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New Exact Solution for the (2+1)-Dimensional Dispersive Long Wave

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Abstract

First Integral method obtains some exact solution of non-integrable equations as well as integrable ones. This article is concerned with First Integral method for solving the solution of dispersive long wave system. It is worth mentioning that this method is based on the theory of commutative algebra in which division theorem is of concern. To recapitulate, this investigation has resulted in two exact soliton solutions of the given system. In addition, some figures of partial solutions are provided for direct-viewing analysis. The method can also be extended to other types of nonlinear evolution equations in mathematical physics.

Key words

First Integral method; Exact solution; Dispersive long wave (2+1)-dimensional

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INTRODUCTION

In various fields of science and engineering, many problems can be described by non-linear partial differential (PDEs). The investigation of exact solutions to nonlinear evolution has become an interesting subject in nonlinear science field. To find some exact soliton solutions in high dimensions ((2+1)- and (3+1)-dimensions) is much more difficult than in (1+1)-dimensions.

Moreover, since the time when the soliton concept was first introduced by Zabusky in 1965[1], the study of the solutions of Partial Differential Equations (PDEs) has enjoyed an intense period of activity over the last forty years from both theoretical and numerical points of view. Additionally, nonlinear evolution equations have been the subject of study in various branches of mathematical-physical sciences such as physics, biology, chemistry, plasma, optical fibers and computer technology.

In recent years, other methods have been developed, such as the Backlund transformation method[2], Darboux transformation[3], tanh method[4],[5], extended tanh function method[6], modified extended tanh function method[7], the generalized hyperbolic function[8], the variable separation method[9] and First Integral method was first proposed by Feng in 2002[10], recently this powerful method is widely used by many researchers for example[11],[12]. Abbasbandy and Shirzadi anticipated the first integral method to solve modified Benjamin-Bona-Mahoney equation[13]. The aim of this paper is to find exact soliton solutions for the dispersive long wave (2+1)-dimensional system[14] by the first integral method.

1. FIRST INTEGRAL METHOD

Consider a general nonlinear partial differential equation in the form:

$$F\left(u, u_t, u_x, u_{tt}, u_{xx}, u_{tx}, \ldots\right) = 0, \tag{1}$$

Where $u = u(x, t)$ is the solution of nonlinear PDE equation (1). Furthermore, the transformations which are used are as follows:

$$u\left(x, t\right) = U\left(\xi\right), \quad \xi = k_1(x + k_2 y - k_3 t). \tag{2}$$

Where c is constant. Using the chain rule, it can be found that

$$\frac{\partial}{\partial t}\left(.\right) = -k_1 k_3 \frac{\partial}{\partial \xi}\left(.\right), \quad \frac{\partial}{\partial x}\left(.\right) = k_1 \frac{\partial}{\partial \xi}\left(.\right), \quad \frac{\partial^2}{\partial x^2}\left(.\right) = k_1^2 \frac{\partial^2}{\partial \xi^2}\left(.\right), \ldots. \tag{3}$$

At present, equation (3) is employed to change the nonlinear PDE equation (1) to nonlinear ordinary differential equation

$$G(U(\xi), U_{\xi}(\xi), U_{\xi\xi}(\xi), \ldots) = 0 \tag{4}$$

Next, a new independent variable is introduced as:

$$X\left(\xi\right) = u\left(\xi\right), \quad Y = \frac{\partial u(\xi)}{\partial \xi}. \tag{5}$$

This yields a system of nonlinear ODEs

$$\begin{aligned} X_{\xi}\left(\xi\right) &= Y\left(\xi\right), \\ Y_{\xi}\left(\xi\right) &= F_1\left(X\left(\xi\right), Y\left(\xi\right)\right). \end{aligned} \tag{6}$$

If it is revealed that the integrals to equation (6) are under the same conditions of the qualitative theory of ordinary differential equation[15], then general solutions to (6) can be solved directly. However, it is generally so difficult for us to realize this even for one first integral, because for a given plane autonomous system, there is no systematic theory that can tell us how to find its first integrals, nor is there a logical way for telling us what these first integrals are. Thus, Division Theorem is used to obtain one first integral of (6) equation. Now, let us recall the Division Theorem:

Division Theorem:

Suppose that $P(w, z)$ and $q(w, z)$ are polynomials in $C[w, z]$ and $P(w, z)$ is irreducible to $C[w, z]$. If $q(w, z)$ vanishes through all zero points of $P(w, z)$, then there exists a polynomial $G(w, z)$ in $C(w, z)$ such that

$$q\left(w, z\right) = P(w, z)G(w, z)$$

See [16].

