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The Impact of Rumor Transmission on Product Pricing in BBV Weighted Networks

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Abstract

This paper simulates rumor transmission in the weighted social network of customers using multi-intelligent modeling method. By comparing different strategies through social network analysis method, evaluation index of the source node controlling is found. Operational strategies of order quantity and price of products are formulated based on the simulation results of rumor transmission. Finally, this paper analyzes the impact of the related factors of social network, rumor transmission, product price and product order quantity on rumor transmission and enterprise income

Key words: BBV weighted network; Rumor transmission; Operation strategy; Multi-agent simulation

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INTRODUCTION

The booming Internet and social networks have contributed to the rapid spread of information. In this context, the focus of government and enterprise crisis public relations is to control the influence of negative rumor transmission on social stability and enterprise operational income. However, spontaneity and uncertainty of rumor propagation make it impossible for enterprises to master the process of information dissemination. The impact of rumor spread is usually difficult to assess and predict, so it is difficult for enterprises to formulate the optimal operation strategy.

In recent years, complex networks have become the focus of research. But for simplifying the problemresearchers reduced complex network to unweighted network. Such as ER random graphs (Erdds & Renyi, 1959), the WS small world model (Watts & Strogatz, 1998) and the BA scale-free network (Barabási & Albert, 1999). Although these models reflect some characteristics of complex networks, they are limited to the scope of unweighted network. Unfortunately, many networks in real life are weighted networks. For example, researchers have different degrees of collaboration in scientist collaboration networks and roads have different traffic flow in urban traffic networks (Liu, Yan, & Wang, 2007). In weighted networks, the edges between nodes represent not only the connection between nodes, but also the degree of closeness. According to these researches. Barrat proposed the BBV network model in 2004 (Barrat, Barthélemy, & Vespignani, 2004). Since the BBV network model is in good agreement with actual network systems, it becomes a common network model in researches of weighted network. The research of this paper is based on BBV network model.

Rumor transmission has also aroused extensive research. Considering the memory effect, social strengthen effect and other factors, Lü (2011) found that information in regular networks spread faster and wider than in random networks. Sathe (2008) and Zhao (2011) studied the spread of rumors on the Social Journal platform and analyzed the influence of network average degree and forgetting rate on rumor transmission. Zhao (2013) studied the propagation characteristics and threshold of rumors in BBV weighted networks. Huang (2013) found that the probability of information browsing satisfied the pow-law distribution by collecting relevant data from micro-blog. Zhang (2015) emphasized the cumulative effect of memory and used the memory function $p(t)$ instead of the constant memory rate in the traditional model. Qian (2015) studied the influence of rumor acquired from other channels on rumor propagation in uniform network and ER scale-free network based on the SIR model.

Based on the limitations of existing research and the needs of practice and theoretical research, this paper puts forward the following research questions:

(a) What's the impact of the structure of consumer's social relationship weighted network on the spread of rumor?

(b) How to identify an influential individual as the source node in the consumer social relationship weighted network, so as to effectively control the speed and scope of the rumor?

(c) What are the key factors that influence rumor transmission and enterprise operation revenue? How should enterprises formulate rumor control strategy and operational strategy to minimize their loss?

1. PRICING DECISION MODELING BASED ON RUMOR TRANSMISSION

1.1 Pricing Decision Problem Based on Rumor Transmission

Assuming that the perceived value v of the potential consumers of the product obeys the uniform distribution of [0,1], that is v ~Uniform[0,1]. When the price of the product p is below or equal to the perceived value v, the potential consumer will buy it. On the contrary, they will not buy when the price is higher than the perceived value.

