3.47 | 16 | 1.75 | 2 | 14.00 | 15 | 1.87 | 5 | 5.60 | 8 | 3.75 | 7 | 4.29 | 17 | 1.76 | 11 | 2.72 |
| 5 | 7 | 4.29 | 20 | 1.75 | 1 | 23.0 | 024 | 2.16 | 18 | 1.56 | 7 | 4.00 | 6 | 4.67 | 3 | 9.34 | 17 | 1.76 | 4 | 7.50 | 7 | 4.29 | 9 | 3.33 |

Table 7
Evening Service Rate

|  |  |  |  | ay 1 |  |  |  |  |  |  | ay 1 |  |  |  |  |  |  |  |  | ay 1 | 111 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $C_{i}$ <br> $\stackrel{\rightharpoonup}{0}$ | $\begin{aligned} & \sum_{i}^{-1} \\ & \text { En } \end{aligned}$ | $\stackrel{\sim}{*}$ | $\sum_{\substack{N}}^{N}$ | $\stackrel{\sum_{0}^{\infty}}{\infty}$ | ${ }^{*}$ | $\begin{aligned} & \sum_{i}^{H} \\ & \text { Hi } \end{aligned}$ | $\sim^{*}$ | $\begin{aligned} & \sum_{7}^{F} \\ & \end{aligned}$ | А |  | ${ }^{*}$ | $\sum_{\substack{\infty}}^{\infty}$ | © |  | $\sum_{i}^{\pi}$ | $\stackrel{ \pm}{*}$ |  | $\sim^{*}$ |  |  | $\sum_{\substack{\infty \\ \hline}}^{\infty}$ | $\stackrel{\sim}{*}$ | $\sum^{2}$ | - |

[^0]
## APPENDIX E

## TRAFFIC FLOW RATIO

Table 8
Traffic Flow Ratio－Morning and Evening

| C | Day 1 |  |  |  | Day 11 |  |  |  |  | Day 111 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & \text { E } \\ & \text { Zun } \end{aligned}$ | $\underset{\text { N }}{\substack{n \\ ~ N}}$ | $\sum_{\substack{\infty \\ \hline}}^{\infty}$ | $\sum_{\substack{\pi}}^{\substack{2}}$ |  | $\underset{\text { N }}{\substack{n \\ ~ N}}$ | $\sum_{\substack{n\\}}$ | $\begin{aligned} & \underbrace{\pi}_{n} \\ & \text { n} \end{aligned}$ |  | N | $\sum_{\substack{\infty \\ \hline}}$ | 䂞 |
| 1 | 0.19 | 0.10 | 0.14 | 0.25 | 0.28 | 0.20 | 0.20 | 0.23 | 0.19 | 0.14 | 0.19 | 0.21 |
| 2 | 0.22 | 0.22 | 0.14 | 0.24 | 0.25 | 0.20 | 0.17 | 0.20 | 0.21 | 0.12 | 0.19 | 0.13 |
| 3 | 0.14 | 0.21 | 0.14 | 0.14 | 0.30 | 0.23 | 0.24 | 0.23 | 0.17 | 0.19 | 0.19 | 0.21 |
| 4 | 0.15 | 0.22 | 0.14 | 0.25 | 0.29 | 0.23 | 0.20 | 0.18 | 0.21 | 0.09 | 0.17 | 0.19 |
| 5 | 0.16 | 0.22 | 0.14 | 0.25 | 0.21 | 0.16 | 0.11 | 0.22 | 0.27 | 0.05 | 0.21 | 0.21 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | NT N | 等 | 式 | 年 | N | 毕 | 式 | 式 | N్ |  | － |
| 1 | 0.30 | 0.26 | 0.14 | 0.28 | 0.26 | 0.20 | 0.21 | 0.20 | 0.19 | 0.19 | 0.21 | 0.19 |
| 2 | 0.33 | 0.32 | 0.14 | 0.25 | 0.19 | 0.23 | 0.29 | 0.13 | 0.21 | 0.21 | 0.18 | 0.16 |
| 3 | 0.21 | 0.29 | 0.22 | 0.26 | 0.17 | 0.18 | 0.21 | 0.23 | 0.18 | 0.18 | 0.14 | 0.11 |
| 4 | 0.32 | 0.29 | 0.28 | 0.26 | 0.23 | 0.14 | 0.20 | 0.20 | 0.12 | 0.12 | 0.22 | 0.23 |
| 5 | 0.22 | 22 | 0.14 | 0.33 | 0.26 | 0.23 | 0.23 | 0.16 | 0.16 | 0.18 | 0.24 | 0.14 |

