# Measles Trends Dynamic Forecasting Model Based on Grey System Theory 

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#### Abstract

Measles is a kind of acute respiratory infectious disease, one of the most common acute respiratory diseases among children, which seriously harms children's health with highly infectious. Its popularity within the community can be formed without universal vaccination in the densely populated areas. Therefore, the establishment of early warning mechanisms of measles to predict trends of measles, to provide scientific basis for decision making for the relevant departments to prevent and control measles, is a significant public health work. A forecast method of measles trends was given based on information mining and grey system theory. Using this method, combined with China's actual situation, an evaluation of a measles epidemic trends gray system GM $(1,1)$ model was constructed. The model evaluates and predicts the trends of the incidence of measles in China well.


Key words: Grey System Theory; Dynamic prediction; Measles

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## INTRODUCTION

Measles is a kind of acute respiratory infectious diseases, to which measles vaccination is the most efficient way. Measles is the most common acute respiratory disease among children, which seriously harms children's health with highly infectious. Its popularity within the community can be formed without universal vaccination in the densely populated areas, with a high prevalence in every 2-3 years. It clinically present with fever, upper respiratory inflammation and eye conjunctivitis, and it is characterized by red maculopapule to the skin, measles mucosa spot on buccal mucosa, and Legacy pigmentation with chaff bran sample desquamation after rash withdrawing. Measles patients are the only source of infection, and Children turn to be infectious from the $7^{\text {th }}$ day after contacting with the measles to $5^{\text {th }}$ days after a rash. The virus existing in the secretions of eye conjunctiva, oral cavity, nasal cavity, pharynx and trachea, etc., can be spread by coughing, speaking and sneezing. This disease is highly infectious with more than $90 \%$ susceptible contact incidence. According to statistics, highest incidence occurred among children from 1 to 5 -year-old. Live attenuated measles vaccine can reduce morbidity, while the onset age extended backwards because of its unabiding immunity. At present, most incidences can be found in unvaccinated preschool children, teenagers and young people with immunity failure, which even can form the popularity in communities. Therefore, the establishment of early warning mechanisms of measles to predict trends of measles, to provide scientific basis for decision making for the relevant departments to prevent and control measles, is a significant public health work.

This paper is based on information mining and gray system theory, giving a measles epidemic trend analysis method. A measles epidemic trends GM $(1,1)$ model based on information mining and gray system theory was constructed combing with China's actual situation, and was used to analyze and forecast the incidence of measles trends.

## 1. THEORY AND METHODS

Gray system theory ${ }^{[1]}$ was put forward earliest based on the macro forecast and decision about society and economy, in which GM $(1,1)$ model was the most widely used grey model.

### 1.1 The Traditional Modeling Method of GM $(1,1)$ Model

Setting time series for $t=\left\{t_{1}, t_{2}, \ldots, t_{\mathrm{n}}\right\}$, the corresponding original data sequence is

$$
\begin{equation*}
x^{(0)}=\left\{x^{(0)}\left(t_{1}\right), x^{(0)}\left(t_{2}\right), \ldots, x^{(0)}\left(t_{\mathrm{n}}\right)\right\} \tag{1}
\end{equation*}
$$

Setting $\Delta t_{k}=t_{k}-t_{k-1}$, when $\Delta t_{k}=$ const, sequence (1) is equal-space sequence. When $\Delta t_{k} \neq$ const, sequence (1) is non-equal-space sequence. One-accumulated generate sequence of original data sequence (1) is

$$
\begin{equation*}
x^{(1)}=\left\{x^{(1)}\left(t_{1}\right), x^{(1)}\left(t_{2}\right), \ldots, x^{(1)}\left(t_{\mathrm{n}}\right)\right\} \tag{2}
\end{equation*}
$$

Wherein

$$
\left\{\begin{array}{c}
x^{(1)}\left(t_{1}\right)=x^{(0)}\left(t_{1}\right)  \tag{3}\\
x^{(1)}\left(t_{k}\right)=x^{(1)}\left(t_{1}\right)+\sum_{i=2}^{k} x^{(0)}\left(t_{i}\right) \Delta t_{i} \\
k=2,3, \cdots, n
\end{array}\right.
$$