According to this assumption, the purchase decision of a consumer is only affected by price. In a market where the quantity of potential consumer is N, the market demand for a product can be described as D=(1-p)·N, and the revenue of the product is π=p·D=p·(1-p)·N. Therefore, the optimal pricing of the product and the profit of the product are as follows:

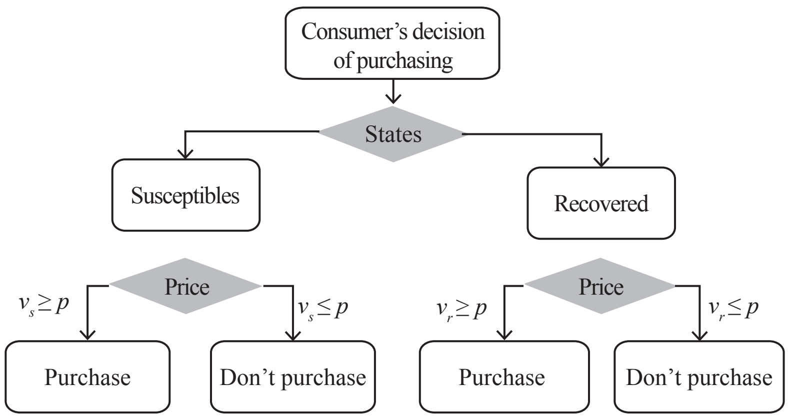

Figure 1
Consumer Purchasing Decision Tree

$$p^* = \frac{1}{2}, \quad D^* = \frac{1}{2}N, \quad \pi^* = \frac{1}{4}N.$$

According to the above hypotheses, consumers consider mainly the influence of rumors and the price of products, and the two factors are independent of each other. The consumer purchasing decision tree is shown in Figure 1.

Where v_s and v_r respectively represents the perceived value of a health consumer and an immune consumer. Since the immune population is affected by rumors, the perceived value of these consumers is surely lower than the that of the healthy population, and there is a certain correlation between them. That is, $v_s=\mu v_r$, $0<\mu<1$, v_s~Uniform[0,1] , v_r~Uniform[0,μ].

According to the decision rules of consumers, when the product price is p, the market demand function is as follows:

$$D(p) = (1-p)\cdot(1-R_{\text{final}})\cdot N + (\mu - p)\cdot R_{\text{final}}\cdot N = [1-p-(1-\mu)R_{\text{final}}]\cdot N.$$

where R_{final} is the density of the immune population after the transmission of a rumor, $0<R_{\text{final}}\leq 1$, $0\leq p\leq 1$, $0<\mu<1$.

Finally, the optimal product pricing decision of the enterprise is to maximize the product returns:

$$\pi = [1-(1-\mu)R_{\text{final}}]\cdot N\cdot p - N\cdot p^2.$$

According to the analysis of the mathematical model, the optimal price and product revenue are as follows:

$$p^* = \frac{1}{2}[1-(1-\mu)R_\infty],$$

$$\pi^* = \frac{N}{4}[1-(1-\mu)R_\infty]^2.$$

The final coverage of rumor spread is negatively related to the optimal pricing of enterprises. When there is no rumor diffusion in the extreme value, enterprises can get the maximum profit with higher pricing strategies. But when rumors exist, enterprises need to determine product prices according to the actual range of rumor spreading.

1.2 Multi-Agent Simulation Modeling of Rumor Propagation

This paper uses the SIR model of virus propagation to describe the dynamic process of rumor spreading in consumer social relations network. According to the model, individuals are divided into three types in the social network with node size *N*. They are: (a) Susceptibles: They have never been exposed to rumors, and they tend to become disseminator after they get into contact with rumors. (b) Infectives: Person who actively spread rumors in a network. (c) Recovered: They have heard rumors but lost interest in the spread of rumors and are no longer involved. Parameter λ means propagation probability to accept the rumor and becomes a disseminator; α refers to immune probability to change into an immune person. The classical rumor SIR propagation model can be represented by Figure 2.

$$\boxed{S} \xrightarrow{\lambda} \boxed{I} \xrightarrow{\alpha} \boxed{R}$$

Figure 2
SIR Model

Using $s(t)$, $i(t)$ and $r(t)$ to represent the density of the group of the health, disseminator and the immune in the network at *t* time, they satisfy the following equation at any time:

$$s(t)+i(t)+r(t)=1\ ,$$

$$\begin{cases} \dfrac{\mathrm{d}s(t)}{\mathrm{d}t} = -\lambda s(t)i(t) \\ \dfrac{\mathrm{d}i(t)}{\mathrm{d}t} = \lambda s(t)i(t) - \alpha i(t) \\ \dfrac{\mathrm{d}r(t)}{\mathrm{d}t} = \alpha i(t) \end{cases} .$$