## APPENDIX F

## DEGREE OF SATURATION

Table 9
Degree of Saturation－Morning and Evening

| $C_{i}$ | Day 1 |  |  |  | Day 11 |  |  |  |  | Day 111 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\underset{\text { I }}{\substack{7 \\ \hline}}$ | ${\underset{\sim}{Z}}_{N}^{N}$ | $\sum_{\substack{\infty}}^{\infty}$ |  | $\underset{\text { Zu}}{\substack{T}}$ | $\sum_{\substack{N}}^{N}$ | $\sum_{\substack{n\\}}^{\infty}$ |  |  |  | $\sum_{\substack{\infty \\ \hline}}^{\infty}$ | － |
| 1 | 0.86 | 0.63 | 1.00 | 0.79 | 0.21 | 0.86 | 0.86 | 1.00 | 0.91 | 0.67 | 0.90 | 1.00 |
| 2 | 1.00 | 0.95 | 1.00 | 0.75 | 0.12 | 0.89 | 0.75 | 0.88 | 1.00 | 0.57 | 0.93 | 0.63 |
| 3 | 0.64 | 0.92 | 1.00 | 0.44 | 1.31 | 1.00 | 1.07 | 1.00 | 0.80 | 0.90 | 0.80 | 1.00 |
| 4 | 0.67 | 0.93 | 1.00 | 0.80 | 1.25 | 1.00 | 0.87 | 0.80 | 1.00 | 0.43 | 0.94 | 0.91 |
| 5 | 0.71 | 1.10 | 1.00 | 0.79 | 1.11 | 0.71 | 0.50 | 1.00 | 1.29 | 0.24 | 1.00 | 1.00 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 式 |  | 10 | $\xrightarrow[y]{y}$ | 式 | $\begin{aligned} & \text { N } \\ & \text { N1 } \\ & \end{aligned}$ | 钲 | 式 | 式 | N | Nom | 式 |
| 1 | 1.33 | 1.13 | 1.00 | 0.85 | 1.14 | 0.90 | 0.91 | 0.89 | 0.94 | 0.86 | 1.00 | 0.92 |
| 2 | 1.50 | 1.38 | 1.00 | 0.81 | 0.85 | 1.00 | 0.26 | 0.57 | 1.00 | 1.23 | 0.88 | 0.77 |
| 3 | 0.94 | 1.25 | 1.60 | 0.81 | 0.73 | 0.78 | 0.91 | 1.00 | 0.87 | 0.77 | 0.67 | 0.55 |
| 4 | 1.42 | 1.25 | 2.00 | 0.80 | 1.00 | 0.63 | 0.87 | 0.89 | 0.57 | 0.55 | 1.06 | 1.10 |
| 5 | 1.50 | 0.93 | 1.00 | 1.00 | 1.13 | 1.00 | 1.00 | 0.73 | 0.77 | 0.82 | 1.15 | 0.67 |

