CSCanada
Management Science and Engineering
Vol. 10, No. 1, 2016, pp. 76-82
DOI:10.3968/8221

ISSN 1913-0341 [Print]
ISSN 1913-035X [Online]
www.cscanada.net
www.cscanada.org

Optimization of Project Portfolio Selection Considering Interactions Among Multiple Projects

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Received 2 November 2015; accepted 18 January 2016
Published online 16 March 2016

Abstract

In conditions of capital constraints, a single-objective nonlinear 0-1 integer programming model is proposed based on grey theory. First, application of Grey Theory deals with uncertainty of attribute weights' values given by experts and projects' scores under different attributes. Second, we construct two multi-attribute utility objective functions by comparing situations of considering interactions and without interactions, and two new multi-project portfolio optimization models are established. Finally, a numerical example illustrates effectiveness and practicality of the proposed model.

Key words: Multi-project portfolio; Grey theory; Grey numbers; Interactions

Xu, L., Li, J. M., & Zhao, Y. M. (2016). Optimization of Project Portfolio Selection Considering Interactions Among Multiple Projects. *Management Science and Engineering*, *10*(1), 76-82. Available from: URL: http://www.cscanada.net/index.php/mse/article/view/8221 DOI: http://dx.doi.org/10.3968/8221

INTRODUCTION

With the deepening of economic globalization, enterprises require diverse investment and management, at the same time investors often face many investment opportunities. However, how to pick out projects appropriately are becoming a hot issue that investors concerned about due to the limited funds or other resources. Nowadays multi-project selection has been studied by many scholars. Carlsson and Fullér (2001) put forward trapezoid fuzzy numbers to estimate future cash flows, and a mixed integer linear programming model is used to select R&D projects. Supposing that cash flows for each year and the amount of investment are fuzzy variables, Zhang, Mei, and Lu (2011) proposed a new composite index that measured benefit and risk, then established a project portfolio model and solved it by using genetic algorithm. Shou, Wang, Li, and Yao (2014) constructed a mathematical model of portfolio selection and scheduling in conditions of limited resources. In addition, scoring, AHP, multi-attribute utility theory and other methods have been widely used in the multi-project portfolio.

However, these studies have neither considered projects' interactions, nor considered investors' preference for projects' selection criteria. Interaction between projects was first proposed by Baker and Freland, and when two or more projects are selected for investment, it will produce positive or negative interactions between projects in the project portfolio (Baker & Freeland, 1975). Carlsson and Fuller (1995) pointed out that negligence of projects' interactions would result in an undesirable outcome. Therefore, it is necessary to analyze projects' interactions in portfolio selection. In recent years, studies on projects' interactions have gradually been a cause for concern. Hassanzadeh, Nemati, and Sun (2014) proposed multi-objective 0-1 integer programming model of R&D projects considering projects' interactions, where robust optimization dealt with uncertain information. Fox, Baker, and Bryant (1984) constructed a multi-objective 0-1 programming model based on benefit-dependent, risk-dependent and technology-dependent. Yu, Wang, and Wen (2012) established a multi-level project portfolio selection model considering projects' interactions, in which two examples were solved by genetic algorithm.

In addition, information that investors are able to obtain is uncertain and the grasped historical data is limited in business operations. Grey theory is a discipline which studies uncertain phenomenon, where part of the information is clear and part is unclear. In recent 30 years, the rapidly developed Grey Theory is widely used in many fields, such as economy, finance, agriculture, medicine and so on. Wong (2015) added the concept of fuzzy set and grey theory to random Markov process to improve the forecasting performance of the model. Rupak (2015) proposed a multi-attribute decision model of project portfolio in uncertain environment. Therefore, with considering projects' interactions, the paper established a multi-attribute utility decision model against uncertainty of the project implementation process, and solved it by genetic algorithms.