Suppose that an enterprise chooses a customer as the initial disseminator of the rumor in the potential consumer social network, the initial conditions are $s(t)=(N-1)/N$, $i(0)=1/N$, $r(0)=0$. In the process of propagation, the number of disseminators gradually increases and then some of them change into the immune. When the number of propagators in the network is zero, the propagation process stops and the whole system reaches a stable state. In the mathematical model of rumor propagation, the density of the immune population at the end of the propagation is recorded as R_∞ and we have $i(\infty)=0$, $s(\infty)=1-r(\infty)=1-R_\infty$. Finally, the density of the immune R_∞ can be obtained from the following equation:

$$\mathrm{d}r(t) = \frac{\alpha}{\lambda}\mathrm{d}s(t) - \frac{\alpha}{\lambda i(t)}\mathrm{d}s(t)\ .$$

According to the initial and final state conditions of the propagation, we can get the transcendental equation from the above differential equation:

$$R_\infty = 1 - e^{-\varepsilon R_\infty}\ .$$

This shows that there is no propagation threshold for information spreading over a homogeneous network, that is, rumors can always spread through the network and affect a fixed proportion of people. Particularly, when $\lambda=\alpha=1$, the density of the final immune node can be obtained from the equation, that is $R_\infty=0.7968$.

This paper extends the premise of the hypothesis of homogeneity and uniform mixing in the mathematical model of rumor propagation, and establishes a multi-agent model of rumor propagation. We assume that consumers have relatively stable social relations, have a certain number of neighbors and specific network positions, and information can only be spread among connected individuals.

The multi-agent simulation process of rumor propagation is as follows (shown in Figure 3):

2. MODEL VERIFICATION

2.1 Verification of BBV weighted network characteristics

This paper uses software R to generate the BBV weighted network and the parameters are set up as shown in Table 1. Here we assume that the number of potential consumers of a product is 5,000 and each consumer is associated with an average of 2 individuals. New consumers have an increase rate of 1 to the original consumers.

Table 1
BBV Weighted Network Parameter

Parameter	Value
Number of vertices *N*	5,000
Number of linked neighbors on each side of a vertex *K*	2
The fraction of weight which is induced by the new edge onto the others *δ*	1

The edge weight, point weight and the degree of the BBV network all satisfy the power law distribution, in which the point weight and the degree are positively correlated. The edge weight is, where. The point weight is, where. The degree is, where. The relativity of weight and degree is.

The parameters of the BBV weighted network are set up as above-mentioned and its property is shown in Table 2. The actual values of the degree distribution, edge weight distribution and point weight distribution are close to the theoretical value.

