VIM Solution for Mixed Convection over Horizontal Moving Porous Flat Plate

Hamed Shahmohamadi[a],* and Kasra Ataee[a]

[a]Wolfson School of Mechanical and Manufacturing Engineering, Loughborough University, UK.

* Corresponding author.
  Address: Wolfson School of Mechanical and Manufacturing Engineering, Loughborough University, Loughborough LE11 3TU, Leicestershire, United Kingdom;
  E-Mail: H.Shahmohamadi@lboro.ac.uk

Abstract: The non-viscous, laminar mixed convection boundary-layer flow over a horizontal moving porous flat plate, with chemical reaction, is considered. The governing equations are expressed in non-dimensional form and the series solutions of coupled system of equations are constructed for velocity, temperature and concentration functions using variational iteration method. The investigated parameters are: buoyancy parameter, chemical reaction parameter, order of chemical reaction, Prandtl number and Schmidt number.

Key words: Mixed convection; Heat and mass transfer; Moving porous flat plate; Chemical reaction; Variational iteration method

1. INTRODUCTION

A convection situation involving both natural and forced convection is commonly referred as mixed convection; in porous media has been an important topic because of its wide range of application in engineering and science. In mixed convection flows, the forced convection as well as the free convection effects is of comparable magnitudes. Thus, mixed convection occurs if the effect of buoyancy forces on a

forced flow or the effect of forced flow on a buoyant flow is significant. The laminar mixed convection, which is encountered in various applications in thermal engineering and science, received a special attention from researchers in the past decades. Examples of these applications include solar energy systems, boilers, compact heat exchangers and cooling of electronic devices and etc. Incropera [1], Aung [2] and Gebhart *et al.* [3] were researches that investigated the mixed convection with either uniform wall temperature or wall heat flux thermal boundary condition. Srinivas and Muthiraj [4] studied mixed convection in a vertical porous channel, Datta *et al.* [5] presented non-similar solution of mixed convection in the presence of surface mass transfer over a horizontal flat plate. Laminar mixed in the entrance region of a vertical circular tube for assisted and opposed air flows with the using of a uniform wall heat flux boundary condition [6], turbulent mixed convection flow over a inclined flat channel [7] are two more examples of mixed convection. Recently a number of papers [8–11] have appeared on the combined free and forced convection in a porous medium involving horizontal surface. Kandasamy *et al.* [12] investigated the effect of chemical reaction on heat and mass transfer over a wall in presence of suction or injection. Hayat *et al.* [13] studied the laminar flow problem of convective heat transfer on a semi-infinite plate in the presence of concentration and chemical reaction. Some articles [14,15] investigated the impact of chemical reaction on moving porous medium flat plate.

Because of the nonlinearities in the reduced differential equation, no analytical solution is available and the nonlinear equations usually are solved numerically subject to boundary conditions, one of which is prescribed at infinity. Numerical methods related to this subject are also given in different types [16]. Numerical methods have included finite difference method [17–19].

The variational iteration method (VIM) was first proposed by He [20,21] and systematically illustrated in 1999 [22] and used to give approximate solutions of the problem of seepage flow in porous media with fractional derivatives. The VIM is useful to obtain exact and approximate solutions of linear and nonlinear differential equations. In this method, general Lagrange multipliers are introduced to construct correction functional for the problems. The multipliers can be identified optimally via the variational theory. There is no need of linearization or discretization, and large computational work and round-off errors are avoided. It has been used to solve effectively, easily and accurately a large class of nonlinear problems with approximation [23–25]. It was shown by many authors [26,27] that this method is more powerful than existing techniques such as the Adomian method [28]. He [29] proposed three standard variational iteration algorithms for solving differential equations, integrodifferential equations, fractional differential equations, differential-difference equations, and fractional/fractal differential-difference equations. The algorithm used in this paper belongs to the variational iteration algorithm-I. In our article the viscous, laminar mixed convection boundary-layer flow from a horizontal moving porous flat plate, with the effect of chemical reaction, is considered. The transformed momentum and thermal boundary-layer and concentration equations are solved by VIM.

2. MATHEMATICAL FORMULATIONS

Consider a continuous moving flat plate with a velocity $\overline{U}(\overline{x})$ along the plate which is along horizontal direction, i.e., x-axis, y-axis is normal to the plate at origin

$(0,0)$. $\overline{Y}w(\overline{x})$, $\overline{C}w(\overline{x})$ are the temperature and concentration at the wall while $\overline{T}_\infty$, $\overline{C}_\infty$ are constant temperature and concentration far away the plate (see Figure 1). Neglecting the viscous dissipation and by considering fluid properties as constant except in buoyancy term, the governing Prandtl boundary-layer equations which are parabolic in nature and simplified form of NaiverCStokes pertinent to the problem are
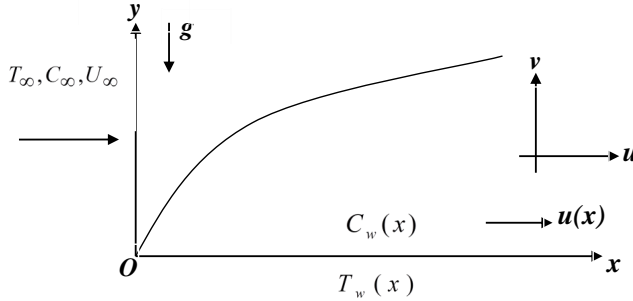

Figure 1
Scheme of the Moving Flat Plate

Continuity equation:
$$\frac{\partial\,\overline{u}}{\partial\,\overline{x}} + \frac{\partial\,\overline{v}}{\partial\,\overline{y}} = 0, \tag{1}$$

x-momentum equation:
$$\overline{u}\frac{\partial\,\overline{u}}{\partial\,\overline{x}} + \overline{v}\,\frac{\partial\,\overline{u}}{\partial\,\overline{y}} = -\frac{1}{\rho}\frac{\partial\,\overline{p}}{\partial\,\overline{x}} + \nu\frac{\partial^{\,2}\overline{u}}{\partial\,\overline{y}^2}, \tag{2}$$

y-momentum equation:
$$-\frac{1}{\rho}\frac{\partial\,\overline{p}}{\partial\overline{y}} + \overline{g}_{\overline{x}}\,\beta_T\left(\overline{T} - \overline{T}_\infty\right) + \overline{g}_{\overline{x}}\,\beta_C\left(\overline{T} - \overline{T}_\infty\right) = 0, \tag{3}$$