1. PRELIMINARIES

Grey theory which founded by Deng Julong in 1982 is a new cross-sectional discipline, and the system will be divided into white system, gray system and black system (Deng, 1989). White system's interior features are fully known, but the situation of black system is completely unknown, so we have little knowledge of information within black system. In real life, most of the systems are grey systems between black and white systems. Then we will explain briefly several definitions and calculations of Grey Theory, including the definitions of grey system and grey numbers and their algorithms.

Definition 2.1 (Deng, 1989) A grey system is defined as a system containing uncertain information presented by a grey number and grey variables.

Definition 2.2 (Deng, 1989) Let X be the universal set and the set of all real numbers, then a grey system G of X is defined by the two mappings $\overline{\mu_G}$ and $\underline{\mu_G}$ where $\overline{\mu_G}$: $X\to[0,1]$, $\underline{\mu_G}$:$X\to[0,1]$, and $\mu_G \geq \underline{\mu_G}.\overline{\mu_G}$ and $\underline{\mu_G}$are the upper and lower membership functions in G respectively.

Definition 2.3 (Deng, 1989) Grey number is defined as a number whose range is known, but whose exact number is unclear. In general, white number, gray number and black number are classified according to the uncertainty level of information. Let $\otimes x = [\underline{x}, \overline{x}] = \left\{ x \middle| \underline{x} \leq \overline{x}; \underline{x}, \overline{x} \in \Re \right\}$. Then $\otimes x$ is defined as follows that has two real parts $\overline{x}$and $\underline{x}$.

If $\underline{x}\to$-∞and $\overline{x}\to\infty$, then $\otimes x$ is black number, which means without any meaningful information.

If $\overline{x}=\underline{x}$, then $\otimes x$ is white number, which means with complete information.

Otherwise, $\otimes x$ is grey number, which means with uncertain information.

Definition 2.4 (Deng, 1989) Let $\otimes x=[\underline{x}, \overline{x}]$, $\otimes y=[\underline{y},\overline{y}]$ be two grey numbers, then the arithmetic operations between $\otimes x$, $\otimes y$are defined as follows:

a) $\otimes x+\otimes y=[\underline{x}+\underline{y}, \overline{x}+\overline{y}]$,

b) $\otimes x-\otimes y=[\underline{x}-\overline{y}, \overline{x}-\underline{y}]$,

c) $\otimes x\times\otimes y=[\min\{\underline{x}\cdot\underline{y}, \underline{x}\cdot\overline{y}, \overline{x}\cdot\underline{y}, \overline{x}\cdot\overline{y}\}, \max\{\underline{x}\cdot\underline{y}, \underline{x}\cdot\overline{y}, \overline{x}\cdot\underline{y}, \overline{x}\cdot\overline{y}\}]$,

c) $\otimes x \times \otimes y = [\min\left\{\underline{x}\cdot\underline{y}, \underline{x}\cdot\overline{y}, \overline{x}\cdot\underline{y}, \overline{x}\cdot\overline{y}\right\}, \max\left\{\underline{x}\cdot\underline{y}, \underline{x}\cdot\overline{y}, \overline{x}\cdot\underline{y}, \overline{x}\cdot\overline{y}\right\}]$,

d) $\otimes x \div \otimes y = \otimes x \times [\frac{1}{\underline{y}}, \frac{1}{\overline{y}}], 0 \notin \otimes y$,

e) $k\otimes x=[k\underline{x}, k\overline{x}]$, $k$is any real number.

Definition 2.5 (Liu, Dang, & Fang, 2004) Let $\otimes x=[\underline{x},\overline{x}]$ be any grey number, then $\tilde{\otimes} x = (1-\alpha)\underline{x} + \alpha\overline{x}, \alpha \in [0,1]$ is a weighted-average white number of grey number $\otimes x$. If $\alpha = \frac{1}{2}$, then $\tilde{\otimes} x$ is an equally weighted-average white number of grey number $\otimes x$.

2. MULTI-PROJECT PORTFOLIO BASED ON GREY THEORY

In economic activities, investors often face many investment opportunities, but due to the limited funds or other resources, how to pick out projects appropriately for investment becomes a hotspot that investors concerned about. We will discuss project portfolio selection based on grey number in this section.

