CSCanada

Progress in Applied Mathematics
Vol. 6, No. 1, 2013, pp. [30–39]
DOI: 10.3968/j.pam.1925252820130601.5231

ISSN 1925-251X [Print]
ISSN 1925-2528 [Online]
www.cscanada.net
www.cscanada.org

Starting Order Seven Method Accurately for the Solution of First Initial Value Problems of First Order Ordinary Differential Equations

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Received: April 23, 2013/ Accepted: June 11, 2013/ Published: July 31, 2013

Abstract: In this paper, we developed an order seven linear multistep method, which is implemented in predictor corrector-method. The corrector is developed by method of collocation and interpolation of power series, approximate solutions at some selected grid points, to give a continuous linear multistep method, which is evaluated at some selected grid points to give a discrete linear multistep method of order seven. The predictors were also developed by method of collocation and interpolation of power series approximate solution, to give a continuous linear multistep method. The continuous linear multistep method is then solved for the independent solution to give a continuous block formula, which is evaluated at some selected grid points to give discrete block method. Basic properties of the corrector was investigated and found to be zero stable, consistent and convergent. The efficiency of the method was tested on some numerical experiments and found to compare favorably with the existing methods.

Key words: Predictor; Corrector; Collocation; Interpolation; Approximate solution; Independent solution; Zero stable; Consistent; Convergent

James, A. A., Adesanya, O. A., & Fasasi, M. K. (2013). Starting Order Seven Method Accurately for the Solution of First Initial Value Problems of First Order Ordinary Differential Equations. *Progress in Applied Mathematics, 6*(1), 30–39. Available from http://www.cscanada.net/index.php/pam/article/view/j.pam.1925252820130601.5231 DOI: 10.3968/j.pam.1925252820130601.5231

1. INTRODUCTION

This paper considers numerical method for solving first order initial value problems of the form

$$y' = f(x, y), \quad y(x_0) = y_0, \tag{1}$$

where f is continuous and satisfies Lipschitz's conditions. Problems in the form (1) has wide application in physical sciences, engineering, electronics, medicine, etc. Very often, these problems do not have close solution which necessitate the derivation of numerical method to approximate solution.

Scholars have developed different numerical methods for the solution of initial value problems of ordinary differential equations. Awoyemi (2001), Adesanya *et al.* (2008), Awoyemi *et al.* (2008), Kayode and Adeyeye, (2011), to mention a few, individually proposed a multiderivative method which is implemented in predictor-corrector method. The major setback of this method is that the predictors are in reducing order of accuracy; hence it affects the accuracy of the method.

Scholars later proposed block method to cater for some of the setbacks of predictor-corrector method. This block method has the properties of Runge Kutta method for being self-starting and does not refuse development of separate predictors or starting values. Among the authors that proposed block method are; Jator (2007), Awoyemi *et al.* (2011), Adesanya *et al.* (2012), Zarina *et al.* (2005).

Block method was found to be cost effective and give better approximation. However, the setback of block method is that the interpolation point must not exceed the order of the differential equation, hence block method cannot exceed all the interpolation point, therefore method of lower order are developed.

Adesanya *et al.* (2012) developed a constant order predictor-corrector method in order to cater for some of the setbacks of block method. This method combined the properties of predictor corrector and block method. This method was found to give better approximation than both the predictor–corrector method and block method.

In this paper, we adopted the method proposed by Adesanya *et al.* (2012), to solve a one-step method with six hybrid points.