Energy equation:
$$\overline{u}\frac{\partial\,\overline{T}}{\partial\,\overline{x}} + \overline{v}\frac{\partial\,\overline{T}}{\partial\,\overline{y}} = \alpha\frac{\partial^2\,\overline{T}}{\partial\,\overline{y}^2}, \tag{4}$$

Concentration equation:
$$\overline{u}\frac{\partial\,\overline{C}}{\partial\,\overline{x}} + \overline{v}\frac{\partial\,\overline{C}}{\partial\,\overline{y}} = D\frac{\partial^2\,\overline{C}}{\partial\,\overline{y}^2} - K\left(\overline{C} - C_\infty\right)^n. \tag{5}$$

The boundary conditions are
$$\begin{aligned}
\overline{u} = \overline{U}(\overline{x}), \;\; \overline{v} = \overline{v_w}(\overline{x}), \;\; \overline{T} = \overline{T}_W, \;\; \overline{C} = \overline{C}_W \quad &\text{at} \quad \overline{y} = 0, \\
\overline{u} = \overline{U}_\infty, \;\; \overline{u_{\overline{y}}} = 0, \;\; \overline{T} = \overline{T}_\infty, \;\; \overline{C} = \overline{C}_\infty \quad &\text{at} \quad y \to \infty.
\end{aligned} \tag{6}$$

The existence of stream function $\overline{\psi}$ is a consequence of the incompressibility of the fluid in two dimensional flows. Any solenoidal velocity field in two dimensions

can be expressed as $\overline{u} = \overline{\psi}_{\overline{y}}$, $\overline{v} = -\overline{\psi}_{\overline{x}}$. Introducing stream function, dimensionless temperature and concentration function θ and φ defined by

$$\theta = \frac{\overline{T} - \overline{T}_\infty}{\overline{T}_w - \overline{T}_\infty} = \frac{\overline{T} - \overline{T}_\infty}{\Delta \overline{T}}, \quad \varphi = \frac{\overline{C} - \overline{C}_\infty}{\overline{C}_w - \overline{C}_\infty} = \frac{\overline{C} - \overline{C}_\infty}{\Delta \overline{C}}.$$

For mentioned problem, similarity variables are as follows:

$$
\begin{aligned}
&\eta = \mathrm{Re}^{\frac{1}{2}} \, \frac{\xi}{L^{\frac{5-3n}{7-5n}}} = \mathrm{Re}^{\frac{1}{2}} \, \frac{y}{L} \left(\frac{x}{L}\right)^{\frac{2(n-1)}{7-5n}}, \\
&\psi = \gamma \, \mathrm{Re}^{\frac{1}{2}} \, \left(\frac{x}{L}\right)^{\frac{5-3n}{7-5n}} f(\eta), \\
&\Delta T = A \left(\frac{x}{L}\right)^{\frac{4}{7-5n}}, \quad \Delta C = B \left(\frac{x}{L}\right)^{\frac{4}{7-5n}}, \\
&U = U_0 \left(\frac{x}{L}\right)^{\frac{3-n}{7-5n}}, \quad \theta(\xi) = \theta(\eta), \quad \varphi(\xi) = \varphi(\eta).
\end{aligned}
\tag{7}
$$

where Re is local Reynolds number. With using above transforms, we can obtain:

$$f_{\eta\eta\eta} + \left(\frac{5-3n}{7-5n}\right) f \, f_{\eta\eta} - \left(\frac{n+1}{7-5n}\right) f_\eta \, f_{\eta\eta} - K_1 \left[\left(\frac{2(n-1)}{7-5n}\right) \eta \, \theta_\eta + \left(\frac{4}{7-5n}\right) \theta\right]$$
$$- K_2 \left[\left(\frac{2(n-1)}{7-5n}\right) \eta \, \varphi_\eta + \left(\frac{4}{7-5n}\right) \varphi\right] = 0,$$

$$\tag{8}$$

$$\frac{1}{\mathrm{Pr}} \theta_{\eta\eta} + \left(\frac{5-3n}{7-5n}\right) f \, \theta_\eta - \left(\frac{4}{7-5n}\right) f_\eta \, \theta = 0, \tag{9}$$

$$\frac{1}{Sc} \varphi_{\eta\eta} + \left(\frac{5-3n}{7-5n}\right) f \, \varphi_\eta - \left(\frac{4}{7-5n}\right) f_\eta \, \varphi - k_3 \, \varphi^n = 0, \tag{10}$$

where n is the order of chemical reaction. The boundary conditions become

$$
\begin{aligned}
&f(\eta) = 0, \quad f_\eta(\eta) = 1, \quad \theta(\eta) = 1, \quad \varphi(\eta) = 1, \quad \eta \to 0, \\
&f_\eta(\eta) = 0, \quad f_{\eta\eta}(\eta) = 0, \quad \theta(\eta) = 0, \quad \varphi(\eta) = 0, \quad \eta \to \infty,
\end{aligned}
\tag{11}
$$

Here $K_1 = \dfrac{Gr_T}{\mathrm{Re}^{\frac{1}{2}}}$, $K_2 = \dfrac{Gr_c}{\mathrm{Re}^{\frac{5}{2}}}$ buoyancy parameter and $K_3 = \dfrac{Gr_c^{\frac{1}{2}}}{\mathrm{Re}}$ chemical reaction parameter.

