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Derivatives Pricing Based on Stochastic Control With Transaction Cost, Taxes and Dividends

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Abstract

This paper attempts to apply stochastic control theory, considering option pricing on the situation when payment, tax and transaction costs exist, and finally obtain an interval of price, which not only make it more similar to the reality, but also provide reference for investors to make investment decisions. Our paper improved the uncertain volatility model. Based on the theory of stochastic control, we find the viscous solution of nonlinear partial differential equations by numerical methods when transaction cost and tax exist, and complete the empirical analysis on the actual data of the market, which has proved the value of this model.

Key words: Derivative pricing; UMV model; Stochastic control; Transaction cost; Tax

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INTRODUCTION

BS model is one of the three cornerstones of the modern financial system, it is an important tool for pricing financial derivatives and widely applied in the practice of modern finance. Option is a derivative securities value depends on many factors, including the price of the subject matter, volatility, and risk-free interest rate. Therefore, its pricing model gains great concern in the academic community.

Since Black and Scholes (1995) put forward the pricing formulas of European options and offered the analytical solution of European options in 1973, the derivatives market has been extremely enriched and developed. However, the BS model is based on a rigorous set of theoretical assumptions which does not completely accord to the real market situation. Many scholars have modified the shortage of the BS model, which has improved the theory of option pricing.

One of the important assumptions of BS model is that volatility is a constant, which does not comply with the actual market. As a result, volatility has always been the most important research directions in the study of option pricing problem. According to the time-varying and the agglomeration effect of rates of return on financial assets, Hull and White presented the first SV model of continuous time, Stein presented mean-reversion of Gaussian processes to describe the price action of volatility, which enriched the modelling method of volatility.

Heston (1993) presented the closed-form solution of European options while assumed that the variance follows a CIR process. Stochastic volatility model breaks the assumption of constant volatility which does not meet the reality of the real market, so it corrects the smiling effects and skew effects of implied volatility in BS formula effect in some cases. In addition, some scholars suggest using GARCH clusters model to simulate and forecast volatility. Chu (1996) forecasts the volatility of stock index options by GARCH model, which shows the model has good predictive results. Bollerselev and Mikkelsen used a partial integrable EGarch model to study the long memory of volatility.

Joon validated the pricing ability of GARCH clusters model, BS model and SV model by using the data of KOSPI200 index options market, and found that the pricing ability of continuous-time stochastic volatility models is the best. Avellanda (1995) made the restrictions on changes in the volatility. He assumed the volatility fluctuates in an interval specified by the historical fluctuation of the subject matter of options, and proposed the pricing formula of nonlinear partial differential equations. But he left an open question about the existence of the solutions. Yulin Deng proved the existence of the solution of uncertain volatility problem by applying Optimal control pricing theory of stochastic control theory.

Another assumption of BS model is that the market is smooth, and it ignores the transaction cost and taxes when using the spot price to dynamic duplicate option, which is different from the real market. Therefore, many scholars try to subsume the transaction cost and taxes in the pricing model of options. Leland is the first one who proposed the transaction costs in discrete hedge. He modified the volatility of BS model, proposed the option replication strategy of volatility to fix discrete nodes, and considered the option pricing method concerning transaction cost when $\tilde{a}$ is unchanged.

Hoggard overcome the above problem and let the calculation of transaction cost suits the combination options, which improved the Leland model. Many scholars consider the option pricing problem from the aspect of utility when transaction cost exists. Hodges and Neuberger are the first one who used utility function to deal with the option pricing problem when transaction cost exist. Whalley and Willmot proposed an approximation method to simply the calculation, which effectively improves the practicability of the model. Latter scholars such as Zakamouline considered the option pricing when the rate of transaction cost is fixed based on multiple types of utility function, which further enriched the option pricing method based on utility function.

