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Mean-Variance-Skewness Portfolio Selection Model Based on RBF-GA

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Abstract

The classical Markowitz's mean-variance model in modern investment science uses variance as risk measure while it ignores the asymmetry of the return distribution. This article introduces skewness, V-type transaction costs, cardinality constraint and initial investment proportion, and builds a new class of nonlinear multi-objective portfolio model (mean-variance-skewness portfolio selection model). To solve the model, we develop a genetic algorithm(GA) which contains radial basis function(RBF) neural network, called RBF-GA. The experimental results show that the proposed model is more effective and more realistic than others.

Key words: Portfolio model; Skewness; RBF-GA

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INTRODUCTION

Securities market is full of risks and opportunities, one concern of both individual and institutional investors is how to use the assets reasonably, and maximize the return on investment under guarantee a certain level of risk.

Through the introduction of the mean-variance model, Markowitz took a great step toward solving the portfolio optimization (Markowitz, 1952). It has laid the foundation of modern investment theory. That model was built on a frictionless market, used mean and variance to measure the investment returns and investment risks respectively. Its effectiveness was validated by extensive theoretical and empirical analysis. However, some scholars suggested that the mean-variance model can be applicable only for the returns obey normal distribution. While, in the real world, this may not be true, many empirical results showed that the short-term return is asymmetric distribution with fat tail (Chunhachinda et al., 1997; Li, Qin, & Kar, 2010; de Athayde & Flores, 2004).

Arditti and Levy (1975) introduced that higher moments of return are an important measure index, especially the third moment (skewness). Samuelson (1970) noticed the importance of higher moment in portfolio selection, and indicated that skewness and kurtosis are higher-order statistics to measure return and risk. It expresses the good things will happen when the skewness is positive, which is the investors expected. The investors want maximize portfolio of skewness within the same variance and expected return, in order to gain higher return and reduce risk. Barak, Abessia and Modarres (2013) proposed a fuzzy portfolio mean–variance–skewness model with cardinality constraint and turnover rate, and also develop a hybrid algorithm combinates genetic algorithm and fuzzy simulation to solve the model. Tsaur (2013) developed a fuzzy portfolio model that focuses on different investor risk attitudes.

In this paper, to make the portfolio model fit the real market, we propose a mean-variance-skewness portfolio selection model with V-type transaction costs(Nonlinear transaction costs), cardinality constraint and initial investment proportion. Because of the model that we proposed is hard to solve, so we design a RBF-GA to solve this model. The RBF-GA is the genetic algorithm with radial basis function (RBF) neural network. Finally, we conduct an empirical analysis to illustrate the proposed model, and compare the result of RBF-GA and GA.

1. MEAN-VARIANCE-SKEWNESS PORTFOLIO SELECTION MODEL

1.1 Basic Mean-Variance-Skewness Multi-Objective Portfolio Selection Model

A lot of research shows the importance of the skewness and the rationality of the existence of skewness. In the real world, the return of portfolios is asymmetric, the concept of skewness was introduced to measure the asymmetry of portfolio. Therefore, we consider to use the mean, variance and skewness portfolio selection model to measure the high risk of the portfolios with asymmetric return.

Suppose investors have chosen n kinds of securities to invest, let $r_i(i=1,2,\ldots,n)$ are ith security return, let $\overline{R_i} = E(r_i)$, $\sigma_{ij} = \text{cov}(r_i, r_j)$, $\sigma_i^2 = \text{var}(r_i)$, S_i^3, S_{iij} and S_{ijj} are r_i's skewness and coskewness. Let x_i $(i=1,2,\ldots,n)$ is initial investment proportion. So, the expected value R, the risk V and the skewness S are defined as follows:

$$R(x) = X^T \overline{R} = \sum_{i=1}^{n} x_i \overline{R_i} \ ,$$

$$V(x) = X^T V X = \sum_{i=1}^{n} x_i^2 \sigma_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij} \ (i \neq j) \ ,$$

$$S(x) = E(X^T(R-\overline{R}))^3 = \sum_{i=1}^{n} x_i^3 s_i^3 + 3\sum_{i=1}^{n}(\sum_{j=1}^{n} x_i^2 x_j s_{iij} + \sum_{j=1}^{n} x_i x_j^2 s_{ijj}) \ (i \neq j) \ .$$

In portfolio selection model, the investors expect maximize the return and skewnes, while minimize the variance. The basic mean-variance-skewness multi-objective portfolio selection model can be defined as follows:

Model 1:

$$\begin{cases} \text{Maximize} \ \ R(x) = X^T \overline{R} = \sum_{i=1}^{n} x_i \overline{R_i} \ (\overline{R_i} = \sum_{t=1}^{p} \overline{R_{it}} / p), \\ \text{Minimize} \ \ V(x) = X^T V X = \sum_{i=1}^{n} x_i^2 \sigma_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij} \ (i \neq j), \\ \text{Maximize} \ \ S(x) = E(X^T(R-\overline{R}))^3 = \sum_{i=1}^{n} x_i^3 s_i^3 \\ \qquad + 3\sum_{i=1}^{n}(\sum_{j=1}^{n} x_i^2 x_j s_{iij} + \sum_{j=1}^{n} x_i x_j^2 s_{ijj}) \ (i \neq j), \\ s.t. \\ \qquad \sum_{i=1}^{n} x_i = 1. \end{cases} \tag{1}$$

1.2 Improved Mean-Variance-Skewness Multi-Objective Portfolio Selection Model

Considering the situation of real market, we put V-type transaction costs, cardinality constraint and initial investment proportion in this model. The V-type transaction costs we used can be defined as $C(x) = \sum_{i=1}^{n}(p_i x_i^+ + q_i x_i^-)$, where x_i^+ are the increased proportion on the initial investment proportion, and x_i^- are the reduced proportion on the initial investment proportion. Cardinality constraint expresses that you only can choose K kinds of assets to invest in the n kinds of assets, where $0 \leq K \leq n$. The cardinality constraint can prevent improper management caused by many kinds of investment asset. Many investors have invested assets before, so join the initial investment proportion can make the model more realistic.

The improved mean-variance-skewness multi-objective portfolio selection model which we proposed can be defined as follows:

Model 2:

$$\begin{cases} \text{Maximize} \quad R(x) = X^T\overline{R} - \sum_{i=1}^{n}(p_i x_i^+ + q_i x_i^-) = \sum_{i=1}^{n} x_i \overline{R_i} - \sum_{i=1}^{n}(p_i x_i^+ + q_i x_i^-) \\ \qquad (\overline{R_i} = \sum_{t=1}^{p} \overline{R_{it}} / p), \\ \text{Minimize} \quad V(x) = X^T V X = \sum_{i=1}^{n} x_i^2 \sigma_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij} \;\; (i \neq j), \\ \text{Maximize} \quad S(x) = E(X^T(R - \overline{R}))^3 = \sum_{i=1}^{n} x_i^3 s_i^3 \\ \qquad + 3\sum_{i=1}^{n}(\sum_{j=1}^{n} x_i^2 x_j s_{iij} + \sum_{j=1}^{n} x_i x_j^2 s_{ijj}) \;\; (i \neq j), \\ s.t. \\ \quad \sum_{i=1}^{n} x_i = 1, \\ \quad \sum_{i=1}^{n} z_i \leq K, \\ \quad \varepsilon_i z_i \leq x_i \leq \delta_i z_i, \;\; i = 1,...,n \\ \quad z_i \in [0,1]. \;\; i = 1,...,n \\ \quad x_i = x_{0i} + x_i^+ - x_i^-, \; i = 1,...,n \\ \quad x_i^+ \geq 0, x_i^- \geq 0. \; i = 1,...,n \end{cases} \tag{2}$$

Model 2 is an improved multi-objective portfolio selection model based on mean-variance-skewness, however the multi-objective portfolio selection model is hard to solve, so we use the weighted method to transform the multi-objective portfolio selection model to single objective portfolio selection model. Model 3 can be presented as follows:

Model 3:

$$\begin{cases} \text{Minimize} \quad Z(x) = -\lambda_1 R(x) + \lambda_2 V(x) - \lambda_3 S(x) \\ \qquad = -\lambda_1[\sum_{i=1}^{n} x_i \overline{R_i} - \sum_{i=1}^{n}(p_i x_i^+ + q_i x_i^-)] + \lambda_2[\sum_{i=1}^{n} x_i^2 \sigma_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij} \;\; (i \neq j)] \\ \qquad - \lambda_3[\sum_{i=1}^{n} x_i^3 s_i^3 + 3\sum_{i=1}^{n}(\sum_{j=1}^{n} x_i^2 x_j s_{iij} + \sum_{j=1}^{n} x_i x_j^2 s_{ijj}) \;\; (i \neq j)] \;, \\ s.t. \\ \quad \sum_{i=1}^{n} x_i = 1, \\ \quad \sum_{i=1}^{n} z_i \leq K, \\ \quad \varepsilon_i z_i \leq x_i \leq \delta_i z_i, \;\; i = 1,...,n \\ \quad z_i \in [0,1]. \;\; i = 1,...,n \\ \quad x_i = x_{0i} + x_i^+ - x_i^-, \; i = 1,...,n \\ \quad x_i^+ \geq 0, x_i^- \geq 0. \; i = 1,...,n \end{cases} \tag{3}$$