2. METHODOLOGY

2.1. Derivation of Constant Order Predictor

We consider a power series approximate solution in the form;

$$y(x) = \sum_{j=0}^{s+r-1} a_j x^j, \tag{2}$$

where r and s are the number of interpolation and collocation points respectively. The first derivative of (4.2.1) gives

$$y'(x) = \sum_{j=1}^{s+r-1} j a_j x^{j-1}. \tag{3}$$

Substituting (3) into (1) gives

$$f(x,y) = \sum_{j=1}^{s+r-1} j a_j x^{j-1}. \tag{4}$$

Interpolating (2) at x_{n+r}, $r = \frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}$; and Collocating (4) at x_{n+s}, $s = 0, \frac{1}{3}, \frac{2}{3}, 1$, gives a system of non linear equation in the form

$$AX = U \tag{5}$$

where

$$A = [a_0, a_1, a_2, a_3, a_5, a_6, a_7]^T$$

$$U = [y_{n+\frac{1}{6}}, y_{n+\frac{1}{3}}, y_{n+\frac{1}{2}} y_{n+\frac{2}{3}}, f_n, f_{n+\frac{1}{3}}, f_{n+\frac{2}{3}}, f_{n+1}]^T$$

$$X = \begin{bmatrix} 1 & x_{n+\frac{1}{6}} & x^2_{n+\frac{1}{6}} & x^3_{n+\frac{1}{6}} & x^4_{n+\frac{1}{6}} & x^5_{n+\frac{1}{6}} & x^6_{n+\frac{1}{6}} & x^7_{n+\frac{1}{6}} \\ 1 & x_{n+\frac{1}{3}} & x^2_{n+\frac{1}{3}} & x^3_{n+\frac{1}{3}} & x^4_{n+\frac{1}{3}} & x^5_{n+\frac{1}{3}} & x^6_{n+\frac{1}{3}} & x^7_{n+\frac{1}{3}} \\ 1 & x_{n+\frac{1}{2}} & x^2_{n+\frac{1}{2}} & x^3_{n+\frac{1}{2}} & x^4_{n+\frac{1}{2}} & x^5_{n+\frac{1}{2}} & x^6_{n+\frac{1}{2}} & x^7_{n+\frac{1}{2}} \\ 1 & x_{n+\frac{2}{3}} & x^2_{n+\frac{2}{3}} & x^3_{n+\frac{2}{3}} & x^4_{n+\frac{2}{3}} & x^5_{n+\frac{2}{3}} & x^6_{n+\frac{2}{3}} & x^7_{n+\frac{2}{3}} \\ 0 & 1 & 2x_n & 3x^2_n & 4x^3_n & 5x^4_n & 6x^5_n & 7x^6_n \\ 0 & 1 & 2x_{n+\frac{1}{3}} & 3x^2_{n+\frac{1}{3}} & 4x^3_{n+\frac{1}{3}} & 5x^4_{n+\frac{1}{3}} & 6x^5_{n+\frac{1}{3}} & 7x^6_{n+\frac{1}{3}} \\ 0 & 1 & 2x_{n+\frac{2}{3}} & 3x^2_{n+\frac{2}{3}} & 4x^3_{n+\frac{2}{3}} & 5x^4_{n+\frac{2}{3}} & 6x^5_{n+\frac{2}{3}} & 7x^6_{n+\frac{2}{3}} \\ 0 & 1 & 2x_{n+1} & 3x^2_{n+1} & 4x^3_{n+1} & 5x^4_{n+1} & 6x^5_{n+1} & 7x^6_{n+1} \end{bmatrix}$$

Solving (5) for the unknown coefficients using Gaussian elimination method gives a continuous hybrid linear multistep method in the form

$$y(t) = \alpha_{\frac{1}{6}} y_{n+\frac{1}{6}} + \alpha_{\frac{1}{3}} y_{n+\frac{1}{3}} + \alpha_{\frac{1}{2}} y_{n+\frac{1}{2}} + \alpha_{\frac{2}{3}} y_{n+\frac{2}{3}} + h\left(\sum_{j=0}^{1} \beta_j f_{n+j} + \beta_{\frac{1}{3}} f_{n+\frac{1}{3}} + \beta_{\frac{2}{3}} f_{n+\frac{2}{3}}\right) \tag{6}$$

