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Study on Fuzzy Factor GARCH Model

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Abstract

As known to all, securities market is easy to be affected by subjective consciousness and its largest characteristic is uncertainty. Similar to financial market, securities market can be influenced by politics, economy and society, causing return on assets and investment risk to change constantly and to be difficult to describe. As a result, it forms strong fuzziness. Based on the hypothesis of fuzzy rationality, fuzzy rule if-then is applied in the paper to improve traditional factor GARCH model through the thought of fuzzy mathematics.

Firstly, Gaussian membership function is constructed for stock return to calculate the covariance matrix of multiple fuzzy number of return on assets and principal component analysis (PCA) and statistics means are utilized to model revenue volatility. Finally, applying fuzzy factor GARCH model to revenue volatility of stock and to estimate various parameters.

It mainly conducted research on volatility of revenue of financial data by means of factor GARCH model in previous studies but neglected the fuzziness of financial market. In this paper, perspective of fuzziness takes place of randomness and relevant techniques including fuzzy mathematics, analysis of time series and PCA are used to transform factor GARCH model.

In the Markowitz model for portfolio investment, it implicitly assumes that investors perform one-period investment: there are only a certain number of capital funds before making decision and not any securities, but it is not consistent with the real situation. The logical hypothesis is that the process when investors make decision on portfolio investment is their readjusting volume of holdings of different securities and meanwhile remaining capital funds unchanged.

For securities that generate relatively steady revenue and have low risk exist in the financial market, portfolio needs to meet the requirement of securities market and investors. In this paper, former mean of return is taken as the expected return of assets and evaluation of risk matrix and upper and lower probability of quadratic programming model of portfolio is created to help investors to use investment strategy in a better way.

Key words: Fuzzy factor GARCH model; Principal Component Analysis (PCA); Markowitz model for portfolio

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INTRODUCTION

In the course of financial investment, the problem what investors have always paid attention is how to handle portfolio to maximize rate of return on investment and minimize investment risk. Kraft and Engel (1982) depended on time series tools to describe time dependence of conditional variance and initially proposed general auto-regression conditional heteroscedasticity (ARCH). In this period, many efforts had been made to produce lots of ARCH models and time series tool was used to represent time varying conditional variance sequence.

Traditional portfolio model can work out a certain figure for investors' reference based on different volatility models. As it is always required to consider many assets in the practical investment decision and return on assets is difficult to provide accurately and investment risk is also affected by numerous elements, the establishment of fuzzy number for return on assets and final estimated interval for investors are of great significance. Return on assets of fuzzy investment will be put forward in the paper and fuzzy risk matrix will be created.

Scholars have proposed many simplified multivariate ARCH (GARCH)model, such as constant conditional correlation model (Bollerslev, 1987), diagonal VECH model (Bollerslev, Engel, & Wooldridge, 1998), BEEK model (Bollerslev, Engel, & Kro-ner, 1995), multi-factor GARCH model (Engel, Ng, & Roths-Child, 1990), etc.. Multi-factor GARCH model is a special type of BEEK model, and can solve problem that unknown parameters are redundant if factors are chosen in a proper way.

In the paper, PCA is applied to put forward a fuzzy interval number for investors and provide choice for them during making decision. By obscuring return on assets, Gaussian membership function is taken advantage to create investment risk matrix and deviation from the center is taken to measure risk in each of the confidence interval of future return to form the target function with total risk minimization and revenue maximization.

In economic study, many people prefer to use variance or standard deviation of return to measure risk, and the bigger the variance is, the larger the risk is (Q. W. Yu & Y. Yu, 2009). Markowitz theory for portfolio lays the foundation for modern portfolio and risk management, which have been widely applied in practice. However, in the case when the risk structure among assets is unknown, this model cannot be directly applied in practice.

In order to better fit aggregation effect and leverage effect, scholars proposed multivariate GARCH model. Multivariate GARCH model can not only effectively simulate the dynamic fluctuation of assets but also can describe the relevant dynamic structure of fluctuation among assets (Yue & Ding, 2009).

"Portfolio" theory was initially proposed by Markowitz in 1952 and focused on relation between return and risk of risk assets to discuss selection of optimal portfolio in the economic system the first time (Wang, 2001). Zhang Pu and Peng Yucheng gave solution in the case when covariance matrix is undefined in a new algorithm on solving Markwoitz model with inversible covariance matrix; other scholars had made experimental research on application of this theory in Chinese securities market, and the conclusion they drawn all proved that Markowitz theory for portfolio could effectively diversify risk.

