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Heterogeneous Arrival and Departure M/M/1 Queue with Vacation and Service Breakdown

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Abstract
This paper deals with the study of M/M/1 queue with heterogeneous arrival and departure with the provision of server vacations and breakdowns. Customer arrive service facilities with poison process and exponential service time distribution. In this paper we find the mean queue length, mean waiting time in queue and system, average number of customers in the system. The generating function method is used to find these measures of performance. The numerical results are obtained to cite the applicability of model in the real life situations.

Key words: Heterogeneous; Generating function; Vacation; Breakdown; Repair

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INTRODUCTION

Several authors studied the queuing system with heterogeneous arrival and heterogeneous service. Ke and Wang[1] studied the characteristics for the heterogeneous batch arrival queue with server startup and breakdown and obtained the steady state behavior of the system size distribution at stationary point of time as well as the queue size distribution at departure point of time. Later Ke[2] made the contribution to the control policy of M/G/1 queue with server vacations, startup and breakdowns. The system characteristics of such a model are analyzed and the total expected cost function per unit time was developed to determine the optimal threshold of N policies at a minimum cost. Chan et al.[3] developed M/M/1 queue with service breakdowns and customer discouragement and calculated the expected queue size at the end of a working-repair cycle wherein the system is shown to have a stationary distribution if the probability of discouragement is positive. Liu et al.[4] demonstrated stochastic decomposition structures of the queue length and waiting time in an M/M/1/WV queue, and obtained the distributions of the additional queue length and additional delay. Also they established the relationship between the stochastic decomposition properties of the working vacation queue and those of the standard M/G/1 queue with general vacations. Omey and Gulck[5] worked on multiple vacations and server breakdowns. Their approach consists of maximizing an entropy function subject to constraints, where the constraints are formed by some known exact results. Haridass and Arumuganathan[6] analyzed the operating characteristics of an M X/ G / 1 queuing system with unreliable server and single vacation. They studied the model by the embedded Markov chain technique and level crossing analysis and obtained probability generating function of the steady state system size at an arbitrary time. They also derived expressions for the expected number of customers in the system, expected length of busy period and idle period. Servi and Finn[7] studied classical single server vacation model which was generalized to consider a server which works at a different rate rather than completely stops during the vacation period and presented the formulae for the mean, variance, and distribution of the number and time in the system. Frey and Takahash[8] considered an M/GI/1/N queue with vacation time and exhaustive service discipline focusing only on the service completion epochs and presented a simple analysis for the queue length distribution at an arbitrary time as well as for the waiting time distribution. Zhang et al.[9] treated two-threshold policies for an M/G/1 queue with two types of generally distributed random long and short vacations in which the server observes the queue length upon his returning from a vacation. Gray et al.[10]] analyzed a multiple-vacation queueing model, where the service station is subject to breakdown while in operation. Service resumes immediately after a repair process, and a vacation starts at the end of each busy period. Wartenhorst[11] studied the effects of machine breakdown and limited repair capacity on the performance of a system that had to provide service continuously. He considered a system consisting of N stations, each serving its own stream of customers. Leung and Lucantoni[12] gave the insight into two vacation models with constant time-limited service and vacation-dependent, time-limited service for the performance analysis of stations in timed-token networks. They employed the matrix analytic method to solve the vacation models for the queue length distribution and the moments of the sojourn time. Reddy et al.[13] derived the system size distribution, expected length of idle and busy period of a MX/G (a, b)/1 queuing system with N-policy with the provisions of multiple vacations and setup times. Wang[14] proposed state-dependent queuing model wherein the service rate is adjusted at both epochs of customer arrivals and departures .He also studied an alternative queuing model by using an embedded Markov chain technique in which the server changes its service rates, or service types, only at the beginning of service and obtained the probability generating functions and queue length for the steady-state condition. Buzacott[15] constructed the structure of a service system matched to the requirements of customers and made a categorization of service system based on an analysis of their relative performance. Choi et al.[16] considered an M/G/1 queuing system with multiple types of feedback, gated vacations and FCFS policy where the first service of a new customer is either successful or unsuccessful and customers are served in the order of joining the tail of the queue. By using Laplace Stieltjes transform they obtained joint probability generating function and the total response time of system for new and old customers at steady state condition. Gupta and Sikdar[17] studied a single server finite-buffer bulk-service queue and they obtained the distributions of the number of customers in the queue at arbitrary service completion and vacation termination epochs. Chang and Choi[18] developed a finite-buffer discrete-time GeoX/GY/1/K+B queue with multiple vacations and derived a set of linear equations to compute the steady-state departure-epoch probabilities based on the embedded Markov chain technique and presented numerically stable relationships for the steady-state probabilities of the queue lengths at three different epochs: departure, random, and arrival. Recently Park et al.[19] analyzed the single-server two-phase queueing system with a fixed-size batch policy and obtained the explicit formula for optimum batch size and cost of the system. Jain and Jain[20] proposed the multiple vacations queueing system with the provision of single server and gave the expressions for stationary queue length distribution, expected length of busy period, the expected length of working vacation period, the mean waiting time and average delay.