Because of the dimension of three objectives in Model 3 is different, and it can influence the result of Model 3. To solve this problem, we made the dimensionless processing on Model 3, and get Model 4:

Model 4:

$$\begin{cases} \text{Minimize} \quad Z(x) = -\lambda_1 \dfrac{R(x) - R\min}{R\max - R\min} + \lambda_2 \dfrac{V(x) - V\min}{V\max - V\min} - \lambda_3 \dfrac{S(x) - S\min}{S\max - S\min} \\ \quad = -\lambda_1 \dfrac{[\sum_{i=1}^{n} x_i \overline{R_i} - \sum_{i=1}^{n} (p_i x_i^+ + q_i x_i^-)] - R\min}{R\max - R\min} + \lambda_2 \dfrac{[\sum_{i=1}^{n} x_i^2 \sigma_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij} \ (i \neq j)] - V\min}{V\max - V\min} \\ \quad - \lambda_3 \dfrac{[\sum_{i=1}^{n} x_i^3 s_i^3 + 3\sum_{i=1}^{n}(\sum_{j=1}^{n} x_i^2 x_j s_{iij} + \sum_{j=1}^{n} x_i x_j^2 s_{ijj}) \ (i \neq j)] - S\min}{S\max - S\min}, \\ s.t. \\ \quad \sum_{i=1}^{n} x_i = 1, \\ \quad \sum_{i=1}^{n} z_i \le K, \\ \quad \varepsilon_i z_i \le x_i \le \delta_i z_i, \quad i = 1,\dots,n \\ \quad z_i \in [0,1]. \ i = 1,\dots,n \\ \quad x_i = x_{0i} + x_i^+ - x_i^-, \ i = 1,\dots,n \\ \quad x_i^+ \ge 0, x_i^- \ge 0. \ i = 1,\dots,n \end{cases} \tag{4}$$

Where Rmax is the maximum of the expected value as the objective in Model 1, it can be defined as follows:

Model 5:

$$\begin{cases} \text{Maximize} \quad R(x) = X^T \overline{R} - \sum_{i=1}^{n} (p_i x_i^+ + q_i x_i^-) = \sum_{i=1}^{n} x_i \overline{R_i} - \sum_{i=1}^{n} (p_i x_i^+ + q_i x_i^-) \\ \qquad (\overline{R_i} = \sum_{t=1}^{p} \overline{R_{it}} / p), \\ s.t. \\ \quad \sum_{i=1}^{n} x_i = 1, \\ \quad \sum_{i=1}^{n} z_i \le K, \\ \quad \varepsilon_i z_i \le x_i \le \delta_i z_i, \quad i = 1,\dots,n \\ \quad z_i \in [0,1]. \ i = 1,\dots,n \\ \quad x_i = x_{0i} + x_i^+ - x_i^-, \ i = 1,\dots,n \\ \quad x_i^+ \ge 0, x_i^- \ge 0. \ i = 1,\dots,n \end{cases} \tag{5}$$

Through the same way, we can also gat Rmin, Vmax, Vmin, Smax and Smin.

It's easy to know that the dimensionless Model 4 has more applicability than Model 3. The following researches are all based on Model 4.

2. GENETIC ALGORITHM APPROACH

Model 4 happens to be a non-liner programming. It can be shown that it is NP-hard. That is why it cannot be solved analytically when its size becomes large. Therefore, we develop a genetic algorithm to be able to obtain a near-optimal solution within a reasonable time. Genetic algorithm was introduced by Holland in 1969, inspired by Darwin's theory, and then generalized by De Jong (1975) and Goldberg (1989). It is one of the most practical and popular metaheuristics for solving complicated models. The principle of genetic algorithm is a programming technique that mimics biological evolution as a problem-solving strategy.

However, the genetic algorithm often gets local optimal solution. To overcome this shortcoming, we propose using weighted modification method of artificial neural network.

Radial basis function (RBF) neural network is an extremely powerful neural network type. A RBF network can be regarded as a feed-forward network composed of three layers do neurons with different roles. It has better performance and strong capability of global search (Broomhead & Lowe, 1988).