where

$$\alpha_{\frac{1}{6}} = \frac{1}{91}\begin{pmatrix} 171072t^7 - 59852t^6 + 830304t^5 + 208032t^3 - \\ 32544t^2 - 578496 \end{pmatrix}$$
$$\alpha_{\frac{1}{3}} = \frac{1}{169}\begin{pmatrix} 892296t^7 - 2819772^6 + 3398598t^5 + 534006^3 - \\ 56212t^2 - 1952289 \end{pmatrix}$$

$$\alpha_{\frac{1}{2}} = \frac{1}{1183}\left(\begin{array}{c} -13856832t^7 + 4425321t^6 - 54517536t^5 - 9774432t^3 + \\ 1220832t^2 + 32729152 \end{array}\right)$$
$$\alpha_{\frac{2}{3}} = \frac{1}{1183}\left(\begin{array}{c} 5386824t^7 - 167301036^6 + 19933398t^5 + 3331974t^3 - \\ 404262t^2 - 11541498 \end{array}\right)$$
$$\beta_0 = \frac{1}{4732}\left(\begin{array}{c} 59292t^7 - 222264t^6 + 340713t^5 + 128613t^3 - \\ 34046t^2 + 4732t - 277140 \end{array}\right)$$
$$\beta_{\frac{1}{3}} = \frac{1}{4732}\left(\begin{array}{c} 5088420t^7 - 17057628t^6 + 22328055t^5 + 4668525t^3 - \\ 633177t^2 - 1440895 \end{array}\right)$$
$$\beta_{\frac{2}{3}} = \frac{1}{676}\left(\begin{array}{c} 232308t^7 - 705672t^6 + 821475t^5 + 132387t^3 - \\ 15858t^2 - 466140 \end{array}\right)$$
$$\beta_1 = \frac{1}{4732}\left(\begin{array}{c} 16524t^7 - 43092t^6 + 44685t^5 + 6171t^3 - \\ 703t^2 + 232225 \end{array}\right)$$

Evaluating (6) at $t = 1$ gives

$$y_{n+1} = -\frac{384}{91}y_{n+\frac{1}{6}} - \frac{1375}{169}y_{n+\frac{1}{3}} + \frac{54400}{1183}y_{n+\frac{1}{2}} - \frac{24600}{1183}y_{n+\frac{2}{3}} + \tag{7}$$

$$h\left(-\frac{25}{1183}f_n - \frac{3600}{1183}f_{n+\frac{1}{3}} + \frac{375}{169}f_{n+\frac{2}{3}} + \frac{90}{1183}f_{n+1}\right)$$

2.2. Derivation of Predictor

Collocating (4) at x_{n+s}, $s = 0(\frac{1}{6})1$ and interpolating (2) at x_n gives a system of non-linear equations of the form (5) where

$$A = [a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7]^T$$

$$U = [y_n, f_n, f_{n+\frac{1}{6}}, f_{n+\frac{1}{2}}, f_{n+\frac{2}{3}} f_{n+\frac{5}{6}}, f_{n+1}]^T$$