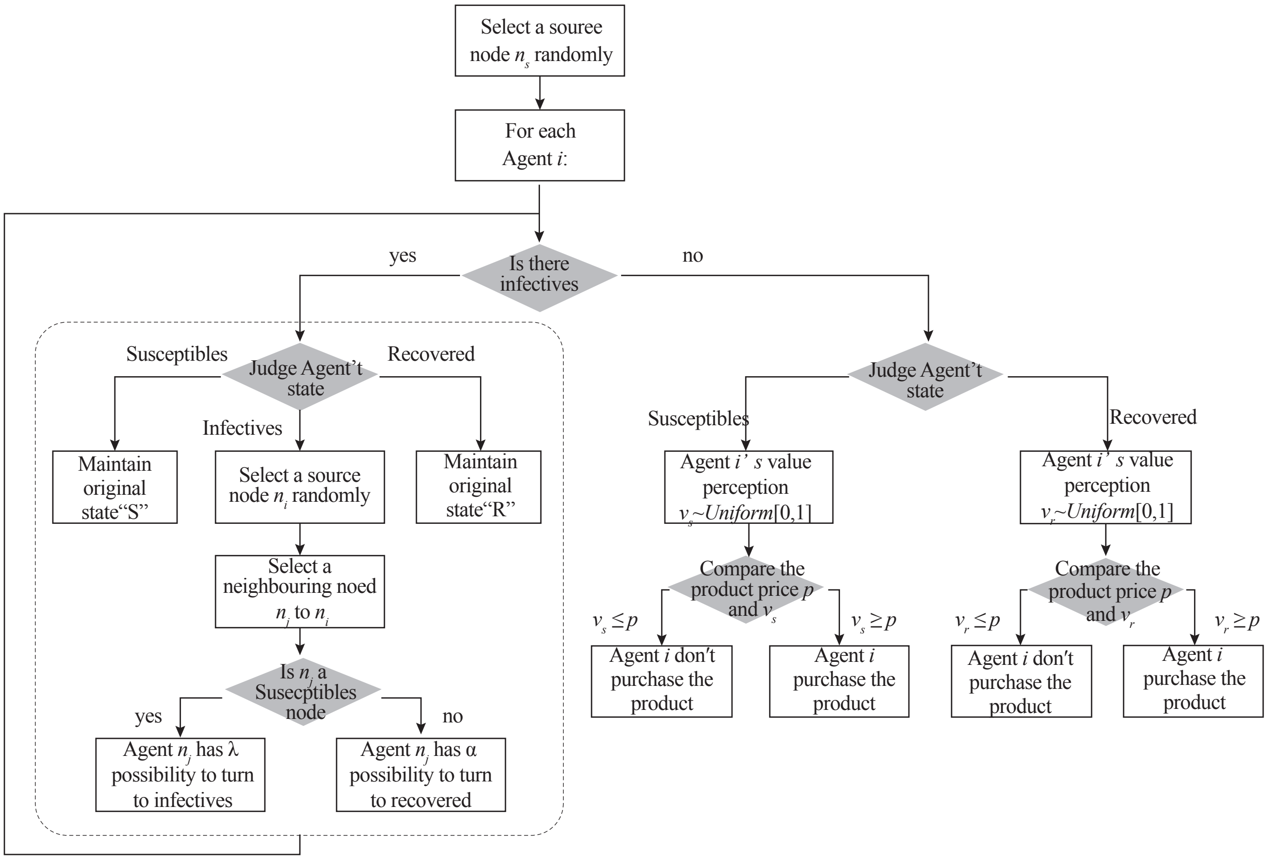

Figure 3
Multi-Agent Simulation Logical Diagram

Table 2
Properties of the BBV Network

Network	Average degree	Max. degree	Min. degree	γ_d	γ_s	γ_w
BBV	4	721	2	2.15	2.24	2.83
Theoretical value	—	—	—	2.33	2.33	3.00

The distribution of the point degree in the BBV weighted network is shown in Figure 4. It shows that the distribution of the point degree of the generated network is also close to the power law distribution. From the above analysis of the parameters of the network, we can see that the social network used to construct the simulation model of rumor spreading is accurate and effective.

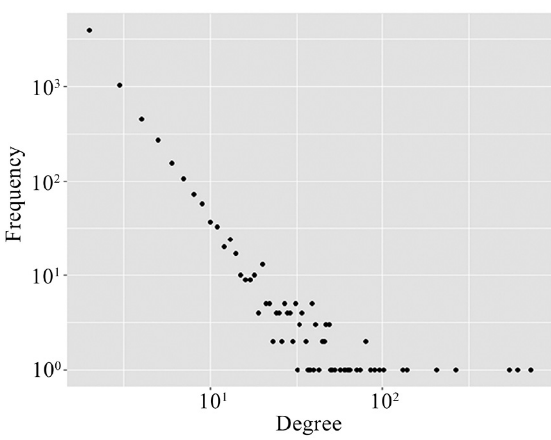

Figure 4
Distribution of the Point Degree of BBV Weighted Network

2.2 Verification of Rumor Propagation Model in BBV Weighted Network

The model in this paper is implemented on the Netlogo 5.2.0 multi-agent simulation platform. Before the simulation experiment, the model is initialized and the BBV weighted network generated by the *R* script is imported. The ratio of immune probability α and propagation probability λ is set to 1.0. Rumor propagation experiment on social relation network is repeated M=10^3 times. Here the source nodes are selected randomly.

Results of the 10^3 times experiments were statistically analyzed and the probability density distribution graph was drawn. As shown in Figure 5, the density is in bimodal distribution. The results show that when the parameters are set as the above table, information may spread out and affect a certain proportion of the population, with a maximum value of 0.010781. And it may not spread out, with the density of the immune close to 0.