In summary, the option pricing model is the core of modern finance. The most fundamental idea of it is to perfectly replicate the option by dynamically adjusting the spot positions under series of assumptions. In the past decades, people have improved and enriched their theoretical results, which have extremely prospered the derivative market. As a key issue of modelling volatility, the uncertain volatility does not take transaction cost and tax into account, which weakens its pricing ability. The existing literature does not has enough investigation about the existence of solution when transaction cost and tax exist, which leaves us space to improve. This paper attempts to apply stochastic control theory, considering option pricing on the situation when payment, tax and transaction cost exist, and finally obtain an interval of price, which make it more close to the reality. This paper improves the uncertain volatility model. Based on the theory of stochastic control, we find the viscous solution of nonlinear partial differential equations by numerical methods when transaction cost and tax exist, and complete the empirical analysis on the actual data of the market.

1. MODEL DEVELOPMENT AND ANALYSIS

Supposing that the securities market satisfies the following assumptions:

(a) The market is complete, in the risk neutral measure, the stock price S in discrete time satisfies the following stochastic differential equation

$$\frac{\delta S}{S} = r\delta t + \sigma\delta W ,$$

where r is the risk-free interest rate, σ is the volatility, both of them are constant, W is the standard Brown motion, δt is the rebalancing time.

(b) The underlying continuously dividend before the maturity, the dividend rate q is constant.

(c) The income (including dividend) from option trading should pay tax as general income, the general income tax rate is τ; capital gains tax is required for stock trading, the capital gains tax rate is C.

(d) Transaction costs exists, it is the direct costs are incurred by the tradeof financial products from the investors, transaction rate is k.

Constructing investment portfolio as $\Pi = V - \Delta S$.

The share of the investment portfolio does not change during period $(t, t+\delta t)$. The return in the risk neutral measure is risk-free interest rate r, and it also needs to pay the general income tax , thus we have

$$\frac{\Pi_{t+\mathrm{d}t} - \Pi_t}{\Pi_t} = r(1-\tau)\delta t .$$

Options trading requires payment of dividends and transaction costs

$$\delta\Pi = (1-\tau)\delta V - (1-c)\Delta\delta S - \Delta S q - k|\omega| S .$$

By Ito's formula

$$\delta V = \frac{\partial V}{\partial S}\mathrm{d}S + \left(\frac{1}{2}\partial^2 S^2\frac{\partial^2 V}{\partial S^2} + \frac{\partial V}{\partial t}\right)\delta t ,$$

let $\Delta = \frac{1-\tau}{1-c}\frac{\partial V}{\partial S}$ to keep the portfolio risk-free under the risk neutral measure

$$\delta\Pi = (1-\tau)\left(\frac{\partial V}{\partial S} + \frac{1}{2}\partial^2 S^2\frac{\partial^2 V}{\partial S^2}\right)\delta t - k|\omega|S - \Delta S q\delta t .$$

Where ω the change of the stock share, thus we have

$$\omega = \frac{1-\tau}{1-c}\left[\frac{\partial V(S+\mathrm{ds}, \mathrm{t}+\mathrm{dt})}{\partial S} - \frac{\partial V(S,t)}{\partial S}\right] = \frac{1-\tau}{1-c}\left[\frac{\partial^2 V}{\partial S^2}\mathrm{d}S + \frac{\partial^2 V}{\partial S\partial t}\mathrm{d}t + \cdots\right]$$
$$= \frac{1-\tau}{1-c}\left[\sigma S\varepsilon\sqrt{\delta t}\frac{\partial^2 V(S,t)}{\partial S^2} + \mu S\delta t\frac{\partial^2 V(S,t)}{\partial S^2} + \frac{\partial^2 V}{\partial S\partial t}\delta t\right] ,$$

neglecting the high order items of $\sqrt{\mathrm{d}t}$, we have

$$\omega = \frac{1-\tau}{1-c}\frac{\partial^2 V}{\partial S^2}\sigma S\varepsilon\sqrt{\delta t} + \mathrm{o}\left(\sqrt{\delta t}\right) ,$$

$$E(|\varepsilon|) = \int_{-\infty}^{\infty}|\varepsilon|\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\varepsilon^2}\mathrm{d}\varepsilon = \frac{1}{\sqrt{2\pi}}\left(\int_{-\infty}^{0}-\varepsilon\, e^{-\frac{1}{2}\varepsilon^2}\mathrm{d}\varepsilon + \int_{0}^{\infty}\varepsilon\, e^{-\frac{1}{2}\varepsilon^2}\mathrm{d}\varepsilon\right) = \sqrt{\frac{2}{\pi}} ,$$