$$X = \begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 \\ 0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 & 5x_n^4 & 6x_n^5 & x_n^6 \\ 0 & 1 & 2x_{n+\frac{1}{6}} & 3x_{n+\frac{1}{6}}^2 & 4x_{n+\frac{1}{6}}^3 & 5x_{n+\frac{1}{6}}^4 & 6x_{n+\frac{1}{6}}^5 & 7x_{n+\frac{1}{6}}^6 \\ 0 & 1 & 2x_{n+\frac{1}{3}} & 3x_{n+\frac{1}{3}}^2 & 4x_{n+\frac{1}{3}}^3 & 5x_{n+\frac{1}{3}}^4 & 6x_{n+\frac{1}{3}}^5 & 7x_{n+\frac{1}{3}}^6 \\ 0 & 1 & 2x_{n+\frac{1}{2}} & 3x_{n+\frac{1}{2}}^2 & 4x_{n+\frac{1}{2}}^3 & 5x_{n+\frac{1}{2}}^4 & 6x_{n+\frac{1}{2}}^5 & 7x_{n+\frac{1}{2}}^6 \\ 0 & 1 & 2x_{n+\frac{2}{3}} & 3x_{n+\frac{2}{3}}^2 & 4x_{n+\frac{2}{3}}^3 & 5x_{n+\frac{2}{3}}^4 & 6x_{n+\frac{2}{3}}^5 & 7x_{n+\frac{2}{3}}^6 \\ 0 & 1 & 2x_{n+\frac{5}{6}} & 3x_{n+\frac{5}{6}}^2 & 4x_{n+\frac{5}{6}}^3 & 5x_{n+\frac{5}{6}}^4 & 6x_{n+\frac{5}{6}}^5 & 7x_{n+\frac{5}{6}}^6 \\ 0 & 1 & 2x_{n+1} & 3x_{n+1}^2 & 4x_{n+1}^3 & 5x_{n+1}^4 & 6x_{n+1}^5 & 7x_{n+1}^6 \end{bmatrix}$$

Solving for the unknown parameters $a_j s$ and substituting back into (2) gives a continuous hybrid linear multistep method in the form

$$y(x) = \alpha_0 y_n + h\left[\sum_{j=0}^{1}\beta_j(x)f_{n+j} + \beta_{\frac{1}{6}}f_{n+\frac{1}{6}} + \beta_{\frac{1}{3}}f_{n+\frac{1}{3}} + \beta_{\frac{1}{2}}f_{n+\frac{1}{2}} + \beta_{\frac{5}{6}}f_{n+\frac{5}{6}}\right] \tag{8}$$

where $\alpha_0 = 1$

$$\beta_0 = \frac{1}{840}\left(7776t^7 - 31757t^6 + 52920t^5 - 46305t^4 + 22736t^3 - 6174t^2 + 840t\right)$$

$$\beta_{\frac{1}{6}} = -\frac{1}{35}\left(1944t^7 - 7560t^6 + 11718t^5 - 9135t^4 + 3654t^3 - 630t^2\right)$$

$$\beta_{\frac{1}{3}} = \frac{1}{280}\left(38880t^7 - 143640t^6 + 207144t^5 - 145215t^4 + 49140t^3 - 6300t^3\right)$$

$$\beta_{\frac{1}{2}} = -\frac{1}{105}\left(19440t^7 - 68040t^6 + 91476t^5 - 58590t^4 - 17780t^3 - 2100t^2\right)$$

$$\beta_{\frac{2}{3}} = \frac{1}{280}\left(38880t^7 - 12820t^6 - 161784t^5 - 96705t^4 + 27720t^3 - 3150t^2\right)$$

$$\beta_{\frac{5}{6}} = -\frac{1}{35}\left(1944t^7 - 6048t^6 + 7182t^5 - 4095t^4 + 1134t^3 - 126t\right)$$

$$\beta_1 = \frac{1}{280}\left(7776t^7 - 22680t^6 + 25704t^5 - 14175t^4 + 3836t^3 - 420t^2\right)$$

$$t = \frac{x - x_n}{h}, f_{n+j}$$

Solving for the independent solution at the selected grid points gives a continous block formula of the form

$$y_{n+k} = y_n + h\left[\sum_{j=0}^{1}\sigma_j(x)f_{n+j} + \sigma_{\frac{1}{6}}f_{n+\frac{1}{6}} + \sigma_{\frac{1}{3}}f_{n+\frac{1}{3}} + \sigma_{\frac{1}{2}}f_{n+\frac{1}{2}} + \sigma_{\frac{5}{6}}f_{n+\frac{5}{6}}\right] \tag{9}$$