In this paper we study the M/M/1 queuing model where customers arrive in the system in Poisson process and they are served exponentially. When the system is either empty or the server itself breaks down we term it as the server vacation. For the queue length longer than N customers the server works with faster service rate and it works with service rate < when there are up to N customers in the system.

1. ASSUMPTIONS AND NOTATIONS

λ_v = Arrival rate during vacation

λ_a = Arrival rate during active service

λ_b = Arrival rate during breakdown

ν = vacation rate

μ_1= service rate when the queue length is $\leq$ N

μ_2 = service rate when the queue length is > N, $\mu_1 < \mu_2$

b = Breakdown rate

r = Repair rate

$\rho_1 = \frac{\lambda_a}{\mu_1}$ $(i \leq N)$

$\rho_2 = \frac{\lambda_a}{\mu_2}$ $(i > N)$

$\rho_0 = \frac{\lambda_v}{\lambda_v + \nu}$

Where $\lambda_v, \lambda_a, \lambda_b, \mu_1, \mu_2, b, r, \nu \geq 0$

2. MATHEMATICAL MODEL

The states of mode are as follows:

(0, i) is the state in which i customers in the queue and the server is on the vacation,

$i \geq 0$. Its probability is P (0, i)

(1, i) is the state in which i customers in the system during active service $i \geq 1$. Its probability is P (1, i)

(2, i) is the state in which i customers in the system during repair process, $i \geq 1$. Its probability is P (2, i)

The following partial generating functions are used for our model

$$F_0(z) = \sum_{i=0}^{\infty} P(0,i)z^i = \sum_{i=0}^{N} P(0,i)z^i + \sum_{i=N+1}^{\infty} P(0,i)z^i = F_{01} + F_{02}$$

$$F_1(z) = \sum_{i=1}^{\infty} P(1,i)z^i = \sum_{i=1}^{N} P(1,i)z^i + \sum_{i=N+1}^{\infty} P(1,i)z^i = F_{11} + F_{12}$$

$$F_2(z) = \sum_{i=0}^{\infty} P(2,i)z^i = \sum_{i=1}^{N} P(2,i)z^i + \sum_{i=N+1}^{\infty} P(2,i)z^i = F_{21} + F_{22}$$

The balance equations for the queue length distribution are:

$\lambda_v P(0, 0) = \mu_1 P(1, 1)$ (1)

$(\nu + \lambda_v)P(0, i) = \lambda_v P(0, i-1);\ i \geq 1$ (2)

$(\lambda_\alpha + \mu_1 + b)P(1, 1) = \nu P(0, 1) + \mu_1 P(1, 2) + rP(2, 1)$ (3)

$(\lambda_\alpha + \mu_1 + b)P(1, i) = \lambda_\alpha P(1, i-1) + \nu P(0, i) + \mu_1 P(1, i + 1) + rP(2, i);\ 2 \leq i \leq N$ (4)

$(\lambda_\alpha + \mu_1 + b)P(1, N) = \lambda_\alpha P(1, N-1) + \nu P(0, N) + rP(2, N) + \mu_2 P(1, N + 1);\ i = N$ (5)

$(\lambda_\alpha + \mu_2 + b)P(1, i) = \lambda_\alpha P(1, i-1) + \nu P(0, i) + \mu_2 P(1, i + 1) + rP(2, i);\ \ i > N$ (6)

$(\lambda_b + r)P(2, 1) = bP(1, 1)$ (7)

$(\lambda_b + r)P(2, i) = bP(1, i) + \lambda_b P(2, i - 1);\ i \geq 2$ (8)