3. BASIC CONCEPTS OF VIM

To illustrate the basic concepts of VIM, we consider the following differential equation:

$$\mathcal{L}u + \mathcal{N}u = g(x), \tag{12}$$

where $\mathcal{L}$, $\mathcal{N}$ and $g(x)$ are the linear operator, the nonlinear operator and a heterogeneous term, respectively.

Assuming $u_0(x)$ is the solution of $\mathcal{L}u = 0$, we can write down an expression to correct the value of some special point, for example at $x = 1$

$$u_{\mathrm{cor}}(1) = u_0(1) + \int_0^1 \lambda \left[\mathcal{L} \, u_0 + \mathcal{N} \, u_0 - g\right] \mathrm{d}x, \tag{13}$$

where λ is a general Lagrange multiplier, which can be identified optimally via variational theory, the second term on the right is called the correction. He [20,21] has modified the above method into an iteration method in the following way:

$$u_{\mathrm{m}+1}(x_0) = u_m(x_0) + \int_0^{x_0} \lambda \left[\mathcal{L}\,u_m + \mathcal{N}\,\tilde{u}_m - g \right] \mathrm{d}x, \tag{14}$$

with $u_0(x)$ as initial approximation with possible unknowns, $\tilde{u}_m$ is considered a restricted variation and is chosen suitably to satisfy the restricted variation conditions, i.e. $\delta \tilde{u}_m = 0$. For arbitrary of x_0, we can rewrite Equation (14) as follows:

$$u_{\mathrm{m}+1}(x) = u_m(x) + \int_0^x \lambda \left[\mathcal{L}\,u_m(\xi) + \mathcal{N}\,\tilde{u}_m(\xi) - g(\xi) \right] \mathrm{d}\xi, \quad m \geq 0. \tag{15}$$

It is obvious that the successive approximations u_j, $j \geq 0$ can be established by determining λ, a general Lagrange's multiplier, which can be identified optimally via the variational theory. As mentioned before, the function $\tilde{u}_m$ is a restricted variation which means $\delta \tilde{u}_m = 0$. Therefore, we first determine the Lagrange multiplier λ which will be identified optimally via integration by parts. The successive approximations $u_{m+1}(x)$, $m \geq 0$ of the solution $u(x)$ will be readily obtained upon using the obtained optimal Lagrange multiplier and by using admissible function u_0. Once λ is determined, then several approximations $u_j(x)$, $j \geq 0$ follow immediately. Consequently, the exact solution may be obtained using

$$u = \lim_{m \to \infty} u_m. \tag{16}$$

4. VIM SOLUTIONS

In order to obtain VIM solution of Equations (8)–(10), we construct a correction functions which reads

$$f_{m+1}(\eta) = f_m(\eta) + \int_0^\eta \lambda_1 \left[\frac{\partial^4 f_m(\tau)}{\partial \tau^4} + \left(\frac{5 - 3n}{7 - 5n} \right) \tilde{f}_m(\tau) \frac{\partial^3 \tilde{f}_m(\tau)}{\partial \tau^3} \right.$$
$$- \left(\frac{n+1}{7-5n} \right) \frac{\partial \tilde{f}_m(\tau)}{\partial \tau} \frac{\partial^2 \tilde{f}_m(\tau)}{\partial \tau^2} - K_1 \left[\left(\frac{2(n-1)}{7-5n} \right) \tau \frac{\partial \tilde{\theta}_m(\tau)}{\partial \tau} + \left(\frac{4}{7-5n} \right) \tilde{\theta}_m(\tau) \right]$$
$$\left. - K_2 \left[\left(\frac{2(n-1)}{7-5n} \right) \tau \frac{\partial \tilde{\phi}_m(\tau)}{\partial \tau} + \left(\frac{4}{7-5n} \right) \tilde{\phi}_m(\tau) \right] \right] d\tau, \tag{17}$$

$$\theta_{m+1}(\eta) = \theta_m(\eta) + \int_0^\eta \lambda_2 \left[\frac{1}{\mathrm{Pr}} \frac{\partial^2 \theta_m(\tau)}{\partial \tau^2} \right.$$
$$\left. + \left(\frac{5-3n}{7-5n} \right) \tilde{f}_m(\tau) \frac{\partial \tilde{\theta}_m(\tau)}{\partial \tau} - \left(\frac{4}{7-5n} \right) \frac{\partial \tilde{f}_m(\tau)}{\partial \tau} \tilde{\theta}_m(\tau) \right] d\tau, \tag{18}$$

$$\phi_{m+1}(\eta) = \phi_m(\eta) + \int_0^\eta \lambda_3 \left[\frac{1}{Sc} \frac{\partial^2 \phi_m(\tau)}{\partial \tau^2} + \left(\frac{5-3n}{7-5n} \right) \tilde{f}_m(\tau) \frac{\partial \tilde{\phi}_m(\tau)}{\partial \tau} \right.$$
$$\left. - \left(\frac{4}{7-5n} \right) \frac{\partial \tilde{f}_m(\tau)}{\partial \tau} \tilde{\phi}_m(\tau) - K_3 \tilde{\phi}_m^n(\tau) \right] d\tau, \tag{19}$$

where λ_1, λ_2 and λ_3 are the general Lagrange multipliers and $\tilde{f}_m(\tau)$, $\tilde{\theta}_m(\tau)$ and $\tilde{\phi}_m(\tau)$ are considered as restricted variations, i.e. $\delta\tilde{f}_m(\tau) = 0$, $\delta\tilde{\theta}_m(\tau) = 0$ and $\delta\tilde{\phi}_m(\tau) = 0$. To find the optimal values of λ_1, λ_2 and λ_3, we have