Suppose there are N projects to select from, let $P=\{P_1, P_2,\cdots, P_N\}$ is an optional projects' set of N possible project alternatives, and when $x_i=1$ is defined when the project i is selected, in contrast when $x_i=0$ is defined when the project i is not selected. Besides, Let $C=\{C_1,C_2,\cdots, C_N\}$ be the set representing the assets required for the candidate projects and I be the present value of the total investment amount. In order to make full use of initial capital, the used amount of investment must be Q at least, so constraints can be defined as: $Q \leq \sum_{i=1}^{N} C_I x_i \leq M$.

First, the object set of evaluation $P=\{P_1,P_2,\cdots, P_N\}$ and the set of attributes $A=\{A_1, A_2,\cdots, A_M\}$ should be selected. A group of experts in accordance with information about the purpose of evaluation and the evaluation object selected the set of attributes. The paper selects benefit, risk and feasibility as project attributes based on Lean (Yu, Wang, & Wen, 2012).

In the implementation process of projects, the impact of different attributes on projects is different. The previous group of experts gives corresponding weights of every attribute. Clear numbers the experts' reviews are defined as are inaccurate because of uncertainty of human judgment, therefore, we define values of every attribute weights as grey numbers. Let $(\otimes w_j^1, \otimes w_j^2 \cdots \otimes w_j^k)$ be the vector of attribute weights, and $\otimes w_j^d = \left[\underline{w}_j^d, \overline{w}_j^d\right]$. Meanwhile, we give grey numbers and corresponding linguistic variables in order to standard experts' scoring, which shown in Table 1. Then the attribute weight can be calculated by simple arithmetic average according to Equation 1.

Table 1
The Grey Numbers of Attribute Weights and Corresponding Linguistic Variables

Linguistic variables	Grey numbers of attribute weights
Very Low (VL)	[0,0.1]
Medium Low (ML)	[0.1,0.3]
Low (L)	[0.3,0.4]
Medium (M)	[0.4,0.6]
High (H)	[0.6,0.7]
Medium High (MH)	[0.7,0.9]
Very High（VH）	[0.9,1.0]

$$\otimes w_j = \frac{\left[\otimes w_j^1 + \otimes w_j^2 + \cdots + \otimes w_j^k\right]}{k}. \quad (1)$$

The groups of experts score each project on different attributes, let $(\otimes r_{ij}^1, \otimes r_{ij}^2 \cdots \otimes r_{ij}^k)$ be the vector of scores which experts give to the i^{th} project on j^{th} attribute. We define scores of each project on different attributes as grey numbers, and give grey numbers and corresponding linguistic variables in order to standard experts' scoring, which shown in Table 1. Then the score of the i^{th} project on j^{th} attribute can be calculated by simple arithmetic average according to Equation 2.

Table 2
Grey Numbers of Each Project on Different Attributes and Corresponding Linguistic Variables

Linguistic variables	Grey numbers of each project on different attributes
Very Poor (VP)	[0,1]
Medium Poor (MP)	[1,3]
Poor (P)	[3,4]
Medium (M)	[4,6]
Good (G)	[6,7]
Medium Good (MG)	[7,9]
Very Good (VG)	[9,10]

$$\otimes r_{ij} = \frac{(\otimes r_{ij}^1 + \otimes r_{ij}^2 + \cdots + \otimes r_{ij}^k)}{k}. \quad (2)$$

According to scores of each project on different attributes, the grey decision matrix can be constructed D as

$$D = \begin{bmatrix} \otimes r_{11} & \otimes r_{12} & \cdots & \otimes r_{1M} \\ \otimes r_{21} & \otimes r_{22} & \cdots & \otimes r_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes r_{M1} & \otimes r_{M2} & \cdots & \otimes r_{NM} \end{bmatrix}.$$