From the equation (1) we get

$P(1, 1) = \frac{\lambda_a}{\mu_1} P(0, 0)$

Substituting the value of P(1, 1) from (9) in the equation (7) we get

$(\lambda_b + r)P(2, 1) = \frac{b\lambda_v}{\mu_1} P(0, 0)$

$\therefore\ P(2, 1) = \frac{b\lambda_v}{\mu_1(\lambda_b + r)} P(0, 0)$ (10)

Again from equation (2)

$P(0, i) = \frac{\lambda_v}{(\lambda_v + \nu)} P(0, i-1)$

$\Rightarrow\ P(0, i) = \rho_0^i P(0,0)$ (11)

The generating functions:

We have

$F_0(z) = \sum_{i=0}^{\infty} P(0,i)z^i$

$F_0(z) = \frac{1}{1-\rho_0 z} P(0, 0)$ (12)

Multiplying equation (4) by zi and sum for i = 2, 3, 4,…, N - 1

$$(\lambda_\alpha + \mu_1 + b)\sum_{i=2}^{N-1} P(1,i)z^i = \lambda_a \sum_{i=2}^{N-1} P(1,i-1)z^i + \nu\sum_{i=2}^{N-1} P(0,i)z^i + \mu\sum_{i=2}^{N-1} P(1,i+1)z^i + r\sum_{i=2}^{N-1} P(2,i)z^i;\ 2 \leq i < N \quad (13)$$

Similarly, multiplying equation (5) by z^N

$(\lambda_\alpha + \mu_1 + b)P(1, N)z^N = \lambda_\alpha P(1, N-1)z^N + \nu P(0, N)zN + \mu_2 P(1, N + 1)z^N + rP(2, N)z^N;\ i = N$ (14)

Similarly, multiplying equation (6) by z^i and sum for i= N + 1, N + 2 …

$$(\lambda_\alpha + \mu_2 + b)\sum_{i=N+1}^{\infty} P(1,i)z^i = \lambda_a \sum_{i=N+1}^{\infty} P(1,i-1)z^i + \nu\sum_{i=N+1}^{\infty} P(0,i)z^i + \mu_2 \sum_{i=N+1}^{\infty} P(1,i+1)z^i + r\sum_{i=N+1}^{\infty} P(2,i)z^i$$

$i > N$ (15)

Now adding equations (13), (14) and (15) we get,

$$\Rightarrow (\lambda_\alpha + \mu_1 + b)F_1(z) - (\lambda_a + \mu_1 + b)p(1,1)z - (\mu_1 - \mu_2)F_{12}(z) = \lambda_\alpha z F_1(z) + \nu F_0(z) - \nu P(0,0) - \nu P(0,1)z + \frac{\mu_1}{z} F_1(z) - \mu_1 P(1,1) - \mu_1 P(1,2)z + \left(\frac{\mu_1}{z} - \frac{\mu_2}{z}\right) F_{12}(z) + rF_2(z) - rP(2,1)z \quad (16)$$

Again multiplying equation (8) by zi and sum for i = 2, 3, 4, ...

$\Rightarrow\ (\lambda_b + r)F_2(z) - \lambda_b z F_2(z) = bF_1(z) - bzP(1,1) + (\lambda_b + r)zP(2,1)$ (17)

From equation (7) we have

$(\lambda_b + r)P(2, 1) = bP(1, 1)$

$\therefore$ Equation (17) can be written as

$$F_2(z) = \frac{b}{\lambda_b + r - \lambda_b z} F_1(z) \quad (18)$$

Substituting the value of F_2 (z) from (18) into (16) we get

$$\frac{zQ(z) - Q(z)}{z(\lambda_b + r - \lambda_b z)} F_1(z) = z[(\lambda_a + \mu_1 + b)P(1,1) - \nu P(0,1) - \mu_1 P(1,2) - rP(2,1)] - [\nu P(0,0) + \mu_1 P(1,1)] + \frac{\nu}{1-\rho_0 z} P(0\ 0) + (\mu_1 - \mu_2)\left(1 - \frac{1}{z}\right) F_{12}(z) \quad (19)$$

Where, $Q(z) = \lambda_a\lambda_b z^2 - \lambda_a\lambda_b z - b\lambda_b z - \mu_1\lambda_b z - \lambda_a rz + \mu_1 r + \mu_1\lambda_b$ (20)

For the queue distribution the right hand side of (17) must be zero when z = 1. Since

P(1, 1), P(2, 1), P(0, 1) are given by (9), (10) and (11) we can find $\mu_1 P(1, 2)$ as follows:

When z = 1 in the equation (19) then

$$\mu_1 P(1, 2) = \frac{\lambda_v z(\lambda_b + r - \lambda_b z)}{(1-\rho_0 z)Q(z)} P(0, 0) + \frac{(\mu_1 - \mu_2)(\lambda_b + r - \lambda_b z)}{Q(z)} F_{12}(z) \quad (21)$$

Substituting the value of $\mu_1 P(1, 2)$ from (21) in (19) we get

$$\Rightarrow F_1(z) = \frac{\lambda_v z(\lambda_b + r - \lambda_b z)}{(1-\rho_0 z)Q(z)} P(0, 0) + \frac{(\mu_1 - \mu_2)(\lambda_b + r - \lambda_b z)}{Q(z)} F_{12}(z) \quad (22)$$

Now we determine the nature of roots of Q(z) for the positive λ_b. Referring to equation (20) is quadratic expression whose discriminates Δ satisfies

$\Delta \geq b^2 - 4ac$

$\Delta \geq (\lambda_b\mu_1 + \lambda_b b) + (\lambda_b\lambda_\alpha + \lambda_b r)^2 + 2(\lambda_b\mu_1 + \lambda_b b)(\lambda_b\lambda_\alpha + \lambda_b r) - 4\lambda_\alpha\lambda_b\mu_1(r + \lambda_b) = \lambda_b^2 b^2 + (\lambda_\alpha\lambda_b + \lambda_\alpha r + \lambda_b\mu_1)^2 > 0$

So that Q(z) has two distinct real roots.

In order for the steady state queue length distribution to exist both roots of the equation Q(z) = 0 must be greater than 1. Since in Q(z), the coefficient of z_2 is positive the roots of Q(z) = 0 will be greater than 1if and only if Q(1)> 0 and Q'(z) < 0

Here, $Q(1) = \lambda_\alpha\lambda_b - \lambda_\alpha\lambda_b - b\lambda_b - \mu_1\lambda_b - \lambda_\alpha r - \mu_1 r + \mu_1\lambda_b$

$\Rightarrow\ Q(1) = \mu_1 r - b\lambda_b - \lambda_\alpha r \geq 1$

We must assume that

$\mu_1 r \geq b\lambda_b + \lambda_\alpha r$

$$\Rightarrow \frac{b\lambda_b}{\mu_1 r} + \frac{\lambda_a}{\mu_1} < 1 \quad (23)$$

Now (22) implies that $\mu_1 > \lambda_\alpha$, so if (22) holds then

$\Rightarrow\ Q(1) = \lambda_b(\lambda_\alpha - \mu_1) - b\lambda_b - \lambda_\alpha r < 0$

Thus if we assume that (23) holds then the roots z_1 and z_2 of Q(z) = 0 will be greater than 1.

Using the equations (12), (22) and (18) in the generating function for the queue length distribution, we get

$F(z) = F_0(z) + F_1(z) + F_2(z)$

$$\therefore F(z) = \frac{Q(z) + \lambda_v z(\lambda_b + r + b - \lambda_b z)}{(1-\rho_0 z)Q(z)} P(0,0) + \frac{(\mu_1 - \mu_2)(\lambda_b + r - \lambda_b z + b)}{Q(z)} F_{12}(z) \quad (24)$$

From (24) and normalizing condition F(1) = 1, we get

$$\Rightarrow \text{ P}(0,0) = \frac{(\mu_1 r - b\lambda_b - r\lambda_a)(1-\rho_0) - (\mu_1 - \mu_2)(r+b)(1-\rho_0)F_{12}(1)}{\mu_1 r - b\lambda_b - \lambda_a r + \lambda_v(r+b)} \quad (25)$$

Now as,

$$Q(z) = \lambda_a\lambda_b z^2 - \lambda_a\lambda_b z - b\lambda_b z - \mu_1\lambda_b z - \lambda_a rz + \mu_1 r + \mu_1\lambda_b = 0$$

$$\Rightarrow \mu_1(r+\lambda_b)\frac{1}{z^2} - (\lambda_a\lambda_b + b\lambda_b + \mu_1\lambda_b + \lambda_a r)\frac{1}{z} + \lambda_a\lambda_b = 0 \quad (26)$$

is quadratic in $\frac{1}{z}$ and the factors of the above equation are

$$\mu_1(r+\lambda_b)\left(\frac{1}{z}-\frac{1}{z_1}\right)\left(\frac{1}{z}-\frac{1}{z_2}\right) = 0$$