In this paper, the weighted modification method of artificial neural network can be defined as follows (Yu, Wang, & Lai, 2008):

$$\begin{cases} \frac{dX}{dt} = -\nabla_x L(X^*, \beta^*) = \lambda_1 \overline{R} - 2\lambda_2 VX^* + 3\lambda_3 (R - \overline{R})(X^{*T} VX^*) - I^T \beta^*, \\ \frac{d\beta}{dt} = \nabla_\beta L(X^*, \beta^*) = X^{*T} I - 1. \end{cases} \quad (6)$$

The procedure of the RBF-GA is summarized as follows:

Step 1 (Initialization): Set N as population_size, Pc as crossover probability, Pm as mutation probability. Randomly generated N chromosomes as initial population $X(0)$. Set evolution algebra counter: $t \leftarrow 0$.

Step 2 (Evaluation): Evaluate the function value of each chromosome.

Step 3 (Termination Check): If termination criteria are hold to terminate.

Step 4 (Genetic Operations):

a) Selection. Select $M/2$ pairs of chromosomes as parent chromosomes using roulette method limit $M \geq N$.

b) Crossover. Select N chromosomes as the population according to the fitness of each chromosome. Create M new points (offsprings) from the previously selected parents using two-point crossover method with probability Pc.

c) Mutation. Mutate the offsprings using simple variation method with probability Pm.

Step 5 (Replacement): Evaluate the function value of each offspring, and select N chromosomes with higher fitness in the M offsprings and the N previous chromosomes to be population $X(t+1)$.

Step 6: set $t \leftarrow t+1$, go to Step 2.

3. SIMULATION EXPERIMENTS

3.1 Data Description

In this study, to test the versatility and robustness of the proposed approach, three globally-traded stock market indices (S&P500 for the US, FTSE100 for the UK, and Nikkei 225 for Japan) and three globally traded foreign exchanges (euros (EUR), British pounds (GBP) and Japanese yen (JPY)) against the US dollar (USD) are examined in our empirical experiment. The stock indices and exchange data used in this paper are daily and are obtained from Datastream and Pacific Exchange Rate Service. The entire data set covers the period from January 1, 2010 to September 30, 2013. According to the study by Wang et al. (2008), multi-layered feed-forward neural network(MLFNN) has the best prediction accuracy. This paper also uses the MLFNN to examine the data. The data sets are divided into two periods: the learning sample covers January 1, 2010 to July 31, 2013 while the test learning is from August 1, 2013 to September 30, 2013. For brevity, the original data are not listed in the paper.

We choose the daily excess returns of these indices and exchange rates as forecast variables. The excess returns are defined as $R_t=\log((P_t-P_{t-1})/P_{t-1})-r_{t-1}$, where P_t is the price of the stock index or exchange rate traded at t, and r_t is the risk-free interest rate at time t.

3.2 Parameters Setting

We need to set the parameters when we use the RBF-GA and GA, therefore, we set the size of population is 100, the number of iterations is 2000, $P_c=0.8$, $P_m=0.1$, K=4, $\varepsilon_i=0.1$, $\delta_i=0.4$, $x_{01}=x_{02}=...=x_{1n}=1/n$, $i=1,2,\dots,n$, $p^+= 0.0005$， $p^-= 0.0008$. Where p^+ and p^- are different indicates that the transaction cost of buying is smaller than selling, it is same as the reality.

Transfer the Model 2 to Model 3 need to set up weights $\lambda_i(i=1,2,\dots,n)$. Different weights represent different attitudes of investors. Investors are risk-neutral while $\lambda_1=\lambda_2=\lambda_3=1/3$, and they have same attitudes toward on expected excess returns, risk and expected skewness. Investors are risk-seeking while $\lambda_1=0.5$, $\lambda_2=0.25$, $\lambda_3=0.25$, and the investors are prefer more returns. When $\lambda_1=0.25$, $\lambda_2=0.5$, $\lambda_3=0.25$, investors are risk-averse. When $\lambda_1=0.5$, $\lambda_2=0.5$, $\lambda_3=0$, investors treat the returns as same as risk. To analyze the choice of different investors, this paper sets up six types of weights to make a contrastive study. Table 1 shows the types if weights.

Table 1
Weights Setting

Weights	λ_1	λ_2	λ_3
Condition 1	1/3	1/3	1/3
Condition 2	0.2	0.4	0.4
Condition 3	0.4	0.4	0.2
Condition 4	0.25	0.5	0.25
Condition 5	0.5	0.25	0.25
Condition 6	0.5	0.5	0

The result of *R*max, *R*min, *V*max, *V*min, *S*max and *S*min are shown in Table 2.