$$E(|\omega|) = \frac{1-\tau}{1-c}\sqrt{\frac{2\delta t}{\pi}}S\sigma\frac{\partial^2 V}{\partial S^2} ,$$

$$E(k|\omega|S) = \frac{1-\tau}{1-c}\sqrt{\frac{2\delta t}{\pi}}kS^2\sigma\frac{\partial^2 V}{\partial S^2} ,$$

$$\delta\Pi = (1-\tau)\left(\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2S^2\frac{\partial^2 V}{\partial S^2}\right)\delta t - k|\omega|S - \left[\frac{1-\tau}{1-c}\frac{\partial V}{\partial S}Sq\right]\delta t ,$$

$$E(\delta\Pi) = \left((1-\tau)\left(\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2S^2\frac{\partial^2 V}{\partial S^2}\right)\right)\delta t - \frac{1-\tau}{1-c}\sqrt{\frac{2\delta t}{\pi}}kS^2\sigma\frac{\partial^2 V}{\partial S^2} - \left[\frac{1-\tau}{1-c}\frac{\partial V}{\partial S}Sq\right]\mathrm{d}t .$$

As the market is complete, there is no arbitrage opportunities. Considering paying income tax, we have

$$\left((1-\tau)\frac{\partial V}{\partial t} - \frac{1}{2}\sigma^2S^2\frac{\partial^2 V}{\partial S^2}\right)\delta t + \frac{1-\tau}{1-c}\sqrt{\frac{2\delta t}{\pi}}kS^2\sigma\frac{\partial^2 V}{\partial S^2} - \left[\frac{1-\tau}{1-c}\frac{\partial V}{\partial S}Sq\right]\delta t$$

$$= r(1-\tau)\left(V - \frac{1-\tau}{1-c}\frac{\partial V}{\partial t}S\right)\delta t .$$

Divide both sides by

$$\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2S^2\frac{\partial^2 V}{\partial S^2} - \frac{1}{1-c}\sqrt{\frac{2}{\pi\delta t}}kS^2\sigma\frac{\partial^2 V}{\partial S^2} - \frac{1}{1-c}\frac{\partial V}{\partial S}Sq = rV - r\frac{1-\tau}{1-c}\frac{\partial V}{\partial t}S ,$$

$$\frac{\partial V}{\partial t}+\frac{1}{2}\left[\sigma^2S^2 - \frac{2}{1-c}\sqrt{\frac{2}{\pi\delta t}}kS^2\sigma\right]\frac{\partial^2 V}{\partial S^2} + \left[\frac{1-\tau}{1-c}rS - \frac{1-\tau}{1-c}Sq\right]\frac{\partial V}{\partial s} - rV = 0 ,$$

$$\frac{\partial V}{\partial t}+\frac{1}{2}\left[\sigma^2 - \frac{2}{1-c}\sqrt{\frac{2}{\pi\delta t}}k\sigma\right]S^2\frac{\partial^2 V}{\partial S^2} + \frac{1-\tau}{1-c}(r-q)S\frac{\partial V}{\partial s} - rV = 0 \cdot$$

Let $V(t)=h(S(t))$ represents the payment of an European option at time t.

For the European call option, we have $h(S(T))=\max(S_T-K,0)$. For the European put option, we have $h(S(T))=\max(K-S_T,0)$. So we have a partial differential equation with terminal condition as follows

$$\begin{cases}\dfrac{\partial V}{\partial t}+\dfrac{1}{2}[\sigma^2S^2-\dfrac{2}{1-c}\sqrt{\dfrac{2}{\pi\delta t}}kS^2\sigma]\dfrac{\partial^2 V}{\partial S^2}+\dfrac{1-\tau}{1-c}(r-q)S\dfrac{\partial V}{\partial S}-rV=0 , \\ V(S,T)=h(S(T)) .\end{cases}$$

By Feynman-Kac theorem, we could convert the above PDE to the SDE as follows:

$$\mathrm{d}S = \frac{1-\tau}{1-c}(r-q)S\mathrm{d}t + \sigma^* S\mathrm{d}W , \tag{1}$$