$$
\delta f_{m+1}(\eta) = \delta f_m(\eta) + \delta \int_0^\eta \lambda_1 \left[\frac{\partial^4 f_m(\tau)}{\partial \tau^4} + \left(\frac{5-3n}{7-5n} \right) \tilde{f}_m(\tau) \frac{\partial^3 \tilde{f}_m(\tau)}{\partial \tau^3} \right.
$$
$$
- \left(\frac{n+1}{7-5n} \right) \frac{\partial \tilde{f}_m(\tau)}{\partial \tau} \frac{\partial^2 \tilde{f}_m(\tau)}{\partial \tau^2} - K_1 \left[\left(\frac{2(n-1)}{7-5n} \right) \tau \frac{\partial \tilde{\theta}_m(\tau)}{\partial \tau} + \left(\frac{4}{7-5n} \right) \tilde{\theta}_m(\tau) \right]
$$
$$
\left. - K_2 \left[\left(\frac{2(n-1)}{7-5n} \right) \tau \frac{\partial \tilde{\phi}_m(\tau)}{\partial \tau} + \left(\frac{4}{7-5n} \right) \tilde{\phi}_m(\tau) \right] \right] d\tau,
$$
(20)

$$
\delta\theta_{m+1}(\eta) = \delta\theta_m(\eta) + \delta \int_0^\eta \lambda_2 \left[\frac{1}{\text{Pr}} \frac{\partial^2 \theta_m(\tau)}{\partial \tau^2} + \left(\frac{5-3n}{7-5n} \right) \tilde{f}_m(\tau) \frac{\partial \tilde{\theta}_m(\tau)}{\partial \tau} \right.
$$
$$
\left. - \left(\frac{4}{7-5n} \right) \frac{\partial \tilde{f}_m(\tau)}{\partial \tau} \tilde{\theta}_m(\tau) \right] d\tau,
$$
(21)

$$
\delta\phi_{m+1}(\eta) = \delta\phi_m(\eta) + \delta \int_0^\eta \lambda_3 \left[\frac{1}{Sc} \frac{\partial^2 \phi_m(\tau)}{\partial \tau^2} + \left(\frac{5-3n}{7-5n} \right) \tilde{f}_m(\tau) \frac{\partial \tilde{\phi}_m(\tau)}{\partial \tau} \right.
$$
$$
\left. - \left(\frac{4}{7-5n} \right) \frac{\partial \tilde{f}_m(\tau)}{\partial \tau} \tilde{\phi}_m(\tau) - K_3 \tilde{\phi}_m^n(\tau) \right] d\tau,
$$
(22)

or

$$
\delta f_{m+1}(\eta) = \delta f_m(\eta) + \delta \int_0^\eta \lambda_1 \left[\frac{\partial^4 f_m(\tau)}{\partial \tau^4} \right] d\tau,
$$
(23)

$$
\delta\theta_{m+1}(\eta) = \delta\theta_m(\eta) + \delta \int_0^\eta \lambda_2 \left[\frac{1}{\text{Pr}} \frac{\partial^2 \theta_m(\tau)}{\partial \tau^2} \right] d\tau,
$$
(24)

$$
\delta\phi_{m+1}(\eta) = \delta\phi_m(\eta) + \delta \int_0^\eta \lambda_3 \left[\frac{1}{Sc} \frac{\partial^2 \phi_m(\tau)}{\partial \tau^2} \right] d\tau,
$$
(25)

Its stationary conditions can be obtained as follows

$$
1 - \lambda_1^{(3)}(\tau)\big|_{\tau=\eta} = 0, \ \lambda_1''(\tau)\big|_{\tau=\eta} = 0, \ \lambda_1'(\tau)\big|_{\tau=\eta} = 0, \ \lambda_1(\tau)\big|_{\tau=\eta} = 0, \ \lambda_1^{(4)}(\tau) = 0,
$$
(26)

$$
1 - \lambda_2'(\tau)\big|_{\tau=\eta} = 0, \ \lambda_2(\tau)\big|_{\tau=\eta} = 0, \ \lambda_2''(\tau) = 0,
$$
(27)

$$
1 - \lambda_3'(\tau)\big|_{\tau=\eta} = 0, \ \lambda_3(\tau)\big|_{\tau=\eta} = 0, \ \lambda_3''(\tau) = 0.
$$
(28)

The Lagrange multipliers, can be identified as follows

$$
\lambda_1 = \frac{1}{6}(\tau - \eta)^3,
$$
(29)

$$\lambda_2 = \lambda_3 = (\tau - \eta), \tag{30}$$

and the following variational iteration formula can be obtained

$$f_{m+1}(\eta) = f_m(\eta) + \frac{1}{6} \int_0^\eta (\tau - \eta)^3 \left[\frac{\partial^4 f_m(\tau)}{\partial \tau^4} + \left(\frac{5 - 3n}{7 - 5n} \right) \tilde{f}_m(\tau) \frac{\partial^3 \tilde{f}_m(\tau)}{\partial \tau^3} \right.$$
$$- \left(\frac{n+1}{7-5n} \right) \frac{\partial \tilde{f}_m(\tau)}{\partial \tau} \frac{\partial^2 \tilde{f}_m(\tau)}{\partial \tau^2} - K_1 \left[\left(\frac{2(n-1)}{7-5n} \right) \tau \frac{\partial \tilde{\theta}_m(\tau)}{\partial \tau} + \left(\frac{4}{7-5n} \right) \tilde{\theta}_m(\tau) \right]$$
$$\left. - K_2 \left[\left(\frac{2(n-1)}{7-5n} \right) \tau \frac{\partial \tilde{\phi}_m(\tau)}{\partial \tau} + \left(\frac{4}{7-5n} \right) \tilde{\phi}_m(\tau) \right] \right] d\tau, \tag{31}$$

$$\theta_{m+1}(\eta) = \theta_m(\eta) + \int_0^\eta (\tau - \eta) \left[\frac{1}{\Pr} \frac{\partial^2 \theta_m(\tau)}{\partial \tau^2} + \left(\frac{5 - 3n}{7 - 5n} \right) \tilde{f}_m(\tau) \frac{\partial \tilde{\theta}_m(\tau)}{\partial \tau} \right.$$
$$\left. - \left(\frac{4}{7-5n} \right) \frac{\partial \tilde{f}_m(\tau)}{\partial \tau} \tilde{\theta}_m(\tau) \right] d\tau, \tag{32}$$

$$\phi_{m+1}(\eta) = \phi_m(\eta) + \int_0^\eta (\tau - \eta) \left[\frac{1}{Sc} \frac{\partial^2 \phi_m(\tau)}{\partial \tau^2} + \left(\frac{5 - 3n}{7 - 5n} \right) \tilde{f}_m(\tau) \frac{\partial \tilde{\phi}_m(\tau)}{\partial \tau} \right.$$
$$\left. - \left(\frac{4}{7-5n} \right) \frac{\partial \tilde{f}_m(\tau)}{\partial \tau} \tilde{\phi}_m(\tau) - K_3 \tilde{\phi}_m^n(\tau) \right] d\tau, \tag{33}$$