Calculation formulas for reverting one-accumulated generate sequence (2) to original data sequence (1) is

$$
\left\{\begin{array}{c}
x^{(0)}\left(t_{k}\right)=\frac{x^{(1)}\left(t_{k}\right)-x^{(1)}\left(t_{k-1}\right)}{t_{k}-t_{k-1}}  \tag{4}\\
k=2,3, \cdots, n
\end{array}\right.
$$

When one-accumulated generate sequence (2) was close to inhomogeneous exponential law change, the response function of sequence (2) was the solution of differential equation (5).

$$
\left\{\begin{array}{c}
\frac{d x^{(1)}(t)}{d t}+a x^{(1)}(t)=b  \tag{5}\\
x^{(1)}\left(t_{1}\right)=x^{(0)}\left(t_{1}\right)
\end{array}\right.
$$

The solution was $x^{(1)}(t)=\left(x^{(0)}\left(t_{1}\right)-\frac{b}{a}\right) e^{-a\left(t-t_{1}\right)}+\frac{b}{a}$, in which unknown constants $a$ and $b$ were uncertain parameters. Discrete response function of sequence (2) was

$$
\left\{\begin{array}{c}
x^{(1)}\left(t_{1}\right)=x^{(0)}\left(t_{1}\right)  \tag{6}\\
x^{(1)}\left(t_{k}\right)=\left(x^{(0)}\left(t_{1}\right)-\frac{b}{a}\right) e^{-a\left(t_{k}-t_{1}\right)}+\frac{b}{a}
\end{array}\right.
$$

In equation (6), $k=2,3, \ldots, n$. To determine uncertain parameters $a$ and $b$, we could use difference equation (5).

$$
\left\{\begin{array}{c}
\frac{\Delta x^{(1)}\left(t_{k}\right)}{\Delta t_{k}}+a x^{(1)}\left(t_{k}\right)=b  \tag{7}\\
x^{(1)}\left(t_{1}\right)=x^{(0)}\left(t_{1}\right)
\end{array}\right.
$$

Wherein, with $z^{(1)}\left(t_{\mathrm{k}}\right)=\lambda x^{(1)}\left(t_{\mathrm{k}}\right)+(1-\lambda) x^{(1)}\left(t_{\mathrm{k}-1}\right)$ smoothing $x^{(1)}\left(t_{k}\right)$ of difference equation (7), we could get difference equation after smoothing.

$$
\left\{\begin{array}{c}
\frac{\Delta x^{(1)}\left(t_{k}\right)}{\Delta t_{k}}+a z^{(1)}\left(t_{k}\right)=b  \tag{8}\\
x^{(1)}\left(t_{1}\right)=x^{(0)}\left(t_{1}\right)
\end{array}\right.
$$

In above formula, $z^{(1)}\left(t_{k}\right)$ was called as background value and $l \in[0,1]$ was called as background parameters. At present, there is still no optimum getter for background parameters $l$, in order to be used simply and easily, we generally take background parameters for $1 / 2$ in Reference ${ }^{[2],[3]}$. Substituting one-accumulated generates sequence (2) of original data sequence (1) into above formula, with matrix equation $a$ and $b$ could be determined:

$$
\left[\begin{array}{l}
a \\
b
\end{array}\right]=\left(B^{T} B\right)^{-1} B^{T} Y
$$

Inside $Y=\left[x^{(0)}\left(t_{2}\right), x^{(0)}\left(t_{3}\right), \ldots, x^{(0)}\left(t_{n}\right)\right]^{T}$, $B^{T}=\left[\begin{array}{cccc}-z^{(1)}\left(t_{2}\right) & -z^{(1)}\left(t_{3}\right) & \cdots & -z^{(1)}\left(t_{n}\right) \\ 1 & 1 & \cdots & 1\end{array}\right]$.