Since there are clusters in the BBV network, the spreading of rumors may be limited in clusters. However, rumors may spread to individuals far away through a small number of long-term connections and therefore spread throughout the social network.

When the assumption of homogeneity and uniform mixing of social network in the mean field model of rumor propagation is relaxed, the results of rumor propagation change a lot, including the probability of propagation and the final coverage. Results from traditional mathematical models are too optimistic, which shows the limitations of these models in solving such problems.

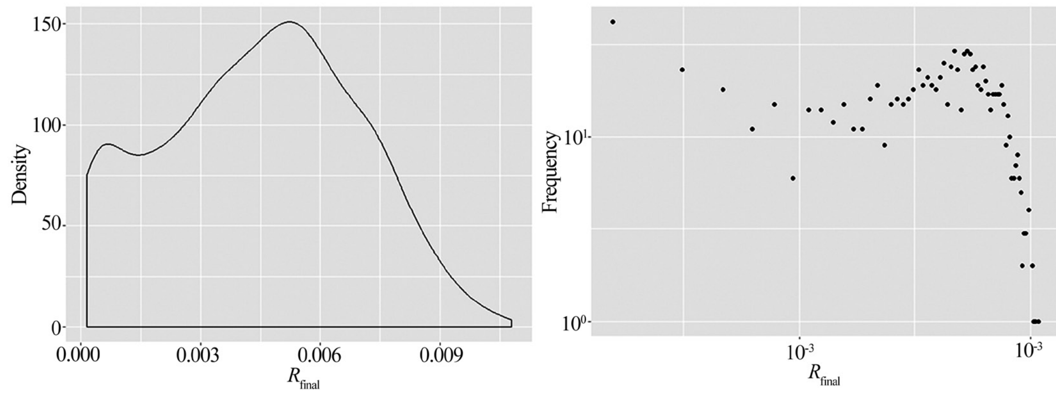

Figure 5
Experimental Result of Model Verification

3. SIMULATION RESULT

3.1 Selection Strategy of Information Source Node

Social network is the channel of information dissemination. Individuals at key positions play a decisive role in the dissemination of information and their positions also reflect how much social capital they possess. This paper mainly considers the difference of social influence determined by the difference of structural position of individual. By comparing the importance evaluation index of different nodes and analyzing the influence of different selection strategies of information source node on rumor propagation.

In order to measure the spread effect of rumors in the consumer network, this paper introduces the following four evaluation indexes:

(a) R_{final}: The density of the final immune population in the network after the propagation. It reflects the influence range of the rumor.

(b) S_{peak}: The peak density of disseminators. It reflects the biggest impact of information dissemination.

(c) T_{final}: The ending time of the propagation. It reflects the length of the whole propagation process.

(d) T_{peak}: The time when the density of the disseminator reaches its peak. It reflects the efficiency of the propagation.

There are three source node selection strategies in this paper. Among these strategies, a random node, the node with biggest out-strength and the node with biggest out-degree are respectively chosen as the first disseminator at the beginning of the propagation. After repeated experiments, the above four indexes: R_{final}, S_{peak}, T_{final}, T_{peak} are used to evaluate the propagation effect.

In order to reduce the influence of the randomness of the social network in the input simulation model, 30 BBV weighted networks are generated and the propagation experiment of each strategy is repeated M=1000 times in these networks. The parameters of the social network and the experiment are shown in Table 3.

Table 3
Parameters of the Simulation Experiment

Type	Parameter	Value
Network	Number of vertices (N)	5,000
	Number of linked neighbors on each side of a vertex (K)	2
	The fraction of weight which is induced by the new edge onto the others (δ)	1
Experiment	Immune probability/propagation probability (α/λ)	1.0
	The negative impact of rumor (μ)	0.5

Firstly, use above four indexes to conduct the variance analysis on the propagation effect of three source node selection strategies. The results show that there are significant differences in the propagation results of different initial node selection strategies (p-value<<0.05). It can be concluded that the strategy used to identify influential individuals as the source of the rumor has a significant impact on the effect of the propagation.