Now we must start with arbitrary initial approximations such that they satisfy the boundary condition. Therefore according to Equation (11) it is straight-forward to choose power initial guesses

$$f_0(\eta) = 1 - e^{-\eta}, \tag{34}$$

$$\theta_0(\eta) = e^{-\eta}, \tag{35}$$

$$\phi_0(\eta) = e^{-\eta}, \tag{36}$$

We use the symbolic software MATHEMATICA to solve the system of integrality equations, Equations (31)–(33), with the initial functions (34)–(36), and successively obtain $f_1(\eta)$, $\theta_1(\eta)$ and $\phi_1(\eta)$. In the same way, we can obtain $f_2(\eta)$, $\theta_2(\eta)$, $\phi_2(\eta)$ and etc.

5. RESULT AND DISCUSSION

Figure 1 shows the boundary-layers, in this case, there are three types of boundary-layer, concentration boundary-layer, thermal boundary-layer and velocity boundary-layer. The velocity, temperature and concentration profiles are obtained by the

8th-order approximation of the VIM and the influence of various physical parameters of the problem on these distributions are discussed in detail and are illustrated graphically through a set of graphs.

From Figure 2 it can be seen that when the first buoyancy parameter (due to temperature gradient) increases, velocity for small values of spanwise coordinate increases and for large values of spanwise coordinate decreases. An increase in first buoyancy parameter leads to a decrease in the thickness of temperature and concentration boundary-layer. These behaviors are clear from Figures 3 and 4. From Figure 5, it observed that for all values of η when the second buoyancy parameter (due to concentration gradient) increase, the velocity increases. Table 1 shows this clearly. Also an increase in second buoyancy parameter leads to a decrease in the thickness of temperature and concentration boundary-layer. These behaviors are obvious from temperature profiles and concentration profiles presented in Figures 6 and 7.

Figures 8–10 shows the effect of chemical reaction parameter on velocity, temperature and concentration profiles, it is clear that with an increasing of K_3 value of $f'(\eta)$ is decreased; we put this result in Table 2.

Figures 11–13 display influence of the order of chemical reaction on the velocity, temperature and concentration profiles, it is clear that with an increasing of n from 1 to 3 all values of $f'(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ are increased, but from 3 to 5 all of these values are decreased; that is shown in Table 3.
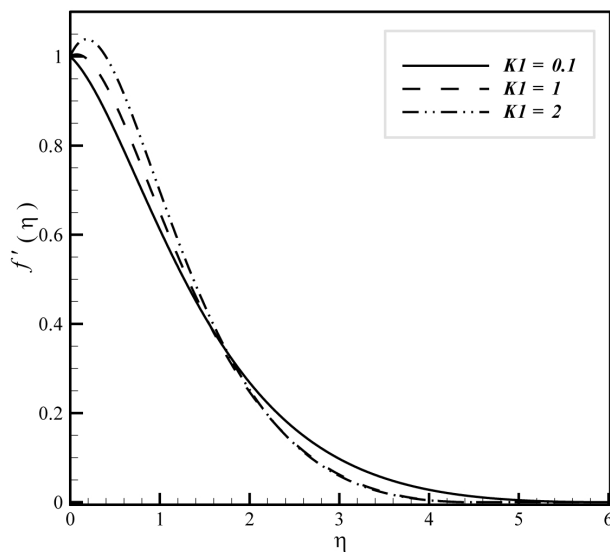

Figure 2
Velocity Profile, $f'(\eta)$ Versus Spanwise Coordinate (η) for Various Buoyancy Parameter, $(K_1 = 0.1, 1, 2)$ when $n = 1$, $K_2 = 1$, $K_3 = 0.5$, $Pr = 1$, **and** $Sc = 0.5$

Figure 3
Temperature Profile, $\theta(\eta)$ Versus Spanwise Coordinate (η) for Various Buoyancy Parameter, ($K_1 = 0.1, 1, 2$) when $n = 1$, $K_2 = 1$, $K_3 = 0.5$, $Pr = 1$, and $Sc = 0.5$
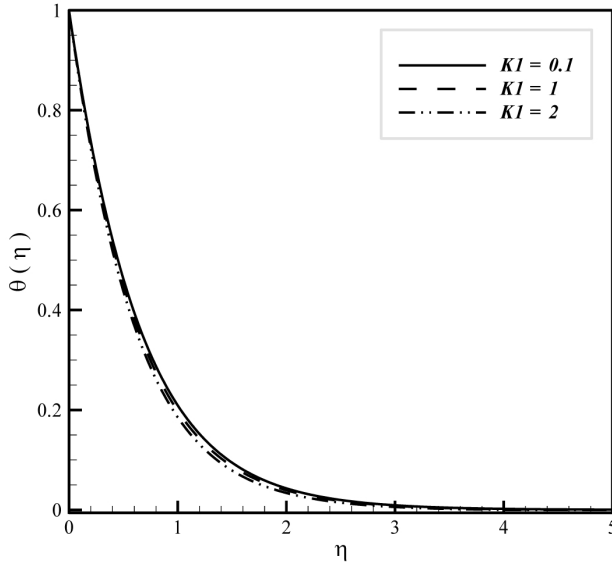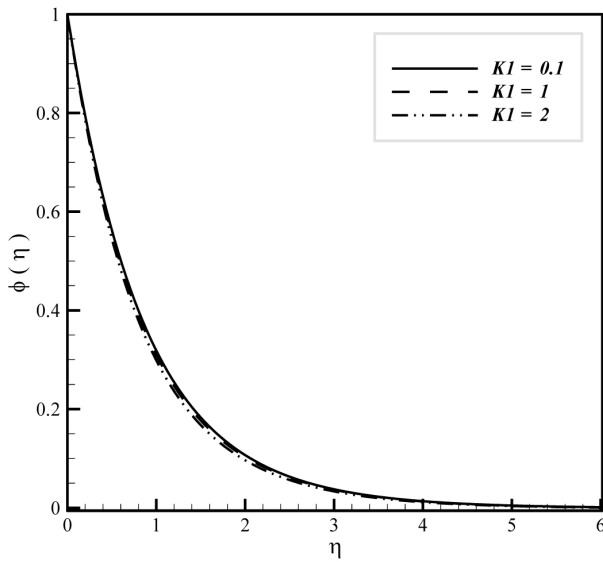