Markowitz model for portfolio investment never considers that some economic resources (like handling charge) will be certainly consumed in the process of security transaction, but investors are always faced with noticeable transaction cost with respect to potential benefits in real securities market. If no attention is given, it will usually lead to the failure of portfolio investment and final failure of investment.

Currently, a majority of portfolio models are based on probability theory, and return on assets is regarded as random variable to reflect the actual condition. In the financial market with many influence factors, especially in the fuzzy and uncertain economic environment, return of the risky asset is also equipped with fuzziness.

In recent years, some scholars have conducted research on fuzzy portfolio. For example, Watada and Ramaswamy proposed portfolio model on the basis of fuzzy decision theory; Tanaka et al. proposed model for security portfolio of fuzzy probability and took advantage of mean, variance and covariance in fuzzy probability to replace the corresponding figures in Markowitz model for portfolio; Tanaka and Guo (1999) put forward portfolio model based on distribution of index probability and Carlsson et al. took return on assets of investors as trapezoidal fuzzy number and put forward portfolio model of fuzzy probability with maximum utility in the case that oversell was not allowed.

1. CONSTRUCTION OF COVARIANCE MATRIX OF FUZZY PORTFOLIO

Random phenomenon is uncertain and should be described through random variables. Numerical characteristics of random variables show probability distribution and data information of the random imagination. Fuzzy phenomenon is also uncertain. As for presentation of fuzzy information, many portfolio models are based on probability theory and return on assets is regarded as random variable. Given varying factors in financial market, risky assets are indeterminate and fuzzy. In this case, it should not be represented by random event, and consequently fuzzy portfolio model based on the theory of fuzzy decision is applied.

It assumes that n (n=1, …, N) return on assets is obscured and corresponds to L rule according to if-then, and Gaussian membership function is created for each rule. At t, parameters are respectively c^lnt and b^lnt; in term t, return of assent is r_{nt}, fuzzy variable.

At, r_{nt}, probability that Membership set of return on assent belongs to fuzzy set F_{lt} is:

$$F_{lt}(r_{nt}) = \exp\left(-\frac{1}{2}\left(\frac{x - c^l{}_{nt}}{b^l{}_{nt}}\right)^2\right), \quad (1)$$

Where l=1,2,..., L.

γ level set when R_{nt} belongs to fuzzy set F_{Lt} is $[r^lnt]^\gamma = [a^l{}_1(\gamma), a_2{}^l(\gamma)]$. It assumes $M^l{}_*(r_{nt})$ is the lower probability of return of the first asset at t, and $M^{l*}(r_{nt})$ is upper probability of return of the first asset at t, then:

$$M^l{}_*(r_{nt}) = 2\int_0^1 \gamma\, a_1{}^l(\gamma)\mathrm{d}\gamma\ , \quad (2)$$

$$M^{l^*}(r_{nt}) = 2\int_0^1 \gamma\, a_2{}^l(\gamma)\mathrm{d}\gamma\ . \quad (3)$$

Where l=1,2, ... L.

$$a_1{}^l(\gamma) = c^l{}_{nt} + 2b^l{}_{nt}\sqrt{\ln\gamma}\,, \tag{4}$$

$$a_2{}^l(\gamma) = c^l{}_{nt} - 2b^l{}_{nt}\sqrt{\ln\gamma}\,. \tag{5}$$

Where l=1,2, ... L.

Equations (4) and (5) are substituted into Equations (2) and (3):

$$M^l{}_*(r_{nt}) = 2\int_0^1 \gamma\left(c^l{}_{nt} + 2b^l{}_{nt}\sqrt{\ln\gamma}\right)\mathrm{d}\gamma\,, \tag{6}$$

$$M^{l^*}(r_{nt}) = 2\int_0^1 \gamma\left(c^l{}_{nt} - 2b^l{}_{nt}\sqrt{\ln\gamma}\right)\mathrm{d}\gamma\,. \tag{7}$$

Where l=1,2, ... L.

$u(r_{jt})$ is the result of continuous multiplication between return on assets and membership of fuzzy set:

$$g_l(r_{nt}) = \frac{u_l(r_{nt})}{\sum_{l=1}^{L} u_l(r_{nt})}\,,$$

$$u_l(r_{nt}) = \prod_{j=1}^{k} F_{lj}(r_{jt})\,.$$

To sum up,

$$M^*(r_{nt}) = \sum_{l=1}^{L} g_l(r_{nt})\, M^{l*}(r_{nt})\,,$$

$$M_*(r_{nt}) = \sum_{l=1}^{L} g_l(r_{nt})\, M^l{}_*(r_{nt})\,.$$

n=1,..., N.