Secondly, the mean value of propagation results on 30 BBV weighted networks and the parameter estimation are obtained and shown in Figure 6 and Table 4. The results show that the selection of the node with the biggest out-strength as the source node, the coverage and the peak of the density of the disseminators are bigger while the time to reach the peak and the propagation time are longe. All the indexes show consistent results. From the perspective of rumor control, it can be concluded that public relations with those individuals who have the strongest points can reduce the negative impact of rumors on enterprises.

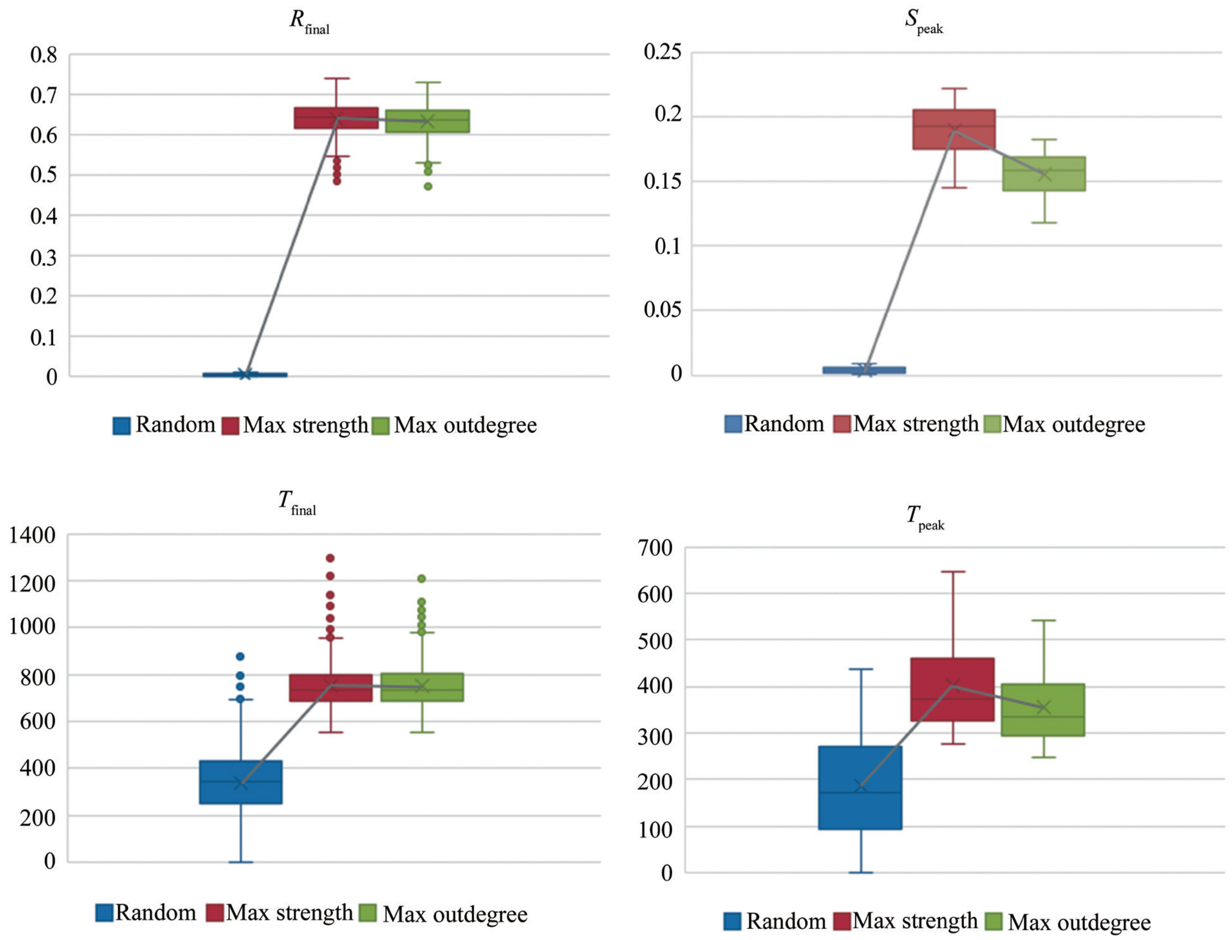

Figure 6
Comparison of Propagation Under Different Source Node Selection Strategies

Table 4
Mean Value of Rumor Propagation Under Different Source Node Selection Strategies

	R_{final}	S_{peak}	T_{final}	T_{peak}
Random	0.00444	0.00355	337.148	168.574
Out strength	0.63873	0.19162	750.532	375.266
Out degree	0.63136	0.15784	748.352	338.558

3.2 Strategy of Product Pricing

The previous paper has pointed out that out-strength is an effective index to identify the key nodes of rumor propagation. Therefore we choose the biggest out strength as the source node of the rumor propagation.