Table 2
Value of Rmax, Rmin, Vmax, Vmin, Smax and Smin

*R*max	*R*min	*V*max	*V*min	*S*max	*S*min
7.16567×10^{-4}	-0.00124	5.7688×10^{-5}	9.49123×10^{-6}	0.28990	-0.75730

3.3 Experiment Results

To illustrate the proposed model, we implement the Model 4 with the data and the parameters, and use RBF-GA and GA methods to solve the Model 4. As shown in Table 3 and Table 4, we can see the proportions of different assets in the portfolio for different conditions based on RBF-GA and GA. From Table 5, we obtain the optimal objective value of the Model 4 through RBF-GA and GA. And Table 6 shows the result of corresponding expected excess returns, risk and expected skewness.

Table 3
Proportions of Different Assets in the Portfolio Based on RBF-GA for Different Weights

	x_1	x_2	x_3	x_4	x_5	x_6
Condition 1	0.26116	0.17209	0.16675	0	0.40000	0
Condition 2	0.24873	0.16667	0	0.18459	0.40000	0
Condition 3	0.26206	0.17128	0.16666	0	0.40000	0
Condition 4	0.26670	0.16668	0	0.16666	0.39996	0
Condition 5	0.22625	0.17177	0.20198	0	0.40000	0
Condition 6	0.36831	0.19051	0.16667	0	0	0.27451

Table 4
Proportions of Different Assets in the Portfolio Based on GA for Different Weights

	x_1	x_2	x_3	x_4	x_5	x_6
Condition 1	0.26149	0.17167	0.16684	0	0.40000	0
Condition 2	0.24016	0.16471	0	0.19513	0.40000	0
Condition 3	0.26608	0.16715	0.16676	0	0.40000	0
Condition 4	0.26765	0.16509	0	0.16726	0.40000	0
Condition 5	0.21493	0.18090	0.20418	0	0.40000	0
Condition 6	0.36794	0.18869	0.16686	0	0	0.27651

Table 5
Optimal Objective Value of Model 4 Based on RBF-GA and GA

	Optimal objective value	
Algorithm	**RBF-GA**	**GA**
Condition 1	0.39052	0.39050
Condition 2	0.35546	0.35540
Condition 3	0.31071	0.31069
Condition 4	0.23451	0.23435
Condition 5	0.45979	0.45973
Condition 6	0.21252	0.21250

Table 6
Value of Corresponding Expected Excess Returns, Risk and Expected Skewness

	Expected excess returns		Risk		Expected skewness	
	RBF-GA	**GA**	**RBF-GA**	**GA**	**RBF-GA**	**GA**
Condition 1	5.1971×10^{-5}	5.1974×10^{-5}	2.2912×10^{-5}	2.2911×10^{-5}	6.9640×10^{-2}	6.9600×10^{-2}
Condition 2	-3.6672×10^{-4}	-3.8074×10^{-4}	1.8039×10^{-5}	1.8029×10^{-5}	1.2531×10^{-1}	1.2869×10^{-1}
Condition 3	5.1619×10^{-5}	5.0634×10^{-5}	2.2899×10^{-5}	2.2864×10^{-5}	6.9570×10^{-2}	6.9070×10^{-2}
Condition 4	-3.4808×10^{-4}	-3.5123×10^{-4}	1.8059×10^{-5}	1.8044×10^{-5}	1.1991E-01	1.2032×10^{-1}
Condition 5	1.0507×10^{-4}	9.9578×10^{-5}	2.4705×10^{-5}	2.4953×10^{-5}	6.5280×10^{-2}	6.4540×10^{-2}
Condition 6	1.4353×10^{-4}	1.4032×10^{-4}	2.3087×10^{-5}	2.3009×10^{-5}	-3.4444×10^{-1}	-3.4606×10^{-1}

From Table 5, we can see that the Optimal objective value of RBF-GA is higher than GA, which indicates that the RBF-GA that we proposed has better performance and capability of global search than GA.

As shown in Table 6, different investors with different risk attitudes have different efficient portfolios. Compare Condition 1-6, the corresponding expected excess returns, risk and expected skewness are obviously very different. In Condition 1, the excess returns, risk and skewness are in the middle level because they have the same weights. While in Condition 2, the excess returns has least weight, so it is smaller but the risk and skewness are better. The investors can choose different portfolio according to their attitudes towards risk.

CONCLUSION

This study proposes a Mean-variance-skewness portfolio selection model based on RBF-GA, while V-type transaction costs, cardinality constraint and initial investment proportion are also considered, which makes the model is more realistic and has strong effective. And the experimental results show that Compare with the traditional GA, the RBF-GA we proposed has better capability of global search. Through the use of the RBF-GA mean-variance-skewness portfolio selection model, investors can construct a portfolio which matches their risk preference.

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