In l fuzzy rule, upper and lower probability of variance of return on assistantis:

$$var^l{}_*(r_{nt}) = 2\int_0^1 \gamma\left(M^l{}_*(r_{nt}) - a_1{}^l(\gamma)\right)^2 \mathrm{d}\gamma, \tag{8}$$

$$var^{l*}(r_{nt}) = 2\int_0^1 \gamma\left(M^{l^*}(r_{nt}) - a_2{}^l(\gamma)\right)^2 \mathrm{d}\gamma. \tag{9}$$

Where l=1,2, ... L.

Substituted into Equations (4) and (5):

$$var^l{}_*(r_{nt}) = 2\int_0^1 \gamma\left(M^l{}_*(r_{nt}) - \left(c^l{}_{nt} + 2b^l{}_{nt}\sqrt{\ln\gamma}\right)\right)^2 \mathrm{d}\gamma, \tag{10}$$

$$var^{l*}(r_{nt}) = 2\int_0^1 \gamma\left(M^{l^*}(r_{nt}) - \left(c^l{}_{nt} - 2b^l{}_{nt}\sqrt{\ln\gamma}\right)\right)^2 \mathrm{d}\gamma\,. \tag{11}$$

Where l=1,2, ... L.

To sum up,

$$var^*(r_{nt}) = \sum_{l=1}^{L} g_l(r_{nt})\, var^{l*}(r_{nt})\,,$$

$$var_*(r_{nt}) = \sum_{l=1}^{L} g_l(r_{nt})\, var^l{}_*(r_{nt})\,.$$

n=1,..., N.

According to above equations, it can respectively obtain Gaussian membership function of N assets at t, and then column vector of lower probability of return on N assets at t can be represented as $M_*(r_{nt})=(M_*(r_{1t}), M_*(r_{2t}),..., M_*(r_{Nt}))^T$, and column vector of upper probability of return on N assets at t can be described as $M^*(r_{nt})=(M^*(r_{1t}), M^*(r_{2t}),..., M^*(r_{Nt}))^T$. Column vector of investment proportion of N portfolio is represented as $X=(x_1, x_2,...,x_N)^T$ and risky matrix of N asset at t is represented as Ω.

At t, upper and lower probability of return for portfolio are respectively represented as $X^TM^*(R_t)^T$ and $X^TM_*(R_t)^T$, and investment risk can be represented as $X^T\Omega_tX$.

Covariance of upper and lower probability of return on assets can be obtained according to definition, and are represented as $cov^*(R_t)$ and $cov_*(R_t)$:

$$cov_*(R_t) = \begin{bmatrix} M_*(R_{11t}) & M_*(R_{12t}) & \cdots & M_*(R_{1Nt}) \\ M_*(R_{21t}) & M_*(R_{22t}) & \cdots & M_*(R_{2Nt}) \\ \vdots & \vdots & \vdots & \vdots \\ M_*(R_{N1t}) & M_*(R_{N2t}) & \cdots & M_*(R_{NNt}) \end{bmatrix}, \tag{12}$$

$$cov^*(R_t) = \begin{bmatrix} M^*(R_{11t}) & M^*(R_{12t}) & \cdots & M^*(R_{1Nt}) \\ M^*(R_{21t}) & M^*(R_{22t}) & \cdots & M^*(R_{2Nt}) \\ \vdots & \vdots & \vdots & \vdots \\ M^*(R_{N1t}) & M^*(R_{N2t}) & \cdots & M^*(R_{NNt}) \end{bmatrix}. \tag{13}$$

The first K main elements of covariance of upper and lower probability of return on assets can be obtained according to PCA. In addition, corresponding eigenvector matrix can be obtained, and eigenvector matrix of $cov_*(R_t)$ is $U_*(\alpha_1,\alpha_2,...,\alpha_k)$, and that of $cov^*(R_t)$ is $U^*(\beta_1,\beta_2,...,\beta_k)$. Therefore, upper and lower probability of column vector of K main elements in T can be respectively represented as:

$$I_{*tk}= U_*{}^TM_*(R_t)\,, \tag{14}$$

$$I^*{}_{tk}= U^{*T}M^*(R_t)\,. \tag{15}$$

Let K main elements be first K factors of sequence GARCH effect of return on assets. After factor column vector is obtained, regression model can be created for factor through upper and lower probability of return on assets. According to arbitrage pricing theory, upper and lower probability of return on assets can be further represented as:

$$M_*(R_t)=\mu_*+s_*I_{*tk}+u_{t*}\,, \tag{16}$$

$$M^*(R_t)=\mu^*+s^*I^*{}_{tk}+u_t{}^*\,. \tag{17}$$

Where μ is return on risk-free and fixed item; $c^*I^*{}_{tk}$ and c_*I_{*tk} in Equations (15) and (16) represent upper and lower probability of systematic risk return, where $I^*{}_{tk}$ and I_{*tk} are results of PCA and respectively represent upper and lower probability of first K main elements in the upper and lower probability of covariance matrix of return on assets, and s is coefficient matrix and can be evaluated through regression method; is unsystematic risk return.

Upper and lower probability of risk return matrix can be obtained from above process:

$$\Omega_t^{\ *} = var(\mu + s^* I^*{}_{tk} + u_t^*) = s^* \sum\nolimits_t^* s^{*T} + \omega_t^{\ *}, \quad (18)$$

$$\Omega_{t_*} = var(\mu + s_* I_{*tk} + u_{t*}) = s_* \sum\nolimits_{*t} s_*^{\ T} + \omega_{t_*}. \quad (19)$$

Where Σ_t^* and Σ_{t*} are respectively upper and lower probability of covariance matrix at t, and $s^*\Sigma_t^* s^{*T}$ and $s_*\Sigma_{*t} s_*^T$ is systematic risk matrix, and ω_t is covariance matrix of random vector at t and represents unsystematic risk matrix.

According to the evaluation of, it assumes $N\times t$-dimensional matrix composed of all residual items in regression of return on N asset at t is $\hat{V}_t = (\hat{v}_1, \hat{v}_2,...,\hat{v}_t)$, and where $\hat{v}_i$ is residual vector in regression of return on assets at $i(1,...t)$. According to multivariate numerical theory, unsystematic risk matrix ω_t can be evaluated through formula $\hat{\omega}_t = \frac{1}{T-1}\times\ (\hat{v}_t\hat{v}_t^T)$.

Systematic risk of return on assets is described by fluctuation of factors. For conditional variance in each period is different, and there is no correlation among factors, i.e. no correlation among $i_1...i_k$, principal covariance matrix Σ_t only has element in diagonal line.

Thus, upper and lower probability of principal covariance matrix at t can be represented as:

$$\sum\nolimits_t^{\ *} = diag^*(\sigma_{t1}^2, \sigma_{t2}^2, ..., \sigma_{tk}^2), \quad (20)$$

$$\sum\nolimits_{t*} = diag_*(\sigma_{t1}^2, \sigma_{t2}^2, ..., \sigma_{tk}^2). \quad (21)$$

2. ESTABLISHMENT OF FUZZY GARCH MODEL

Volatility should be modeled for principal covariance matrix in Equations (20) and (21). If it assumes that return on assets is fuzzy variable, then it is equipped with fuzzy uncertainty and fuzzy GARCH model should be used to model for volatility, and j=1,2...k at t.

$$y_t = \sigma_t \varepsilon_t,$$

$$\sigma_t^{\ 2} = \alpha^l{}_0 + \sum_{i=1}^{q} \alpha^l{}_i\, y_{t-i}{}^2 + \sum_{j=1}^{p} \beta^l{}_j\, \sigma_{t-j}. \quad (22)$$

Where, $R_{it}=y_{t-i}-c_t$, i=1,2,3...,n.

At this moment, return on assets has been obscured and fuzzy set has been established for return on assets for each time series, so principal factor represented by linear of return on assets is also fuzzy. At t, membership of fuzzy set of l rule for return rj of j principal factor is $F_{lj}(rj_t)$.

Where l=1,2...L; j=1,2...k.

Then:

$$u(rj_t) = \prod_{i=1}^{k} F_{lj}(rj_t). \quad (23)$$

As Gaussian membership function is applied, result of Equation (22) is:

$$u(rj_t) = \prod_{j=1}^{n} F_{lj}(rj_t) = \prod_{j=1}^{n} \exp\left(-\frac{1}{2}\left(\frac{rj_t - c^l{}_{jt}}{b^l{}_{jt}}\right)^2\right). \quad (24)$$