Considering that the product pricing model based on rumor spread contains many random factors, it is necessary to repeat the simulation experiments under the same parameter setting and take the average value as the statistical index of the simulation results. Finally, the pricing strategy of an enterprise is to select the optimal price which can maximize the profits of the enterprise by trying all feasible solutions.

The settings of the BBV weighted network and the rumor propagation parameters are shown above. For every product price p, simulation experiments are repeated M=250 times.

As shown in Figure 7, with the increase of product prices, the average market demand gradually decreases. The relationship between them accords with the hypothesis of the general demand relationship. The regression analysis between the average market demand and price in the simulation shows that the linear regression model of the average market demand $D(p)$=3406.15-5003.78p (Adjusted R-squared: 0.9996) is consistent with the theoretical relationship between market demand and price.

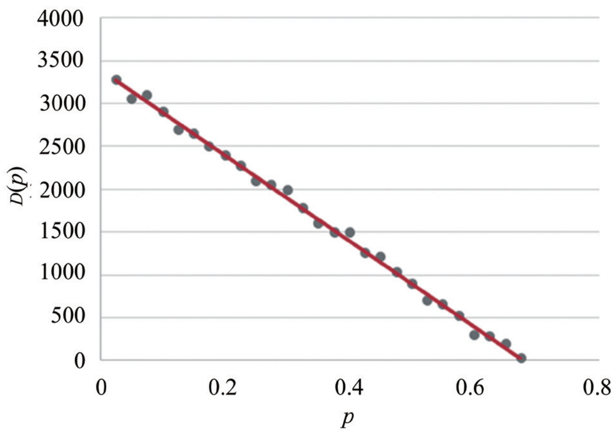

Figure 7
Relationship Between Demand and Price

Figure 8 shows the relationship between the enterprise income and product price. The optimal price of the product and the corresponding market demand and average revenue can be obtained:

$$p^*=0.34,\ D^*=1702,\ \pi^*=578.68\ .$$

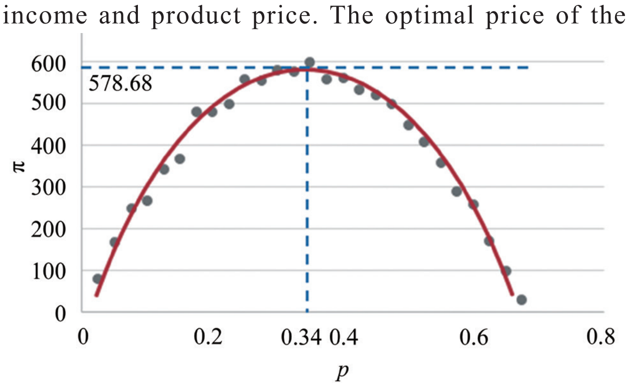

Figure 8
Relationship Between Income and Price

3.3 Parameter Analysis

In the rumor propagation model, the greater the propagation probability λ is, the more likely the potential consumers of the healthy are infected by the rumors. The immune probability α describes the mechanism that the disseminator loses interest in spreading the rumor and stops spreading to the neighboring nodes. When the immune probability α increases, the propagation process will terminate faster and the influence range will be smaller.

Parameter α/λ is changed and the experiment is repeated M=1,000 times for each value. The node with the biggest point degree is chosen as the source node and analysis of the density of the immune node R_{final} after the propagation is conducted.

As shown in Figure 9 and Table 5, the bigger the parameter α/λ is, the smaller the density of the immune node after the propagation is and the more concentrated the distribution is. It means when we increase the parameter α/λ , the coverage of the rumor will be smaller and the probability of spreading will decrease, that is, rumor propagation is suppressed effectively.