In order to ensure $\otimes r_{ij} = \in [0,1]$, $\otimes r_{ij}$ need for standardization by the normalization method. If A_j is a gain attribute, let

$$\otimes r_{ij}^* = \left[\frac{\underline{r}_{ij}}{\max_{1\le i\le N} \bar{r}_{ij}}, \frac{\bar{r}_{ij}}{\max_{1\le i\le N} \bar{r}_{ij}}\right]. \quad (3)$$

If A_j is a loss attribute, let

$$\otimes r_{ij}^* = \left[\frac{\min_{1\le i\le N} \underline{r}_{ij}}{\bar{r}_{ij}}, \frac{\min_{1\le i\le N} \underline{r}_{ij}}{\underline{r}_{ij}}\right]. \quad (4)$$

Therefore, the grey decision matrix D^* is defined as

$$D^* = \begin{bmatrix} \otimes r_{11}^* & \otimes r_{12}^* & \cdots & \otimes r_{1M}^* \\ \otimes r_{21}^* & \otimes r_{22}^* & \cdots & \otimes r_{2M}^* \\ \vdots & \vdots & \ddots & \vdots \\ \otimes r_{M1}^* & \otimes r_{M2}^* & \cdots & \otimes r_{NM}^* \end{bmatrix}.$$

According to attribute weights, with using $\otimes v_{ij} = \otimes r_{ij}^* \times \otimes w_j$, the weighted normalized grey decision matrix X is constructed as:

$$V = \begin{bmatrix} \otimes v_{11} & \otimes v_{12} & \cdots & \otimes v_{1M} \\ \otimes v_{21} & \otimes v_{22} & \cdots & \otimes v_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes v_{M1} & \otimes v_{M2} & \cdots & \otimes v_{NM} \end{bmatrix}.$$

Using the feature of multi-attribute utility theory (MAUT) which can combine qualitative and quantitative research methods, multi-project portfolio model P(1) is constructed as follows with assumption that there are N projects which are independent.

$$\text{p(1)} \begin{cases} \text{Max} \sum_{i=1}^{N} (\sum_{j=1}^{M} \otimes v_{ij}) x_i \\ Q \le \sum_{i=1}^{n} C_i x_i \le I \\ x_i = \{0,1\} \end{cases}.$$

Since the constructed objective function of the model is a grey number, it is necessary to whiten the objective function. Accordingly, the model P(1) can be expressed as:

$$\text{P(1)'} \begin{cases} \text{Max}(1-\alpha) \sum_{i=1}^{N} (\sum_{j=1}^{M} \underline{v}_{ij}) x_i + \alpha \sum_{i=1}^{N} (\sum_{j=1}^{M} \overline{v}_{ij}) x_i \\ Q \le \sum_{i=1}^{n} C_i x_i \le I \\ x_i = \{0,1\} \\ \alpha \in [0,1] \end{cases},$$

where parameter α reflects investors' degree of optimism –pessimism and $\alpha \in [0,1]$. When $\alpha=1$, it reflects the investor is optimistic about investment projects. When $\alpha=0$, it reflects the investor is pessimism about investment projects. When $\alpha = \frac{1}{2}$, it reflects the investor is neutral about investment projects.

However, selecting and excluding one or more projects will result in significant changes of benefits, risks and feasibility in multi-project portfolio different from investment in a single project. Previous studies have considered only the impact of individual projects in different attributes to decision results, while they have ignored the impact of interactions among projects in different attributes to decision results.

Carlsson pointed out that the impact of individual projects in different attributes to decision results are different from the impact of interactions among projects in different attributes to decision results, and the negligence of projects' interactions will result in an undesirable outcome (Carlsson & Fuller, 1995). Therefore, considering interactions of different projects' attributes is very necessary.