Where $\frac{1}{z_1}$ and $\frac{1}{z_2}$ are the roots of the above equation (24)

$$\Rightarrow \mu_1(r+\lambda_b)\left(\frac{1}{z}-\alpha\right)\left(\frac{1}{z}-\beta\right) = 0$$

For z = 1

$$\mu_1(r+\lambda_b)(1-\alpha)(1-\beta) = 0 \quad (27)$$

Here from (24), we have

$$\Rightarrow \text{ P}(0,0) = \frac{F(z)\,Q(z) - (\mu_1-\mu_2)(\lambda_b + r - \lambda_b z + b)F_{12}(z)}{Q(z) + \lambda_v z(\lambda_b + r + b - \lambda_b z)} \times (1-\rho_0 z)$$

When z = 1 using the factors of Q(z) only in the numerator we get

$$\text{P}(0,0) = \frac{\mu_1(r+\lambda_b)(1-\alpha)(1-\beta)(1-\rho_0) - (\mu_1-\mu_2)(1-\rho_0)(r+b)F_{12}(1)}{\mu_1 r - b\lambda_b - r\lambda_a + \lambda_v(r+b)} \quad (28)$$

Now from (24) and (28), we get

$$F(z) = R(z)\left[\frac{\mu_1(r+\lambda_b)(1-\alpha)(1-\beta)(1-\rho_0) - (\mu_1-\mu_2)(r+b)(1-\rho_0)F_{12}(1)}{(1-\rho_0 z)Q(z)}\right] + \frac{(\mu_1-\mu_2)(\lambda_b + r - \lambda_b z + b)}{Q(z)}F_{12}(z) \quad (29)$$

$$\text{Where, } R(z) = \frac{Q(z) + \lambda_v z(\lambda_b + r + b - \lambda_b z)}{\mu_1 r - b\lambda_b - \lambda_a r + \lambda_v(r+b)} \quad (30)$$

Note that R(1) = 1

The case in which $\lambda_b = 0$ is also of interest. In this case no customers are admitted to the queue during a repair process. If $\lambda_b = 0$ then from (20)

$$\Rightarrow Q(z) = \mu_1 r(1-\rho_1 z) \quad (31)$$

Here in (31), when, Q(z) = 0

Then, $\mu_1 r(1-\rho_1 z) = 0$

$$\Rightarrow z = \frac{1}{\rho_1}$$

That is, Q(z) = 0 has a single root $\frac{1}{\rho_1}$

From (24) When $\lambda_b = 0$ and using $Q(z) = \mu_1 r(1-\rho_1 z)$, we get

$$F(z) = \frac{\mu_1 r(1-\rho_1 z) + \lambda_v z(r+b)}{\mu_1 r(1-\rho_1 z)(1-\rho_0 z)}P(0,0) + \frac{(\mu_1-\mu_2)(r_b+b)}{\mu_1 r(1-\rho_1 z)}F_{12}(z) \quad (32)$$

When z = 1 from the normalizing condition F(1) = 1 we have

$$\therefore P(0,0) = \frac{\mu_1 r(1-\rho_1)(1-\rho_0) - (\mu_1-\mu_2)(1-\rho_0)(r_b+b)F_{12}(1)}{\mu_1 r(1-\rho_1) + \lambda_v(r+b)}\text{P}(1, N+1) \quad (33)$$

Now using the value of P(0, 0) from (33) in (32) we get

$$F(z) = R(z)\left[\frac{\mu_1 r(1-\rho_1)(1-\rho_0) - (\mu_1-\mu_2)(1-\rho_0)(r_b+b)F_{12}(1)}{\mu_1 r(1-\rho_1 z)(1-\rho_0 z)}\right] + \frac{(\mu_1-\mu_2)(r+b)}{\mu_1 r(1-\rho_1 z)}F_{12}(z) \quad (34)$$

$$\text{Where, } R(z) = \frac{\mu_1 r(1-\rho_1 z) + \lambda_v z(r+b)}{\mu_1 r(1-\rho_1) + \lambda_v(r+b)} \quad (35)$$