Figure 4
Concentration Profile, $\phi(\eta)$ Versus Spanwise Coordinate (η) for Various Buoyancy Parameter, ($K_1 = 0.1, 1, 2$) when $n = 1$, $K_2 = 1$, $K_3 = 0.5$, $Pr = 1$, and $Sc = 0.5$
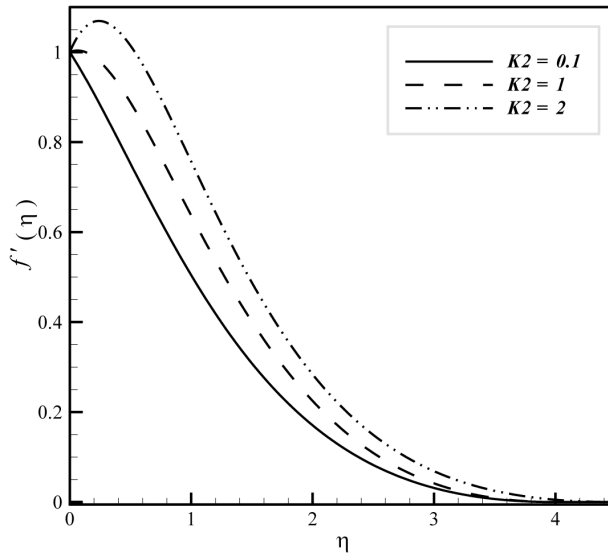
Figure 5
Velocity Profile, $f'(\eta)$ Versus Spanwise Coordinate (η) for Various Buoyancy Parameter, ($K_2 = 0.1, 1, 2$) when $n = 1$, $K_1 = 1$, $K_3 = 0.5$, $Pr = 1$, and $Sc = 0.5$
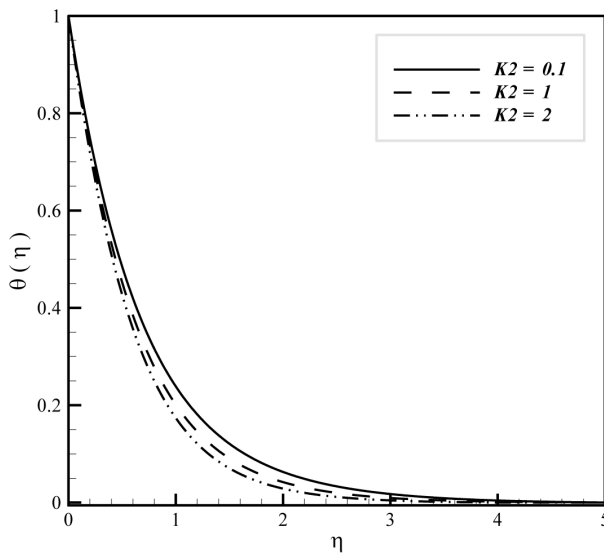

Figure 6
Temperature Profile, $\theta(\eta)$ Versus Spanwise Coordinate (η) for Various Buoyancy Parameter, ($K_2 = 0.1, 1, 2$) when $n = 1$, $K_1 = 1$, $K_3 = 0.5$, $Pr = 1$, and $Sc = 0.5$
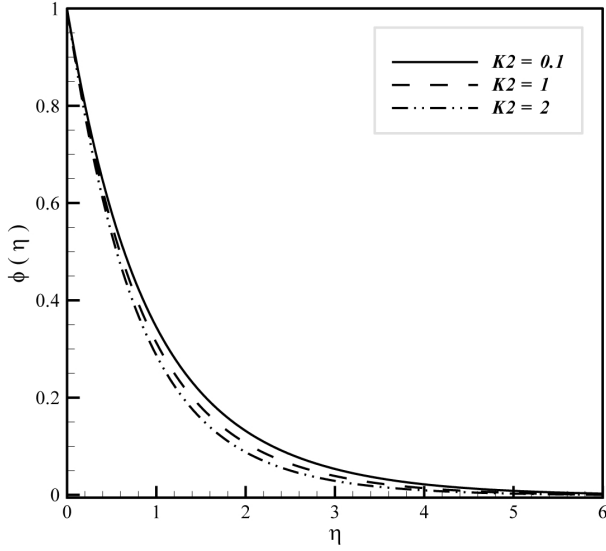
Figure 7
Concentration Profile, $\phi(\eta)$ Versus Spanwise Coordinate (η) for Various Buoyancy Parameter, ($K_2 = 0.1, 1, 2$) when $n = 1$, $K_1 = 1$, $K_3 = 0.5$, $Pr = 1$, and $Sc = 0.5$
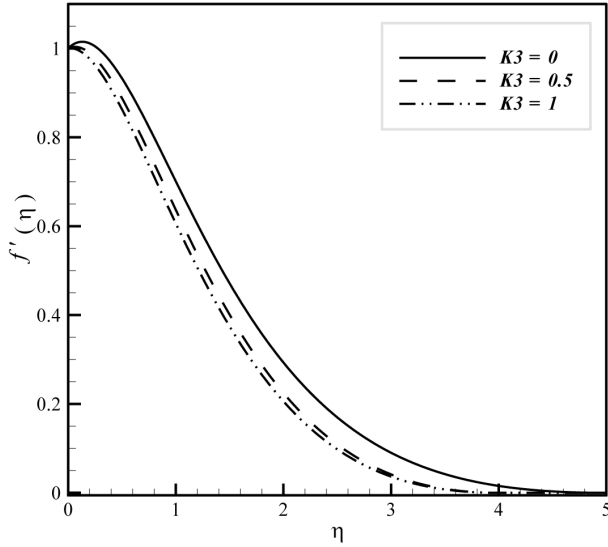