Here the optimal price, market demand and product return are given under different value of α/λ.

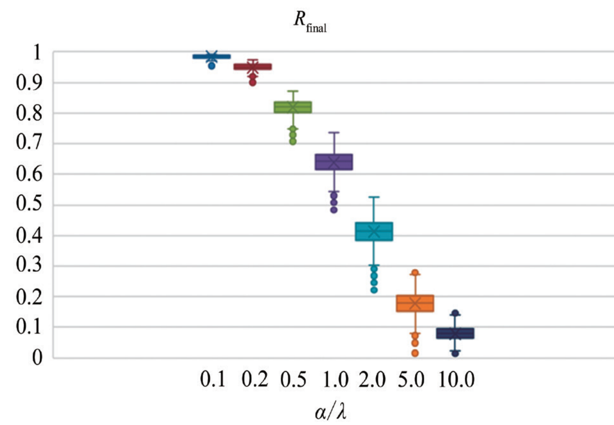

Figure 9
Influence of Propagation Probability on Rumor Propagation

Table 5
The Influence of the Parameter α/λ

α/λ	R_{final}	q^*	Q^*	π^*
0.1	0.98414	0.25397	1,270	322.54
0.2	0.94974	0.26257	1,313	344.75
0.5	0.81774	0.29557	1,478	436.85
1.0	0.63873	0.34032	1,702	579.22
2.0	0.41223	0.39694	1,985	787.93
5.0	0.17706	0.45573	2,279	1,038.62
10.0	0.08025	0.47994	2,400	1,151.85

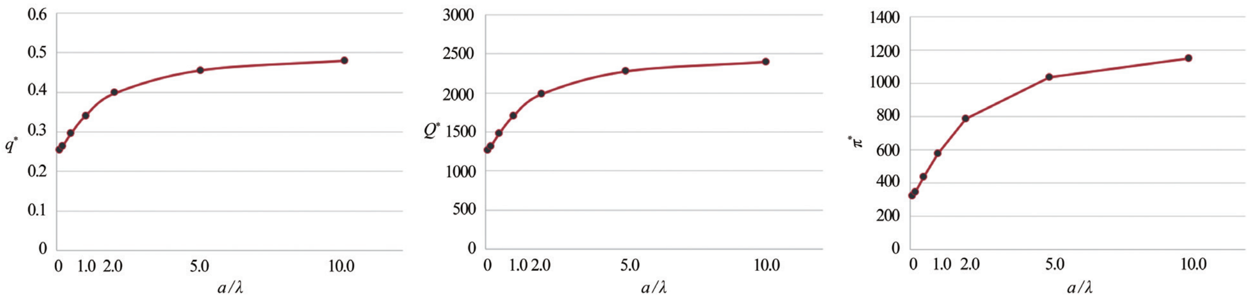

Figure 10
The Influence of Parameter *α*/*λ*

It can be concluded form Figure 10 that although the price of the product decreases when the propagation probability increases, the market demand decreases under the effects of the rumor propagation and therefore the final product revenue decreases. In summary, the spread probability of rumor has an important influence on the final earnings of enterprises, and measures should be taken to reduce it.

The greater the immune probability is, the smaller the effect of rumor propagation on consumer decision making is. Even when products are sell at a lower price, the market demand of the product and the final income of the enterprise are still greatly reduced. Therefore, enterprises should pay attention in reducing the influence of information propagation immune mechanism and thus reduce the negative influence of rumor propagation.

CONCLUSION

Through the model simulation and parameter analysis, we can get the following management inspiration:

According to the analysis of the mean field model of rumor propagation, the final coverage of the rumor (final density of the immune) is a definite value. However, it is proved to be an indeterminate value which is in bimodal distribution according to the simulation. This shows the limitations of the mathematical model.

Compared the strategy of selecting a node randomly or the node with the biggest outdegree, the strategy of selecting the node with the biggest point degree as the source of rumor to curb is better for controlling the negative effects of rumor propagation.

Enterprises should try to reduce consumers' acceptance of rumors and improve their immunity to rumors. According to the analysis of the parameter α/λ, both increasing the immune probability α and reducing the propagation probability λ can make the enterprises gain more orders when the price is reduced by rumors, thus increasing the profits.

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