For $\lambda_b > 0$ the mean queue length L_q can be found by computing F′(1) from (29) and (30)

$$L_q = \frac{\alpha}{(1-\alpha)} + \frac{\beta}{(1-\beta)} + \frac{\rho_0}{(1-\rho_0)} + \frac{\lambda_b(\lambda_a - b - \mu_1) + \lambda_v(r+b-\lambda_b) - r\lambda_a}{\mu_1 r - b\lambda_b - \lambda_a r + \lambda_v(r+b)}$$

$$- \frac{(\mu_1-\mu_2)(r+b)(1-\rho_0)\sum_{i=N+1}^{\infty}P(1,i)}{\mu_1(r+\lambda_b)} \times \left[\frac{\lambda_b(\lambda_a - b - \mu_1) + \lambda_v(r+b-\lambda_b) - r\lambda_a}{(1-\alpha)(1-\beta)(1-\rho_0)\{\mu_1 r - b\lambda_b - \lambda_a r + \lambda_v(r+b)\}} + \frac{\{\alpha(1-\beta)(1-\rho_0) + \beta(1-\alpha)(1-\rho_0) + \rho_0(1-\alpha)(1-\beta)\}}{(1-\alpha)^2(1-\beta)^2(1-\rho_0)^2}\right]$$

$$+ (\mu_1-\mu_2)\left[\frac{(\mu_1 r - b\lambda_b - r\lambda_a)\left\{(r+b)i\sum_{i=N+1}^{\infty}P(1,i) - \lambda_b\sum_{i=N+1}^{\infty}P(1,i)\right\} - \sum_{i=N+1}^{\infty}P(1,i)(r+b)\{\lambda_a\lambda_b - b\lambda_b - \mu_1\lambda_b - r\lambda_a\}}{(\mu_1 r - b\lambda_b - r\lambda_a)^2}\right] \quad (36)$$

The average number of customers in the system L_s can also be obtained by

$$L_s = L_q + \frac{\lambda_v}{\mu_1} + \frac{\lambda_a}{\mu_1} + \frac{\lambda_b}{\mu_1} + \frac{\lambda_v}{\mu_2} + \frac{\lambda_a}{\mu_2} + \frac{\lambda_b}{\mu_2} \quad (37)$$

The average waiting time per customer in the queue and the system are respectively:

$$W_q = \frac{L_q}{\lambda_v} + \frac{L_q}{\lambda_a} + \frac{L_q}{\lambda_b} \quad (38)$$

And

$$W_s = L_s + \frac{1}{\mu_1} + \frac{1}{\mu_2} \quad (39)$$

Similarly for $\lambda_b = 0$ mean queue length L_q can be found by computing F'(1) from (33) and (34)

$$L_q = \frac{\rho_1}{(1-\rho_1)} + \frac{\rho_1}{(1-\rho_1)} + \frac{\lambda_v(r+b) - \mu_1 r\rho_1}{\mu_1 r(1-\rho_1) + \lambda_v(r+b)}$$

$$- \frac{(\mu_1-\mu_2)(1-\rho_0)(r+b)\sum_{i=N+1}^{\infty}P(1,i)}{\mu_1 r\{\mu_1 r(1-\rho_1) + \lambda_v(r+b)\}} \times \left\{\frac{(1-\rho_1)(1-\rho_0)\{-\mu_1 r\rho_1 + \lambda_v(r+b)\} + \{\mu_1 r(1-\rho_1) + \lambda_v(r+b)\}\{\rho_0(1-\rho_1) + \rho_1(1-\rho_0)\}}{(1-\rho_1)^2(1-\rho_0)^2}\right\}$$

$$+\frac{(\mu_1-\mu_2)\,(r+b)}{\mu_1 r}\cdot\frac{(1-\rho_1)i\sum_{i=N+1}^{\infty}P(1,i)+\rho_1\sum_{i=N+1}^{\infty}P(1,i)}{(1-\rho_1)^2} \qquad (40)$$

The average number of customers in the system L_s can also be obtained by

$$L_s = L_q + \frac{\lambda_v}{\mu_1}+\frac{\lambda_a}{\mu_1}+\frac{\lambda_v}{\mu_2}+\frac{\lambda_a}{\mu_2} \qquad (41)$$

The average waiting time per customer in the queue and the system are respectively:

$$W_q = \frac{L_q}{\lambda_v}+\frac{L_q}{\lambda_a} \qquad (42)$$

And

$$W_s = L_s + \frac{1}{\mu_1}+\frac{1}{\mu_2} \qquad (43)$$

3. NUMERICAL RESULTS AND INTERPRETATION

In this section we provide the numerical results using equation (37). MATLAB software has been used to develop the computer program. Six different figures have been shown. Some of the parameters are kept fixed where as some of the parameters are varied which are shown in the following figures.