Let Equation (22) multiply by Equation (24):

$$\sigma_t^{\ 2} = \frac{\sum_{j=1}^{n} u(rj_t)\left(\alpha^l{}_0 + \sum_{i=1}^{q} \alpha^l{}_i\, y_{t-i}{}^2 + \sum_{j=1}^{p} \beta^l{}_j\, \sigma_{t-j}\right)}{\sum_{j=1}^{n} u(r_{jt})}$$

$$= \sum_{j=1}^{n} g(rj_t)\left(\alpha^l{}_0 + \sum_{i=1}^{q} \alpha^l{}_i\, y_{t-i}{}^2 + \sum_{j=1}^{p} \beta^l{}_j\, \sigma_{t-j}\right). \quad (25)$$

Where:

$$g(r_{jt}) = \frac{u(rj_t)}{\sum_{j=1}^{n} u(rj_t)}.$$

Equation (22) is substituted into result:

$$y_t = \left[\frac{\sum_{l=1}^{L}\prod_{j=1}^{n} \exp\left(-\frac{1}{2}\left(\frac{rj_t - c^l{}_{jt}}{b^l{}_{jt}}\right)^2\right)\left(\alpha^l{}_0 + \sum_{i=1}^{q} \alpha^l{}_i\, y_{t-i}{}^2 + \sum_{j=1}^{p} \beta^l{}_j\, \sigma_{t-j}\right)}{\sum_{l=1}^{L}\prod_{j=1}^{n} \exp\left(-\frac{1}{2}\left(\frac{rj_t - c^l{}_{jt}}{b^l{}_{jt}}\right)^2\right)}\right]^{\frac{1}{2}} \varepsilon_t,$$

$$\sigma_t^{\ 2} = \frac{\sum_{l=1}^{L}\prod_{j=1}^{n} u(rj_t)\left(\alpha^l{}_0 + \sum_{i=1}^{q} \alpha^l{}_i\, y_{t-i}{}^2 + \sum_{j=1}^{p} \beta^l{}_j\, \sigma_{t-j}\right)}{\sum_{l=1}^{L}\prod_{j=1}^{n} u(rj_t)}. \quad (26)$$

According to Equation (26), $H_t^* = s^*\Sigma_t^* s^{*T}$; $H_{t*} = s_*\Sigma_{*t} s_*^T$. It can evaluate the upper and lower probability of H_t:

$$\hat{H}_t^{\ *} = \hat{s}^* \hat{\Sigma}_t^{\ *} \hat{s}^{*T},$$

$$\hat{H}_{t_*} = \hat{s}_* \hat{\Sigma}_{t_*} \hat{s}_*^{\ T}.$$

Where $\sum_t{}^* = diag^*(\sigma_{t1}^2, \sigma_{t2}^2, \ldots, \sigma_{tk}^2)$,

$\sum_{t*} = diag_*(\sigma_{t1}^2, \sigma_{t2}^2, \ldots, \sigma_{tk}^2)$.

3. MARKOWITZ MODEL

For securities that generate relatively steady revenue and have low risk exist in financial market, portfolio needs to meet the requirement of securities market and investors.

Return mean before t is taken as estimated value $\hat{R}_t$ of expected return of asset R_t, and estimated value of risky matrix Ω_t at t is $\hat{\Omega}_t = \hat{H}_t + \hat{\omega}_t$. When expected return r of asset portfolio is given, upper probability of quadratic programming model of the portfolio is:

$$\max X^{T*} \hat{\Omega}_t{}^* X^* \quad (27)$$
$$\text{s.t. } X^{T*} \hat{R}_t{}^* = r$$
$$X^{T*} 1_{N\times1} = 1$$

Lower probability of quadratic programming model of portfolio is:

$$\max X^T{}_* \hat{\Omega}_{t*} X_* \quad (28)$$
$$\text{s.t. } X^T{}_* \hat{R}_{t*} = r$$
$$X^T{}_* 1_{N\times1} = 1$$

Operations research method is applied to solve above equation to obtain solution of upper and lower probability portfolio.

CONCLUSION

Factor GARCH is a promotion of multivariate GARCH model, which largely simplifies parameter estimator of multivariate GARCH. However, traditional GARCH model only takes random uncertainty of return risk into account but neglects uncertainty of fuzziness of risk. In the paper, starting from fuzzy modeling of return, factor GARCH model is utilized to conduct PCA for upper and lower probability of covariance matrix with fuzzy return, and then fuzzy GARCH model is constructed for volatility of risky matrix, and consequently Markowitz model for portfolio of upper and lower probability of fuzzy portfolio is obtained. Furthermore, upper and lower probability is planned for investment proportion to receive a fuzzy internal and provide reference for investor when making decision on investment.

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