Figure 8
Velocity Profile, $f'(\eta)$ Versus Spanwise Coordinate (η) for Various Chemical Reaction Parameter, ($K_3 = 0, 0.5, 1$) when $n = 1$, $K_1 = K_2 = 1$, $Pr = 1$, and $Sc = 0.5$

Figure 9
**Temperature Profile, $\theta(\eta)$ Versus Spanwise Coordinate (η) for Various
Chemical Reaction Parameter, $(K_3 = 0, 0.5, 1)$ when $n = 1$, $K_1 = K_2 = 1$,
$Pr = 1$, and $Sc = 0.5$**

Figure 10
**Concentration Profile, $\phi(\eta)$ Versus Spanwise Coordinate (η) for Various
Chemical Reaction Parameter, $(K_3 = 0, 0.5, 1)$ when $n = 1$, $K_1 = K_2 = 1$,
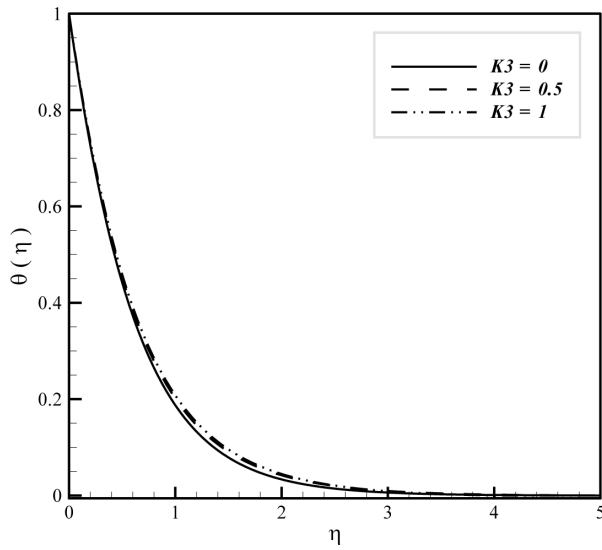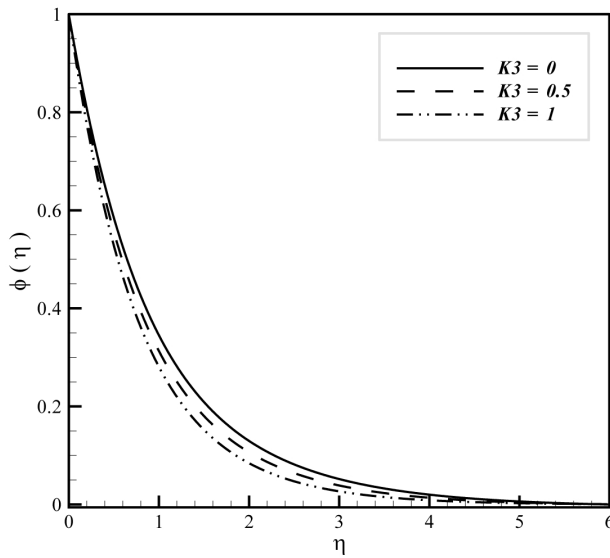$Pr = 1$, and $Sc = 0.5$**

Figure 11
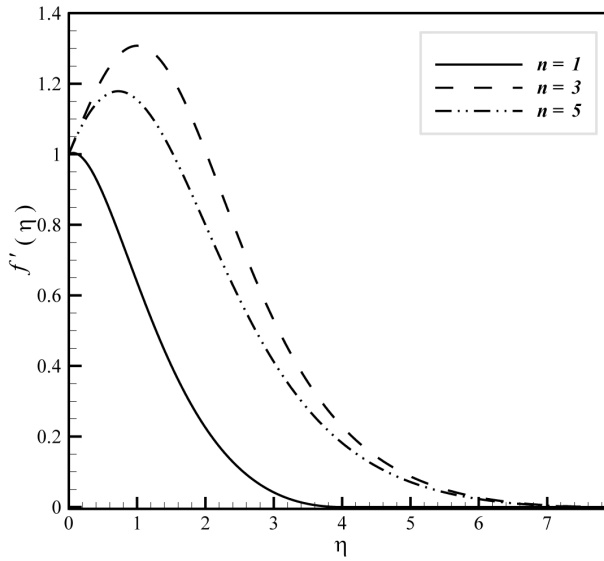**Velocity Profile, $f'(\eta)$ Versus Spanwise Coordinate (η) for Various Order of Chemical Reaction, ($n = 1, 3, 5$) when $K_1 = K_2 = 1$, $K_3 = 0.5$, $Pr = 1$, and $Sc = 0.5$**

Figure 12
Temperature Profile, $\theta(\eta)$ Versus Spanwise Coordinate (η) for Various Order of Chemical Reaction, ($n = 1, 3, 5$) when $K_1 = K_2 = 1$, $K_3 = 0.5$, $Pr = 1$, and $Sc = 0.5$