2. (2+1)-DIMENSIONAL DISPERSIVE LONG WAVE

In this section, it is aimed to discuss the dispersive long wave (2+1)-dimensional system, written in the form of the following equations:

$$\begin{aligned} u_{ty} + v_{xx} + \tfrac{1}{2}\left(u^2\right)_{xy} &= 0, \\ v_t + \left(uv + u + u_{xy}\right)_x &= 0. \end{aligned} \tag{7}$$

The celebrated (2+1)-dimensional dispersive long wave equation are firstly obtained by Boiti et al.[17] and has been found in some studies conducted by Ablowitz and Clarkson[18]. Furthermore, extended homogeneous method was used by Zhang Jie-Feng in 2002 to investigate dispersive long wave (2+1)- dimensional

system[19]. It is necessary to state that equation (7) plays an important role in nonlinear physics. That is to say, some special similarity solutions are also given in[20].
By using the transformation

$$u(x,y,t) = U(\xi), \quad v(x,y,t) = V(\xi), \quad \xi = k(x + ly - \lambda t).$$

equation(7) changes into:

$$\begin{gathered} -l\lambda U_{\xi\xi}(\xi) + kV_{\xi\xi}(\xi) + \tfrac{l}{2}\left(U^2(\xi)\right)_{\xi\xi} = 0, \\ -\lambda V_{\xi}(\xi) + \left(U(\xi)V(\xi) + U(\xi) + klU_{\xi\xi}(\xi)\right)_{\xi} = 0. \end{gathered} \tag{8}$$

Where by two integrating the first equation of the Eq.(8), respect to ξ, it can be found that

$$V(\xi) = \frac{l\lambda}{k}U(\xi) - \frac{l}{2k}U^2(\xi) + d_1 \tag{9}$$

Whit integrating the second equation of the Eq.(8), respect to ξ, it can be obtained that

$$-\lambda V(\xi) + (U(\xi)V(\xi) + U(\xi) + klU_{\xi\xi}(\xi)) = d_2. \tag{10}$$

Substituting (9) whit Eq.(10), the following equation will be achieved:

$$U_{\xi\xi} = \frac{d_2 + \lambda d_1}{kl} + \left(\frac{\lambda^2}{k^2} - \frac{d_1}{kl}\right)U(\xi) + \frac{3\lambda}{2k^2}U^2(\xi) + \frac{1}{2k^2}U^3(\xi), \tag{11}$$

Where d_1 and d_2 are two integration constants. According to the first integral method, by using (5) and (6), it will be determined that

$$\dot{X}(\xi) = Y(\xi), \tag{12}$$

$$\dot{Y}(\xi) = \frac{d_2 + \lambda d_1}{kl} + \left(\frac{\lambda^2}{k^2} - \frac{d_1}{kl}\right)X(\xi) + \frac{3\lambda}{2k^2}X^2(\xi) + \frac{1}{2k^2}X^3(\xi). \tag{13}$$

We suppose that $X(\xi)$ and $Y(\xi)$ are nontrivial solutions of (12) and (13) and $q[X,Y] = \sum_{i=0}^{m} a_i(X)Y^i = 0$ is an irreducible polynomial in the complex domain $C[X,Y]$ such that

$$q[X(\xi), Y(\xi)] = \sum_{i=0}^{m} a_i(X)Y^i = 0, \tag{14}$$

Where $a_i(X)(i = 0, \ldots, m)$ are polynomials of X and $a_m(X) \neq 0$. Equation (14) is called first integral to (12) and (13). Due to Division Theorem, there exists a polynomial $g(X)+h(X)Y$ in the complex domain $C[X,Y]$, such that

$$\frac{dq}{d\xi} = \frac{dq}{dX}\frac{dX}{d\xi} + \frac{dq}{dY}\frac{dY}{d\xi} = (g(X) + h(X)Y)\sum_{i=0}^{m} a_i(X)Y^i \tag{15}$$

In this example, By assuming that $m = 1$ in equation (14), and by equating the coefficients of $Y^i(i = 2,1,0)$ on both sides of equation (15), these will be:

$$\dot{a}_1(X) = a_1(X)h(X), \tag{16}$$

$$\dot{a}_0(X) = a_1(X)g(X) + a_0(X)h(X), \tag{17}$$

$$a_1(X)\left[\frac{d_2 + \lambda d_1}{kl} + \left(\frac{\lambda^2}{k^2} - \frac{d_1}{kl}\right)X(\xi) + \frac{3\lambda}{2k^2}X^2(\xi) + \frac{1}{2k^2}X^3(\xi)\right] = a_0(X)g(X). \tag{18}$$