## APPENDIX G

## AVERAGE WAITING TIME OF VEHICLES PER CYCLE

Table 10
Average Waiting Time of Vehicle per Cycle - Morning and Evening


## APPENDIX H

COMPUTATION OF PARAMETERS: MEAN AND VARIANCE OF OVERALL DELAY

Table 11
Computation of Parameters: Mean and Variance of Overall Delay

| $\frac{\text { Day }}{1}$ | R | $M_{i}$ | $\lambda$ | $\boldsymbol{q}_{\boldsymbol{a}}$ | $S_{a}$ | $C_{a}$ | $Y_{a}$ | $\boldsymbol{X}_{\boldsymbol{a}}$ | E( ${ }^{\text {d }}$ | ( ${ }^{\text {}}$ |  | AR(D) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Rd | $M_{1}$ | 0.22 | 0.047 | 0.280 | 0.060 | 0.17 | 0.77 | 49.48 | 4.36 | 53.84 | 14.89 |
|  |  | $E_{1}$ | 0.22 | 0.122 | 0.453 | 0.099 | 0.27 | 1.23 | 52.65 | 19.77 | 72.42 | 9.87 |
| Rd |  | $M_{2}$ | 0.23 | 0.093 | 0.423 | 0.097 | 0.22 | 0.96 | 57.01 | 7.25 | 64.26 | 115.11 |
|  |  | $E_{2}$ | 0.23 | 0.129 | 0.463 | 0.106 | 0.28 | 1.22 | 57.75 | 20.68 | 78.43 | 9.76 |
| Rd |  | $M_{3}$ | 0.14 | 0.018 | 0.130 | 0.018 | 0.14 | 1.00 | 70.95 | 21.45 | 92.40 | 10.17 |
|  |  | $E_{3}$ | 0.14 | 0.048 | 0.287 | 0.036 | 0.17 | 1.19 | 70.95 | 26.12 | 97.07 | 45.48 |
| Rd |  | $M_{4}$ | 0.32 | 0.067 | 0.285 | 0.091 | 0.24 | 0.73 | 49.54 | 2.65 | 52.19 | 8.35 |
|  |  | $E_{4}$ | 0.32 | 0.126 | 0.454 | 0.145 | 0.28 | 0.87 | 59.98 | 3.48 | 63.46 | 10.23 |
| 11 | Rd | $M_{1}$ | 0.23 | 0.158 | 0.578 | 0.133 | 0.27 | 1.19 | 47.36 | 15.29 | 62.65 | 8.92 |
|  |  | $E_{1}$ | 0.23 | 0.122 | 0.515 | 0.118 | 0.24 | 1.03 | 47.36 | 8.31 | 55.67 | 71.07 |
|  | Rd | $M_{2}$ | 0.23 | 0.044 | 0.221 | 0.050 | 0.20 | 0.86 | 45.45 | 6.84 | 52.29 | 43.42 |
|  |  | $E_{2}$ | 0.23 | 0.055 | 0.286 | 0.066 | 0.19 | 0.84 | 45.20 | 5.20 | 50.40 | 22.21 |
|  | Rd | $M_{3}$ | 0.23 | 0.081 | 0.414 | 0.095 | 0.20 | 0.85 | 45.32 | 4.12 | 49.44 | 15.01 |
|  |  | $E_{3}$ | 0.23 | 0.088 | 0.393 | 0.090 | 0.22 | 0.97 | 46.93 | 7.27 | 54.20 | 279.23 |
|  | Rd | $M_{4}$ | 0.23 | 0.046 | 0.214 | 0.049 | 0.21 | 0.93 | 46.39 | 8.86 | 55.25 | 170.50 |
|  |  | $E_{4}$ | 0.23 | 0.055 | 0.300 | 0.069 | 0.18 | 0.80 | 44.69 | 4.30 | 48.99 | 15.19 |
| 111 | Rd | $M_{1}$ | 0.21 | 0.88 | 0.420 | 0.088 | 0.21 | 1.00 | 57.28 | 9.08 | 66.36 | 7.54 |
|  |  | $E_{1}$ | 0.21 | 0.091 | 0.520 | 0.109 | 0.18 | 1.83 | 69.37 | 3.49 | 72.86 | 11.94 |
|  | Rd | $M_{2}$ | 0.21 | 0.031 | 0.226 | 0.048 | 0.14 | 0.65 | 52.40 | 4.37 | 56.77 | 11.97 |
|  |  | $E_{2}$ | 0.21 | 0.064 | 0.347 | 0.073 | 0.18 | 0.88 | 55.50 | 5.96 | 61.46 | 28.10 |
|  | Rd | $M_{3}$ | 0.21 | 0.073 | 0.387 | 0.081 | 0.19 | 0.90 | 55.79 | 6.05 | 61.84 | 31.95 |
|  |  | $E_{3}$ | 0.21 | 0.116 | 0.587 | 0.123 | 0.20 | 0.94 | 56.38 | 5.58 | 61.96 | 37.89 |
|  | Rd | $M_{4}$ | 0.21 | 0.055 | 0.294 | 0.062 | 0.19 | 0.89 | 55.65 | 6.70 | 62.35 | 41.74 |
|  |  | $E_{4}$ | 0.21 | 0.065 | 0.393 | 0.083 | 0.17 | 0.79 | 54.25 | 3.65 | 57.9 | 12.34 |


[^0]:    1122.50152 .3373 .29262 .00142 .00102 .80112 .5593 .11171 .76122 .50171 .76132 .31 $2122.50162 .1973 .29212 .48132 .15744 .00112 .5574 .00191 .5810 \quad 161.88132 .31$ 3161.88201 .4754 .60163 .25112 .55193 .12122 .3364 .67152 .00112 .73181 .67112 .72 4122.50162 .1973 .29252 .08171 .62783 .50112 .5593 .11142 .14103 .00171 .76103 .00 5162.50142 .5045 .75301 .73151 .65644 .67102 .80112 .55132 .31193 .33201 .50122 .50