Substituting obtained parameters $a$ and $b$ into equation (6), we could get grey system GM $(1,1)$ model of original data sequence $x^{(0)}$ :

$$
\left\{\begin{array}{l}
\hat{x}^{(0)}\left(t_{1}\right)=x^{(0)}\left(t_{1}\right)  \tag{9}\\
\hat{x}^{(0)}\left(t_{k}\right)=\frac{\hat{x}^{(1)}\left(t_{k}\right)-\hat{x}^{(1)}\left(t_{k-1}\right)}{t_{k}-t_{k-1}} \\
\hat{x}^{(1)}\left(t_{k}\right)=\left(x^{(0)}\left(t_{1}\right)-\frac{b}{a}\right) e^{-a\left(t_{k}-t_{1}\right)}+\frac{b}{a}
\end{array}\right.
$$

### 1.2 The Modeling Method of GM $(1,1)$ Model Based on Information Mining

The traditional modeling method of GM $(1,1)$ model had the advantages of simple computation, but its fitting and forecast precision sometimes was poor. Integrating solving parameters and determining boundary value together to discuss (ZHANG, 2007, 2009), we put forward a method based on information mining. GM $(1,1)$ model with this modeling method both can greatly improve the fitting and prediction precision of GM $(1,1)$ model, and keep the advantage of simple computation in the traditional modeling method of GM $(1,1)$ model. For the convenience of the reader, the following is a brief introduction of this method. Firstly, by using the modeling method of traditional (equal-space sequence and non-equal-space sequence) grey system GM $(1,1)$ model, we got grey system GM $(1,1)$ model (9) of original data sequence $x^{(0)}$, then we could call model (9) as rough model.

Finish machining of rough model (9) namely rewrote
third formulas of rough model (9)

$$
\begin{equation*}
\hat{x}^{(1)}\left(t_{k}\right)=\alpha e^{-a\left(t_{k}-t_{1}\right)}+\beta \quad k=2,3, \ldots, n \tag{10}
\end{equation*}
$$

Wherein, $a$ and $b$ were new uncertain parameters.
By using the modeling method of traditional (equalspace sequence and non-equal-space sequence) grey system GM $(1,1)$ model, parameters $a$ could be gotten, then using the accumulated generate sequence and corresponding time series of original data sequence again. Substituting the accumulated generate sequence $x^{(1)}=\left\{x^{(1)}\left(t_{1}\right), x^{(1)}\right.$ $\left.\left(t_{2}\right), \ldots, x^{(1)}\left(t_{\mathrm{n}}\right)\right\}$ and corresponding time series of original data sequence $t=\left\{t_{1}, t_{2}, \ldots, t_{\mathrm{n}}\right\}$ into above formula, we could determine uncertain parameters $a$ and $b$ with matrix equation of linear algebra ${ }^{[4]}$ :

$$
\left[\begin{array}{l}
\alpha \\
\beta
\end{array}\right]=\left(B^{T} B\right)^{-1} B^{T} Y
$$

Wherein $Y=\left[x^{(1)}\left(t_{1}\right), x^{(1)}\left(t_{2}\right), \ldots, x^{(1)}\left(t_{\mathrm{n}}\right)\right]^{T}$,
$B^{T}=\left[\begin{array}{cccc}1 & e^{-a\left(t_{2}-t_{1}\right)} & \cdots & e^{-a\left(t_{n}-t_{1}\right)} \\ 1 & 1 & \cdots & 1\end{array}\right]$.
Substituting parameters $a$ and $b$ into equation (10), we got new grey system GM $(1,1)$ model of original data sequence $x^{(0)}$ :

$$
\begin{equation*}
\hat{\hat{x}}^{(1)}\left(t_{k}\right)=\alpha e^{-a\left(t_{k}-t_{1}\right)}+\beta \quad k=1,2, \ldots, n \tag{11}
\end{equation*}
$$

Table 1
Original Data

|  | Time (year) | The number of incidence (unit: person) | Time (year) |
| :--- | :---: | :---: | :---: |
| 1976 | 2563722 | The number of incidence (unit: person) |  |
| 1986 | 203940 | 1981 | 1015331 |
| 1996 | 76738 | 2001 | 123442 |
| 2002 | 61143 | 2003 | 91253 |
| 2004 | 70583 | 2005 | 123172 |
| 2006 | 100163 |  |  |

The measles epidemic forecast trends gray system
$(1,1)$ model based on information mining method, is GM $(1,1)$ model based on information mining method, is $\hat{\hat{x}}^{(1)}\left(t_{k}\right)=-15720626 e^{-0.206161308\left(t_{k}-t_{1}\right)}+18428783$, wherein $k=$ $1,2, \ldots$, and n .

Figure 1 is a diagram of the curve between the statistics of China from 1976 to 2006 the number of measles incidence and its analog data.