$$\sigma_0 = \frac{1}{840}\left(7776t^7 - 31757t^6 + 52920t^5 - 46305t^4 + 22736t^3 - 6174t^2 + 840t\right)$$

$$\sigma_{\frac{1}{6}} = -\frac{1}{35}\left(1944t^7 - 7560t^6 + 11718t^5 - 9135t^4 + 3654t^3 - 630t^2\right)$$

$$\sigma_{\frac{1}{3}} = \frac{1}{280}\left(38880t^7 - 143640t^6 + 207144t^5 - 145215t^4 + 49140t^3 - 6300t^3\right)$$

$$\sigma_{\frac{1}{2}} = -\frac{1}{105}\left(19440t^7 - 68040t^6 + 91476t^5 - 58590t^4 - 17780t^3 - 2100t^2\right)$$

$$\sigma_{\frac{2}{3}} = \frac{1}{280}\left(38880t^7 - 12820t^6 - 161784t^5 - 96705t^4 + 27720t^3 - 3150t^2\right)$$

$$\sigma_{\frac{5}{6}} = -\frac{1}{35}\left(1944t^7 - 6048t^6 + 7182t^5 - 4095t^4 + 1134t^3 - 126t\right)$$

$$\sigma_1 = \frac{1}{280}\left(7776t^7 - 22680t^6 + 25704t^5 - 14175t^4 + 3836t^3 - 420t^2\right)$$

Evaluating (9) at $t = \frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, 1.$

$$A^{(0)}\mathbf{Y}_m = \mathbf{e}y_n + h\left[\mathbf{d}F(y_n) + \mathbf{dF}(\mathbf{Y}_m)\right] \tag{10}$$

where $A^{(0)} = 6 \times 6$ identity matrix,

$$\mathbf{e} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{Y}_m = \left[y_{n+\frac{1}{6}}, y_{n+\frac{1}{3}}, y_{n+\frac{1}{2}}, y_{n+\frac{2}{3}}, y_{n+\frac{2}{3}}, y_{n+\frac{5}{6}}, y_{n+1}\right]^T$$

$$F(\mathbf{y}_n) = [f_{n-1}\ , f_{n-2}\ , f_{n-3}\ , f_{n-4}\ , f_{n-4}, f_{n-5}\ f_n]^T$$

$$F(\mathbf{Y}_m) = \left[f_{n+\frac{1}{6}}, f_{n+\frac{1}{3}}, f_{n+\frac{1}{2}}, f_{n+\frac{2}{3}}, f_{n+\frac{2}{3}}, f_{n+\frac{5}{6}}, f_{n+1}\right]^T$$

$$\mathbf{b} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \frac{19087}{362880} \\ 0 & 0 & 0 & 0 & 0 & \frac{1139}{22680} \\ 0 & 0 & 0 & 0 & 0 & \frac{137}{2688} \\ 0 & 0 & 0 & 0 & 0 & \frac{143}{2835} \\ 0 & 0 & 0 & 0 & 0 & \frac{3715}{72576} \\ 0 & 0 & 0 & 0 & 0 & \frac{41}{840} \end{bmatrix}$$

$$\mathbf{d} = \begin{bmatrix} \frac{2713}{13120} & \frac{-15487}{120960} & \frac{293}{2835} & \frac{-6737}{120960} & \frac{263}{15120} & \frac{-863}{362880} \\ \frac{47}{189} & \frac{11}{7560} & \frac{166}{2835} & \frac{-269}{7560} & \frac{11}{945} & \frac{-37}{22680} \\ \frac{27}{112} & \frac{387}{4480} & \frac{17}{105} & \frac{-243}{4480} & \frac{9}{560} & \frac{-29}{13440} \\ \frac{232}{945} & \frac{64}{945} & \frac{752}{2835} & \frac{29}{945} & \frac{8}{945} & \frac{-4}{2835} \\ \frac{725}{3024} & \frac{2125}{24192} & \frac{125}{567} & \frac{2875}{24192} & \frac{235}{3024} & \frac{-275}{72516} \\ \frac{9}{35} & \frac{9}{280} & \frac{34}{105} & \frac{9}{280} & \frac{9}{35} & \frac{41}{840} \end{bmatrix}$$