Since $a_1(X)$ is a polynomial of X, then from (16), it may be deduced that $a_1(X)$ is a constant and $h(X) = 0$, we take $a_1(X) = 1$. Balancing the degrees of $g(X), a_1(X)$ and $a_0(X)$, it is concluded that $deg\ g(X) = 1$, only. Suppose that $g(X) = B_0 + A_1X, (A_1 \neq 0)$ then we will find that

$$a_0(X) = \frac{1}{2}A_1X^2 + B_0X + A_0 \tag{19}$$

Where A_0 is an arbitrary integration constant. Substituting $a_0(X), a_1(X)$ and $g(X)$ in equation (18) and setting all the coefficients of powers X to be zero, then a system of nonlinear algebraic equations can be resulted in. Having solved the given equation, the following solutions will be attained:

$$A_1 = \pm\frac{1}{\sqrt{k}}, \quad B_0 = \pm\frac{\lambda}{\sqrt{k}}, \quad A_0 = \mp\frac{d_1+1}{l}\sqrt{k}, \tag{20}$$

Using the conditions (20) in equation (14), it can be searched out that

$$Y(\xi) = -\left(\frac{1}{2\sqrt{k}}X^2(\xi) + \frac{\lambda}{\sqrt{k}}X(\xi) - \frac{d_1+1}{l}\sqrt{k}\right) \tag{21}$$

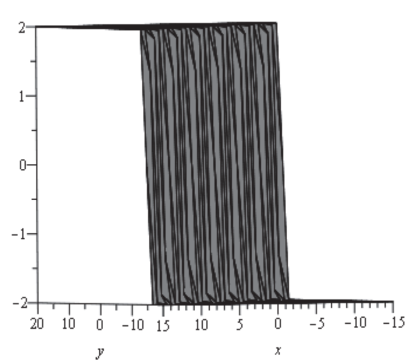
Expression (23) is the first integral of (12). Combining equation (23) with equation (12) we find the exact solution to equation (11) will be found as follows:

$$\begin{gathered} U_1(\xi) = -\lambda + \tfrac{1}{l}\sqrt{2lk(d_1+1)+\lambda^2l^2}\,tanh(\tfrac{1}{2l\sqrt{k}}\sqrt{2lk(d_1+1)+\lambda^2l^2}(\xi+\xi_0)), \\ V_1(\xi) = d_1 + \tfrac{l\lambda}{k}U_1(\xi) - \tfrac{l}{2k}U_1^2(\xi). \end{gathered} \tag{22}$$

Where ξ_0 is an arbitrary integration constant. Then, the exact soliton solution of dispersive long wave (2+1)-dimensional system(7) can be written as:

$$\begin{gathered} u_1(x,y,t) = -\lambda + \tfrac{1}{l}\sqrt{2lk(d_1+1)+\lambda^2l^2}\,tanh\left(\tfrac{1}{2l\sqrt{k}}\sqrt{2lk(d_1+1)+\lambda^2l^2}\,(k(x+ly-\lambda t)+\xi_0)\right), \\ v_1(x,y,t) = d_1 + \tfrac{l\lambda}{k}u_1(\xi) - \tfrac{l}{2k}u_1^2(\xi). \end{gathered} \tag{23}$$

For direct-viewing analysis, we provide the figures of $u_1(x,t)$ and $v_1(x,t)$, where we choose $d_1 = \xi_0 = \lambda = 0, l =$ and $k = 2$.



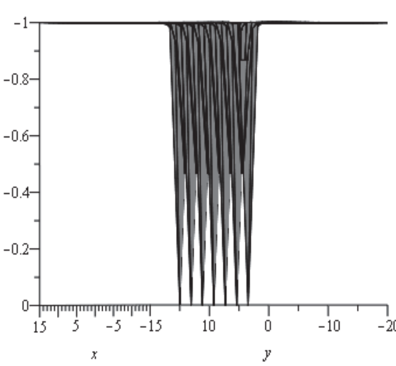


Figure 1 (a) Graphic of the soliton solution $u_1(x,y,t)$ **(b) Graphic of the soliton solution** $v_1(x,y,t)$

In the same way, in the case of (20), it can be acquired from equation (14) that

$$Y(\xi) = -\left(\frac{-1}{2\sqrt{k}}X^2(\xi) - \frac{\lambda}{\sqrt{k}}X(\xi) + \frac{d_1+1}{l}\sqrt{k}\right) \tag{24}$$

Expression (26) is the first integral of (12). Combining equation (26) with equation (12) we find the exact solution to equation (11) will be attained:

$$\begin{gathered} U_2(\xi) = -\lambda - \frac{1}{l}\sqrt{2lk(d_1+1)+\lambda^2 l^2}\,tanh\left(\frac{1}{2l\sqrt{k}}\sqrt{2lk(d_1+1)+\lambda^2 l^2}\,(\xi+\xi_0)\right), \\ V_2(\xi) = d_1 + \frac{l\lambda}{k}U_2(\xi) - \frac{l}{2k}U_2^2(\xi)\,. \end{gathered} \tag{25}$$