Figure 13
Concentration Profile, $\phi(\eta)$ Versus Spanwise Coordinate (η) for Various Order of Chemical Reaction, ($n = 1, 3, 5$) when $K_1 = K_2 = 1$, $K_3 = 0.5$, $Pr = 1$, and $Sc = 0.5$
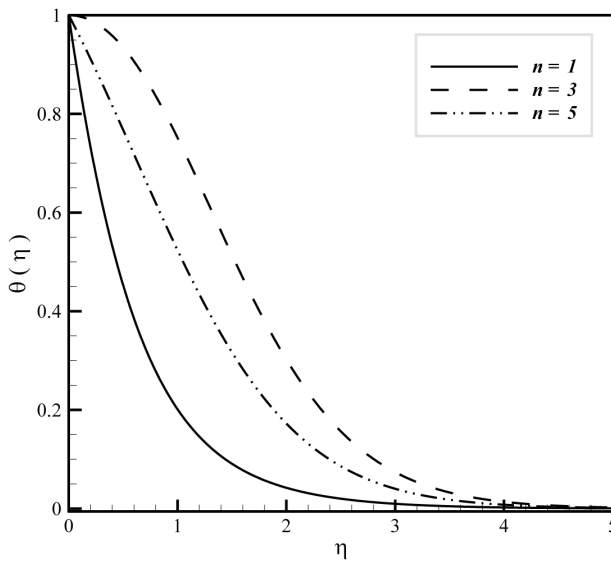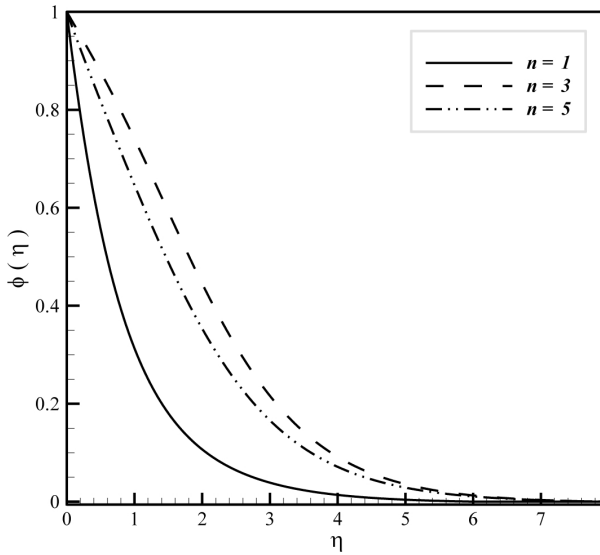
Table 1
Values of $f'(\eta)$ Obtained by the Numerical Method for Different Values of K_2 when $n = 1$, $K_1 = 1$, $K_3 = 0.5$, $Pr = 1$, and $Sc = 0.5$

η	$K_2 = 0.1$	$K_2 = 1$	$K_2 = 2$
0.5	0.752318	0.888225	1.01459
1.5	0.308266	0.401044	0.48788
2.5	0.082255	0.109012	0.149207
3.0	0.0312748	0.0415249	0.0685669
3.5	0.00669622	0.00889407	0.0249377
4.0	8.86262×10^{-8}	2.60744×10^{-7}	0.00513038

6. CONCLUSION

In this letter, the variational iteration method was used for finding the analytic solutions of the system of nonlinear ordinary differential equations derived from similarity transform for the non-viscous, laminar mixed convection boundary-layer flow over a horizontal moving porous flat plate, with chemical reaction. The VIM was used in a direct way without using linearization, perturbation or restrictive assumptions. The method requires less computational work than existing approaches while supplying quantitatively reliable results. Consequently, the present success of the VIM for the highly nonlinear problem of mixed convection boundary-layer

Table 2
Values of $f'(\eta)$ Obtained by the Numerical Method for Different Values of K_3 when $n = 1$, $K_1 = K_2 = 1$, $Pr = 1$, and $Sc = 0.5$

η	$K_3 = 0$	$K_3 = 0.5$	$K_3 = 1$
0.5	0.932821	0.888225	0.863023
1.5	0.471403	0.401044	0.37456
2.5	0.169881	0.109012	0.0979468
3.0	0.0902226	0.0415249	0.0365731
3.5	0.0421181	0.00889407	0.00767876
4.0	0.0155745	2.60744×10^{-7}	1.10409×10^{-7}

Table 3
Values of $f'(\eta)$ Obtained by the Numerical Method for Different Values of n when $K_1 = K_2 = 1$, $K_3 = 0.5$, $Pr = 1$, and $Sc = 0.5$

η	$n = 1$	$n = 3$	$n = 5$
0.5	0.888486	1.21758	1.16146
1.5	0.401471	1.22182	1.01003
2.5	0.109282	0.756287	0.588993
3.0	0.041669	0.52934	0.411802
3.5	0.00893545	0.353544	0.277524
4.0	2.75596×10^{-7}	0.228307	0.181648

flow verifies that the method is a useful tool for nonlinear problems in science and engineering.

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APPENDIX

Nomenclature

u	velocity components in x-direction
v	velocity components in y-direction
$\overline{C}$	concentration
Pr	Prandtl number, $Pr \equiv v/\alpha$
Sc	Schmidt number, $Sc \equiv v/D$
D	mass diffusivity
g	gravitational acceleration
f	dimensionless stream function
n	order of chemical reaction
T	temperature
k	thermal conductivity
K_1, K_2	Buoyancy parameters
K_3	chemical reaction parameter
x	coordinate along a cone ray
y	coordinate normal to cone surface

Greek symbols

α	thermal diffusivity
β_T	coefficient of thermal expansion
β_C	coefficient of concentration expansion
η	similarity coordinate
ψ	stream function
ξ	streamwise coordinate
μ	dynamic viscosity
v	kinematic viscosity
ρ	density of the gas
θ	dimensionless temperature
ϕ	dimensionless concentration

Subscript

∞	condition at infinity
w	condition at the wall
T	denote to temperature
C	denote to concentration