Let $d_{ij}(S_k)$ be the interaction in the j^{th} attribute between i^{th} project and k^{th} project, and $d_{ij}(S_k)\in[-1,1]$. When $d_{ij}(S_k)=0$, it reflects the interaction in the j^{th} attribute between i^{th} project and k^{th} project is nonexistent. When $-1\le d_{ij}(S_k)<0$, it reflects the interaction in the j^{th} attribute between i^{th} project and k^{th} project is negative. When $0<d_{ij}(S_k)\le 1$, it reflects the interaction in the j^{th} attribute between i^{th} project and k^{th} project is positive. Using the feature of multi-attribute utility theory (MAUT), multi-project portfolio model P(2) is constructed as follows with assumption that there are N projects which are interactive.

$$
\text{P(2)}\begin{cases}
\text{Max}(1-\alpha)\{\ \sum_{i=1}^{N}(\sum_{j=1}^{M}\underline{v}_{ij})x_i+\sum_{i=1}^{N}\sum_{k\neq i,k=1}^{N}[\sum_{j=1}^{M}d_{ij}(S_K)(\underline{v}_{ij}+\underline{v}_{kj})]x_ix_k\ \} \\
\qquad +\alpha\{\ \sum_{i=1}^{N}(\sum_{j=1}^{M}\overline{v}_{ij})x_i+\sum_{i=1}^{N}\sum_{k\neq i,k=1}^{N}[\sum_{j=1}^{M}d_{ij}(S_K)(\overline{v}_{ij}+\overline{v}_{Kj})]x_ix_k\ \} \\
Q\le\sum_{i=1}^{n}C_ix_i\le I \\
x_i=\{0,1\} \\
\alpha\in[0,1]
\end{cases}
$$

Since the constructed objective function of the model is a grey number, it is necessary to whiten the objective function. Accordingly, the model P(2) can be expressed as:

$$
\text{P(2)'}\begin{cases}
\text{Max}(1-\alpha)\{\ \sum_{i=1}^{N}(\sum_{j=1}^{M}\underline{v}_{ij})x_i+\sum_{i=1}^{N}\sum_{k\neq i,k=1}^{N}[\sum_{j=1}^{M}d_{ij}(S_K)(\underline{v}_{ij}+\underline{v}_{kj})]x_ix_k\ \} \\
\qquad +\alpha\{\ \sum_{i=1}^{N}(\sum_{j=1}^{M}\overline{v}_{ij})x_i+\sum_{i=1}^{N}\sum_{k\neq i,k=1}^{N}[\sum_{j=1}^{M}d_{ij}(S_K)(\overline{v}_{ij}+\overline{v}_{Kj})]x_ix_k\ \} \\
Q\le\sum_{i=1}^{n}C_ix_i\le I \\
x_i=\{0,1\} \\
\alpha\in[0,1]
\end{cases}
$$

where parameter α reflects investors' degree of optimism–pessimism and $\alpha\in[0,1]$. When $\alpha=1$, it reflects the investor is optimistic about investment projects. When $\alpha=0$, it reflects the investor is pessimism about investment projects. When $\alpha=\frac{1}{2}$, it reflects the investor is neutral about investment projects.

Model P(1)' and P(2)' are nonlinear multi-constrained single objective 0-1 integer programming, and conventional approach is difficult to find an optimum solution. In this paper, we use genetic algorithms to solve it.

3. EMPIRICAL ANALYSIS

Supposing there are N projects to selected from and $P=\{P_1,P_2,\cdots,P_9\}$, The paper selects benefit, risk and feasibility as project attributes based on Lean, and $J=\{A_1$.benefjit; A_2.risk; A_3.feasibility$\}$, where benefit and feasibility are gain attributes and risk is loss attribute. Besides, the assets required for the candidate projects are shown in Table 4, and the present value of the total investment amount is 500 (I=500) and the lower limit of the total investment amount is 300 (Q=300). For comparison purposes, the paper made two calculations of the model with and without consideration of the projects' interaction in different attributes.

Table 4
The Assets Required for the Candidate Projects (Initial Investment Amount IIA)

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9
IIA	90	110	120	100	100	80	100	120	150

According to Table 1, five experts$\{D_1,D_2,\cdots D_5\}$ respectively assign the values of attributes' weights. By using Equation (1), the attribute weight can be obtained and the results are shown in Table 5.