Figure 1
The Tendency Curve of Incidence of Measles with Time

One-inverse accumulated generating above formula, we could get the reducing value of original data sequence $x^{(0)}$

$$
\left\{\begin{array}{cc}
\hat{\hat{x}}^{(1)}\left(t_{k}\right)=\alpha e^{-a\left(t_{k}-t_{1}\right)}+\beta & (k=1,2, \cdots, n)  \tag{12}\\
\hat{x}^{(0)}\left(t_{k}\right)=\frac{\hat{\hat{x}}^{(1)}\left(t_{k}\right)-\hat{\hat{x}}^{(1)}\left(t_{k-1}\right)}{t_{k}-t_{k-1}} & (k=2,3, \cdots, n)
\end{array}\right.
$$

## 2. $G M(1,1)$ MODEL OF MEASLES TRENDS EVALUATION BASED ON INFORMATION MINING

The analysis method of measles trends based on information mining and grey system theory was that: with original data, firstly, modeling GM $(1,1)$ model of measles trends by using grey system theory; then, on the basis of construction grey system GM $(1,1)$ model of measles trends, we constructed GM $(1,1)$ model of measles trends evaluation based on information mining by using the method of information mining.

Table 1 shows the statistical number of measles incidence in China from 1976 to 2006. Data came from Chinese macro data mining system and China health statistical yearbook of CNKI. To save space, we only gave a few representative data.

From 1976 to 2006 the number of measles incidence over time trend graph can be seen, the incidence of measles in China from 1976 to 2006, the number of declining trend. 1976 to 1988, the annual incidence of a downward trend in the number is very obvious, which is stable after 1988.

Figure 2 is a curve diagram between the statistics of China from 1976 to 1996, and the incidence of measles in the number of its analog data.


Figure 2
The Tendency Curve of Incidence of Measles with Time

Figure 3 is the curve between the incidence of measles from 1996 to 2006 the number of statistics and its analog data graph.


Figure 3
The Number of Measles Incidence Trend Versus Time
The number of measles incidence in China showed a slight upward trend, which can be seen thorough the incidence of measles from 1996 to 2006, the number of changes over time trend graphs.

Based on information mining measles epidemic forecast trends gray system $\operatorname{GM}(1,1)$ model $\hat{\hat{x}}^{(1)}\left(t_{k}\right)=-15720626 e^{-0.206161308\left(t_{k}-t_{t}\right)}+18428783$, we h ave forecasted the number of measles incidence in China in 2007-2012. Forecast results suggest that the number of measles incidence in China in 2007-2012 showed a slow upward trend. Specific data are shown in Table 2.

Table 2
Forecast Data

| Time <br> (year) | The number of <br> incidence (unit: <br> person) | Time <br> (year) | The number of <br> incidence (unit: <br> person) |
| :--- | :---: | :---: | :---: |
| 2007 | 97567 | 2008 | 102365 |
| 2009 | 107398 | 2010 | 112680 |
| 2011 | 118220 | 2012 | 124034 |

This may be associated with the frequent movement of the population, suggesting that we should strengthen measles vaccinations mobile populations.

## CONCLUSIONS

By using GM $(1,1)$ model based on the information mining, we gray fitted and dynamic forecasted the number of measles incidence statistics from 1976 to 2006 in

China. The results showed that the measles epidemic in China has been effectively controlled, not trends fulminate.

GM $(1,1)$ model is the most widely used forecasting model in gray dynamic model, mainly used for the fitting and forecasting of the Eigen values of a dominant factor in complex systems, in order to reveal the dominant factor variation and future developments and changes in posture. In the medical field, it is widely used in the prediction of the epidemic ${ }^{[5]}$. But any kind of prediction model has its limitations, GM $(1,1)$ model's shortcomings is that the fitting precision and prediction accuracy is sometimes poor.

This paper gives a modeling way based on information mining gray system GM $(1,1)$ model based on information mining and gray system theory. On the one hand, this way greatly improved GM $(1,1)$ model's fitting precision and prediction accuracy; on the other hand, it maintains the traditional method of modeling the advantage that GM $(1,1)$ model calculation method is simple. A kind of epidemic trend evaluation of GM $(1,1)$ model based on information mining measles was given by this way. The measles epidemic forecasting model was constructed combing with China's actual situation, and was used to analyze and forecast the incidence of measles trends. Case analysis verified the validity and usefulness of the information-based mining method.

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