Where ξ_0 is an arbitrary integration constant. Then, the exact soliton solution of dispersive long wave (2+1)-dimensional system(7) will be achieved:

$$\begin{gathered} u_2(x,y,t) = -\lambda - \frac{1}{l}\sqrt{2lk(d_1+1)+\lambda^2 l^2}\,tanh\left(\frac{1}{2l\sqrt{k}}\sqrt{2lk(d_1+1)+\lambda^2 l^2}\,(k(x+ly-\lambda t)+\xi_0)\right), \\ v_2(x,y,t) = d_1 + \frac{l\lambda}{k}u_2(\xi) - \frac{l}{2k}u_2^2(\xi)\,. \end{gathered} \tag{26}$$

As a final notion, these solutions are considered as new exact soliton solutions for dispersive long wave (2+1) dimensional system.

CONCLUSION

In this study, First Integral method was described to find exact solutions of the dispersive long wave (2+1)-dimensional system. Consequently, two exact soliton solutions were obtained to the dispersive long wave (2+1)-dimensional system. In spite of the fact that these new soliton solutions may be important for physical problems, this study also suggests that one may find different solutions by choosing different methods. Therefore, this method can be utilized to solve many systems of nonlinear partial differential equation arising in the theory of soliton and other related areas of research.

REFERENCES

[1] Zabusky, N J. & Kruskal, MD. (1965). *Phys. Rev. let.* 15.

[2] Miura, M R. (1978). *Backlund Transformation*. Verlag, Berlin: Springer.

[3] Matveev, VB. & Salle, MA. (1991). *Darboux Transformations and Solitons*. Berlin, Heideelberg: Springer.

[4] Parkes, EJ. & Duffy, BR. (1998). An Automated Tanh-Function Method for Finding Solitary Wave Solution to Nonlinear Evolution Equations. *Comput. Phys. Commun, 98*, 288-300.

[5] Evans, DJ. & Raslan, KR. (2005). The Tanh Function Method for Solving Some Important Non-Linear Partial Differential Equation. *Int. J. comput. Math., 82*, 897-905.

[6] Fan, E. (2000). Extended Tanh-Function Method and Its Applications to Nonlinear Equations. *Phys. Lett. A., 277*, 212-218.

[7] Elwakil, SA., EL-Labany, SK., Zahran, SK. & Sabry, MA. (2002). Modified Extended Tanh-Function Method for Solving Nonlinear Partial Differential Equations. *Phys. Lett. 299*, 179-188.

[8] Gao, Y. T. & Tian, B. (2001). Generalized Hyperbolic-Function Method with Computerized Symbolic Computation to Construct the Solitonic Solutions to Nonlinear Equations of Mathematical Physics. *Comput. Phys. Commun., 133*, 158-164.

[9] Tang, XY. & Lou, SY. (2002). Localized Excitations in (2+1)-dimensional Systems. *Phys. Rev. E., 66*, 46601-46617.

[10] Feng, ZS. (2002). The First Integral Method to Study the Burgers-Korteweg-de Vries Equation. *Phys, Lett. A. Math. Gen., 35*, 343-349.

[11] Feng, ZS., & Wang, XH. (2003). The First Integral Method to the Two-Dimensional Burgers-Korteweg-de Vries equation. *Phys, Lett. A., 308*, 173-178.

[12] Yun-Quan, K., & Yu Jun (2004). The First Integral Method to Study a Class of Reaction-Diffusion Equations. Commun. *Theor. Phys.*, *43*, 597-600.

[13] Abbasbandy, S., & Shirzadi A. (2010). The First Integral Method for Modified Benjamin- Bona-Mahony Equation. *Commun Nonlinear Sci Numer Simulat.*, *15*, 1759-1764.

[14] Zhang, Z., & Bi, Q. (2008). Solitary Waves for a Nonlinear Dispersive Long Wave Equation. *Acta Mech Sin.*, *24*, 455-462.

[15] Ding, TR., & Li, CZ. (1996). *Ordinary Differential Equations*. Peking University Press.

[16] Bourbaki, N. (1971). *Commutative Algebra*. Paris: Addison-Wesly.

[17] Boiti, M., epm JJP., & Pempinelli, F. (1987). *Inverse Problem*, *3*, 371.

[18] Ablowitz, MJ., & Clarkson, PA. (1991). Solitons Nonlinear Evolution Equations and Inverse Scattering. *London Mathematical Society Lecture Note Series*, *149*.

[19] Zhang, Jie-feng (2002). Exotic Localized Coherent Structures of the (2+1)-Dimensional Dispersive Long-Wave Equation. *IAPJ. 37*(3), 277-282.

[20] Paquin, G., & Winternitz, P. (1990). *Physica*, *46*, 122.