3. ANALYSIS OF THE BASIC PROPERTIES OF CORRECTOR

3.1. Order of the Method

We define a the linear operator $L\{y(x);h\}$ on (7) as

$$L\{y(x);h\} = y(x) - \alpha_\mu(x)\, y_{n+\mu} + h\left(\sum_{j=0}^{1} \beta(x)\, f_{n+j} + \beta_k(x) f_{n+k}\right) \tag{11}$$

Expanding (7) in Taylor series expansion and comparing the coefficient of h gives

$$\begin{aligned} L\{y(x):h\} = \big(&C_0 y(x) + C_1 h y'(x) + C_2 h^2 y''(x) + ... + C_p h^p y^p(x) \\ &+ C_{p+1} h^{p+1} y^{p+1}(x) + C_{p+2} h^{p+2} y^{p+2}(x) + ...\big) \end{aligned} \tag{12}$$

Definition

The linear operator L and the associated continuous linear multistep method (7) is said to be of order p if $C_O = C_1 = C_2 = ... = C_P = 0$ and $C_{p+1} \neq 0$. C_{p+1} is called the error constant and implies that the local truncation error is given by

$$t_{n+k} = C_{p+1} h^{p+1} y^{p+1}(x) + O\left(h^{p+2}\right) t_{n+k} \tag{13}$$

$C_0 = C_1 = ... = C_6 = 0. = C_7 = 0, C_8 \neq 0$, hence the order of the method is 7 with error constant $c_{p+1} = -1.4678 \times 10^{-10}$.

3.2. Consistency of Our Schemes

3.2.1. Definition

A linear multistep method is said to be consistent if it has order $\rho \geq 1$ and if $\rho(1) = 0$, $\rho'(1) = \sigma(1)$ where $\rho(r)$ is the first characteristic polynomial and $\sigma(r)$ is the second characteristic polynomial for our method.

3.2.2. Consistency

The first and second characteristic polynomial of (7) are given

$$\rho(r) = -\frac{384}{91} z^{\frac{1}{6}} - \frac{3375}{169} z^{\frac{1}{3}} + \frac{54400}{1183} z^{\frac{1}{2}} - \frac{24600}{1183} z^{\frac{2}{3}}$$

$$\sigma(r) = \left(-\frac{25}{1183} - \frac{3600}{1183} r^{\frac{1}{3}} + \frac{375}{169} r^{\frac{2}{3}} - \frac{24600}{1183} r^{\frac{2}{3}}\right)$$

$\rho(1) = 0,\ \rho^{'(1)} = \sigma(1)$ hence is consistent.

3.3. Zero Stability

A linear multistep method is said to be zero stable if the zeros of the first characteristic polynomial $\sigma(r)$ satisfies $|r| = 1$ is simple for our method. The root of the first characteristic polynomial is 0 and 1.

3.4. Convergence

The necessary and sufficient condition for a linear multistep method to be convergent is that it must be consistent and zero stable, hence our method is convergent.

4. NUMERICAL EXAMPLES

4.1. Problem 1

We consider a linear order ordinary differential equation:
$y' = -y,\ y(0) = 1,\ 0 \leq x \leq 1,\ h = 0.1,$
Exact solution $y(x) = e^{-x}$.
This problem was solved by Areo *et al.* (2011).