(i) $r = 1.5$; $\lambda_a = 5{:}1{:}10$; $\lambda_b = 5$; $\lambda_v = 11$; $\mu_2 = 0.6$; $b = 0.2$; $\nu = 0.5$; $s = 0.6$; $i=25$

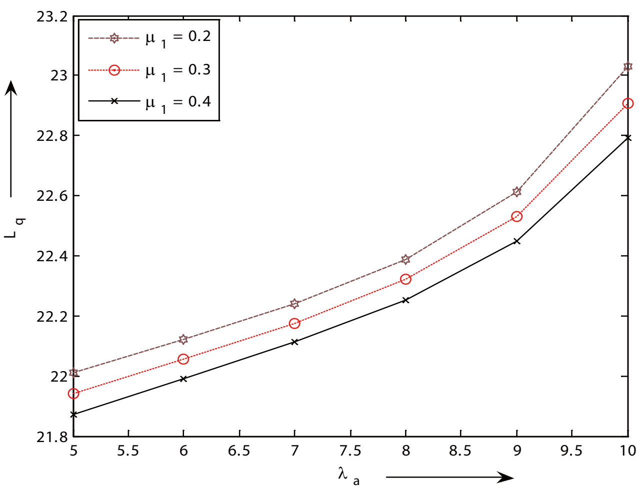

Figure 1
Arrival Rate During Active Service (λ_a) vs. Average Queue Length (L_q) by Varying μ1

(ii) $r = 1.5$; $\lambda_a = 5{:}1{:}10$; $\lambda_b = 5$; $\lambda_v = 11$; $\mu1 = 0.4$; $b = 0.2$; $\nu = 0.5$; $s = 0.6$; $i=25$

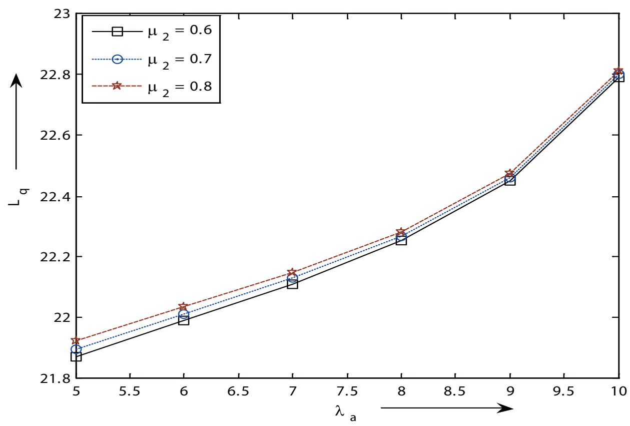

Figure 2
Arrival Rate During Active Service (λ_a) vs. Average Queue Length (L_q) by Varying μ_2

(iii) $\lambda_a = 9$; $\lambda_b = 2$; $\lambda_v = 7$; $\mu_1 = 2{:}1{:}6$; $\mu_2 = 8$; $b = 3$; $\nu = 4$; $s = 0.6$; $i = 25$

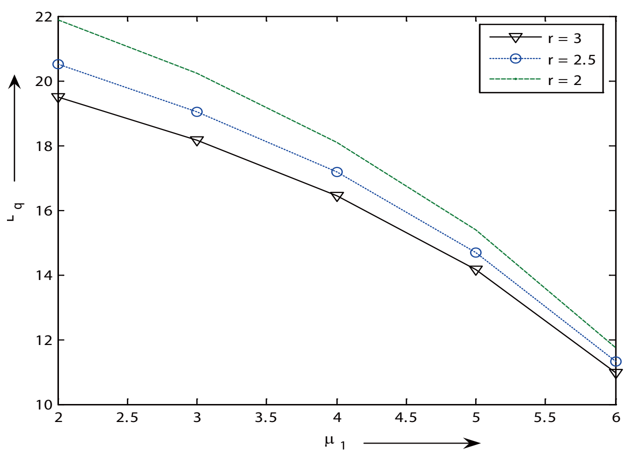

Figure 3
Service Rate μ_1 vs. Average Queue Length (L_q) by Varying Repair Rate (r)

(iv) $r = 3$; $\lambda_a = 9$; $\lambda_b = 2$; $\lambda_v = 7$; $\mu_1 = 2{:}1{:}6$; $\mu_2 = 8$; $\nu = 4$; $s = 0.6$; $i = 25$