Table 5
The Values of Attributes' Weights

	D_1	D_2	D_3	D_4	D_5	$\otimes\omega_j$
Benefit	H	M	MH	H	ML	[0.48,0.64]
Risk	VH	H	VH	VH	VH	[0.84,0.94]
Feasinbility	MH	H	H	MH	VH	[0.70,0.84]

According to Table 2, the five experts score 9 projects on different attributes. By using Equation (1), scores of each project on different attributes can be obtained and the results are shown in Table 6.

Table 6
Scores of Each Project on Different Attributes

A_j	p_i	D_1	D_2	D_3	D_4	D_5	$\otimes r_{ij}$
	P_1	MG	G	G	M	MG	[6.0,7.2]
	P_2	G	MG	MG	VG	G	[7.0,8.4]
	P_3	G	M	MP	MP	P	[3,4.6]
	P_4	P	G	M	M	G	[4.6,6]
Benefit	P_5	MG	MG	G	G	VG	[7,8.4]
	P_6	M	G	G	M	P	[4.6,6]
	P_7	P	P	MP	G	M	[3.4,4.8]
	P_8	MP	P	P	G	G	[3.8,5]
	P_9	VG	G	MG	VG	MG	[7.6,9]

To be continued

Continued

A_j	p_i	D_1	D_2	D_3	D_4	D_5	$\otimes r_{ij}$
	P_1	G	M	M	G	M	[4.8,6.4]
	P_2	P	P	MP	M	MP	[2.4,4]
	P_3	G	MG	MG	G	VG	[7,8.4]
	P_4	M	P	VG	G	G	[5.6,6.8]
Risk	P_5	P	G	G	M	G	[5,6.2]
	P_6	G	MG	MG	G	VG	[7,8.4]
	P_7	VG	G	MG	G	G	[6.8,8]
	P_8	M	MG	G	G	M	[5.4,7]
	P_9	P	MP	MP	VP	MP	[1.2,2.8]

Continued

A_j	p_i	D_1	D_2	D_3	D_4	D_5	$\otimes r_{ij}$
	P_1	G	G	MG	G	VG	[6.8,8]
	P_2	MG	M	VG	G	G	[6.4,7.8]
	P_3	MP	MP	M	P	VP	[1.8,3.4]
	P_4	G	G	MG	M	G	[5.8,7.2]
Feasibility	P_5	P	MP	M	M	P	[3,4.6]
	P_6	G	VG	MG	MG	G	[7,8.4]
	P_7	MG	MG	M	G	MG	[6.2,8]
	P_8	P	MP	M	VP	P	[2.2,3.6]
	P_9	VG	G	MG	G	M	[6.4,7.8]

According to Table 6, the grey decision matrix D can be constructed and the result is shown in Table 7.

Since benefit and feasibility are gain attributes and risk is loss attribute, by using Equations (3) and (4), the normalized grey decision matrix D^* can be constructed and the result is shown in Table 8.

Table 7
Grey Decision Matrix (D)

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9
Benefit	[6.0,7.2]	[7.0,8.4]	[3,4.6]	[4.6,6]	[7,8.4]	[4.6,6]	[3.4,4.8]	[3.8,5]	[7.6,9]
Risk	[4.8,6.4]	[2.4,4]	[7,8.4]	[5.6,6.8]	[5,6.2]	[7,8.4]	[6.8,8]	[5.4,7]	[1.2,2.8]
Feasibility	[6.8,8]	[6.4,7.8]	[1.8,3.4]	[5.8,7.2]	[3,4.6]	[7,8.4]	[6.2,8]	[2.2,3.6]	[6.4,7.8]

Table 8
Normalized Grey Decision Matrix (D^*)

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9
Benefit	[0.667,0.8]	[0.778,0.933]	[0.333,0.511]	[0.511,0.667]	[0.778,0.933]	[0.511,0.667]	[0.378,0.533]	[0.422,0.556]	[0.844,1]
Risk	[0.188,0.25]	[0.3,0.5]	[0.143,0.171]	[0.176,0.214]	[0.194,0.24]	[0.143,0.171]	[0.15,0.176]	[0.171,0.222]	[0.429,1]
Feasibility	[0.810,0.952]	[0.762,0.929]	[0.214,0.405]	[0.690,0.857]	[0.357,0.548]	[0.833,1.000]	[0.738,0.952]	[0.262,0.429]	[0.762,0.929]