Table 1
Table 1 for Problem 1

x	**Exact solution**	**Computed solution**	**Error in new method**	**Error in Areo**
0.1	0.90487418035959	0.90483741801514	1.7444(-11)	2.1(-10)
02	0.81873075307798	0.81873075306219	1.5783(-11)	2.2(-10)
0.3	0.74081822068171	0.74087822066743	1.4281(-11)	6.0(-10)
0.4	0.67032004603563	0.67032004602270	1.2925(-11)	1.0(-10)
0.5	0.60653065971263	0.60653065970093	1.1696(-11)	4.1(-10)
0.6	0.54881163609402	0.54881163608344	1.0580(-11)	7.0(-10)
0.7	0.49658530379140	0.49658530378183	9.5701(-11)	1.5(-10)
0.8	0.44932896411722	0.44932896410856	8.6612(-11)	7.0(-10)
0.9	0.40656965974050	0.40656965973276	7.8371(-11)	1.4(-10)
1.0	0.36787944117140	0.36787944116439	7.0927(-11)	8.0(-10)

Table 2
Table 2 for Problem 11

x	**Exact solution**	**Computed solution**	**Error in new method**	**Error in Areo**
0.1	1.005012520857401	1.005012520875641	1.6554(-11)	2.6067(-11)
02	1.020201340026755	1.020201340070737	4.3981(-11)	8.4790(-11)
0.3	1.046027859908716	1.046027859987168	7.8451(-11)	1.8684(-10)
0.4	1.083287067674958	1.083287067801583	1.2662(-10)	3.5701(-10)
0.5	1.133148453066826	1.133148453263925	1.9709(-10)	6.1004(-10)
0.6	1.197217363131810	1.197217363423617	3.0180(-10)	1.6157(-09)
0.7	1.277621313204886	1.277621313662603	4.5771(-10)	1.6445(-09)
0.8	1.377129776433595	1.377127765025506	6.8954(-10)	2.6158(-09)
0.9	1.499302500056767	1.499502501090387	1.0336(-09)	4.1110(-09)
1.0	1.644872127070013	1.648721272243691	1.5435(-09)	1.5435(-09)

4.2. Problem 11

$y' = xy$, $y(0) = 1$, $h = 0.1$.
Exact solution: $y(x) = e^{\frac{1}{2}x^2}$

4.3. Problem 111

$y' = x - y$, $y(0) = 0$, $0 \leq x \leq 1$, $h = 0.1$.
Exact solution: $y(x) = x + e^{-x} - 1$.
This problem was solved by Areo *et al.* (2011).

Table 3
Table 3 for Problem 112

x	Exact solution	Computed solution	Error in new method	Error in Areo
0.1	0.0048374180359576	0.0048374180151291	1.7443(-11)	0.000
02	0.0187307530779818	0.0187307530621957	1.5786(-11)	0.000
0.3	0.0408182206817178	0.0408182206673440	1.4283(-11)	6.0(-10)
0.4	0.0703200460356394	0.0703200460227154	1.2924(-11)	2.0(-10)
0.5	0.1065306597126334	0.1065306597009394	1.1694(-11)	7.0(-10)
0.6	0.1488116360940265	0.1488116360834448	1.0581(-11)	1.0(-10)
0.7	0.1965853037914095	0.1965853037818356	9.5739(-12)	8.0(-10)
0.8	0.2493289641172218	0.2493289641085604	8.6613(-12)	2.0(-10)
0.9	0.3065696597405995	0.3065696597327599	7.8396(-12)	9.0(-10)
1.0	0.3678794411714430	0.3678794411643524	7.0906(-12)	4.0(-10)

4.4. Discussion of the Result

In this paper, we have considered three numerical examples to test the efficiency of our method. The three problems were solved by Areo *et al.* (2012). They proposed a hybrid method of order seven and adopted classical Runge Kutta method to provide the starting values. The new method gave better approximation because the proposed method is self-starting and does not require starting values

4.5. Conclusion

In this paper, we have proposed an order seven continuous hybrid method for the solution of first order initial value problems in ordinary differential equations. Our method was found to be zero stable, consistent and converges. The numerical examples show that our method gave better accuracy than the existing methods.

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