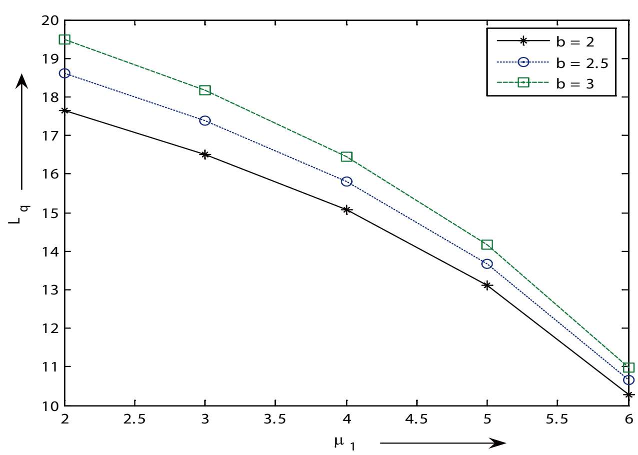

Figure 4
Service Rate μ_1 vs. Average Queue Length (L_q) by Varying Breakdown rate (b)

(v) $\lambda_a = 9$; $\lambda_b = 2$; $\lambda_v = 7$; $\mu_1 = 2$; $\mu_2 = 4:1:8$; b = 2; $\nu = 4$; s = 0.6; i = 2

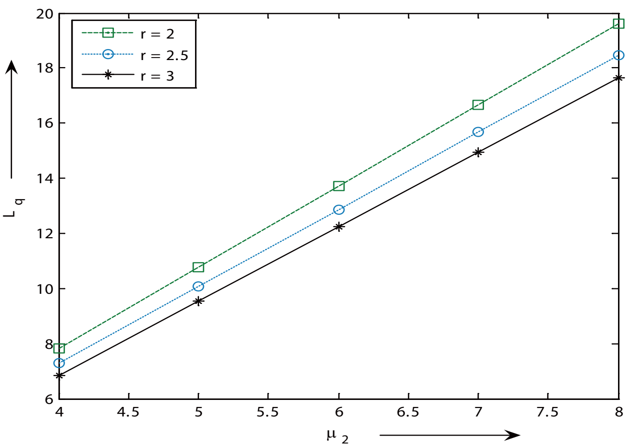

Figure 5
Service Rate μ_2 vs. Average Queue Length (L_q) by Varying Repair Rate (r)

(vi) r = 3; $\lambda_a = 9$; $\lambda_b = 2$; $\lambda_v = 7$; $\mu_1 = 2$; $\mu_2 = 4:1:8$; $\nu = 4$; s = 0.6; i = 25

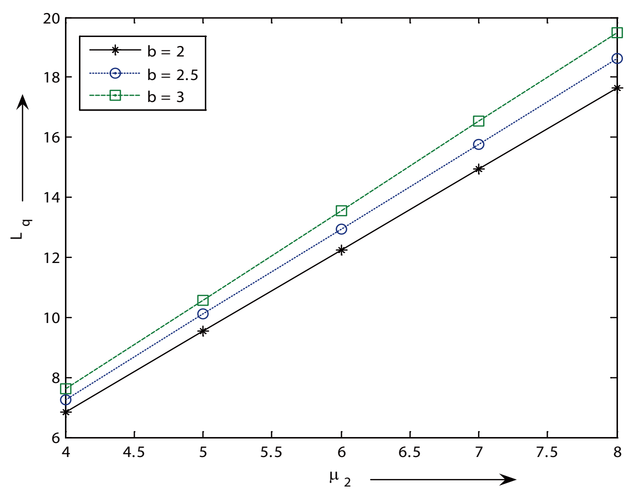

Figure 6
Service Rate μ_2 vs. Average Queue Length (L_q) by Varying Breakdown Rate (b)

Figure 1 displays the correlation between arrival rates (λ_a) during active service vs. mean queue length (L_q) by varying the service rate μ_1. We can observe that for the same arrival rate, as the service rate goes on increasing the mean queue length goes on decreasing. In the figure 2 mean queue length goes on decreasing as we go on increasing the service rate $\mu_2 > \mu_1$. If we increase the value of μ_2 the service rate given by the term ($\mu_1 - \mu_2$) will actually becomes less and hence the mean queue length becomes longer. In the figures 3 and 4 the mean queue length increases and decreases respectively as we go on increasing the breakdown rate and repair rate respectively. The similar result of figures 3 and 4 can be seen in the figures 5 and 6 also.

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