By using Equation $\otimes v_{ij}=\otimes r^*_{ij}\times\otimes w_j$, the weighted normalized grey decision matrix X is constructed and the result is shown in Table 9.multi-project portfolio model P(1)' is constructed as:

Table 9
Weighted Normalized Grey Decision Matrix

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9
Benefit	[0.32,0.512]	[0.373,0.597]	[0.16,0.327]	[0.245,0.427]	[0.373,0.597]	[0.245,0.427]	[0.181,0.341]	[0.203,0.356]	[0.405,0.64]
Risk	[0.158,0.235]	[0.252,0.47]	[0.12,0.161]	[0.148,0.201]	[0.163,0.226]	[0.12,0.161]	[0.126,0.165]	[0.144,0.209]	[0.36,0.94]
Feasibility	[0.567,0.8]	[0.533,0.78]	[0.15,0.34]	[0.483,0.72]	[0.25,0.46]	[0.583,0.84]	[0.517,0.8]	[0.183,0.36]	[0.533,0.78]

$$\text{P(1)'}\begin{cases}\text{Max}(1-\alpha)(1.045x_1+1.158x_2+0.43x_3+0.876x_4+0.786x_5+0.948x_6+0.824x_7+0.53x_8+1.298x_9)\\ \quad +\alpha(1.547x_1+1.847x_2+0.828x_3+1.348x_4+1.283x_5+1.428x_6+1.306x_7+0.925x_8+2.36x_9)\\ 300\le 90x_1+110x_2+120x_3+100x_4+100x_5+80x_6+100x_7+120x_8+150x_9\le 500\\ x_i=\{0,1\}\\ \alpha\in[0,1]\end{cases}$$

Table 10 shows the values of projects' interactions given by five experts, with consideration of projects' interactions in benefit, risk and feasibility, multi-project portfolio model P(2)' is constructed as:

Table 10
Values of Projects' Interactions in Benefit, Risk and Feasibility

	P_1P_2	P_1P_5	P_1P_7	P_2P_6	P_3P_8	P_4P_5	P_5P_9	P_6P_9	P_7P_8	P_8P_9
Benefit	0.2	0.45	0.4	0.65	0.450	0.35	0.55	0.4	0.6	0.55
Risk	-0.15	-0.25	-0.14	-0.15	-0.20	-0.05	-0.10	-0.15	-0.25	-0.15
Feasibility	0	0	0.35	-0.42	-0.20	0	-0.10	0.15	0	0.8

According to model p(1)’and p(2)’, by using genetic algorithm, the multi-project portfolio issues can be solved to obtain maximize utility portfolio. The number of individuals in the population is 100, and the maximum number of iterations is 200. Besides, crossover probability is 0.85, and mutation probability is 0.1. Due to different risk preferences of investors, investors’ degree of optimism–pessimism can be adjusted by controlling parameter α. Interval of parameter α is 0.001. Operating results are shown in Table 11 and 12.

Table 11
Operating Results Without Consideration of Interactions

Parameter	Objective function value	Selected projects
α=0	4.7510	x_1, x_2, x_4, x_6, x_7
α=0.001	4.7512	x_1, x_2, x_4, x_6, x_7
α=0.002	4.7514	x_1, x_2, x_4, x_6, x_7
⋮	⋮	⋮
α=0.6667	6.1195	x_1, x_2, x_4, x_6, x_7
α=0.6668	6.1197	x_1, x_4, x_5, x_6, x_7
α=0.6669	6.1200	x_1, x_4, x_5, x_6, x_7
α=0.6670	6.1202	x_1, x_4, x_5, x_6, x_7
⋮	⋮	⋮
α=1	6.9400	x_1, x_4, x_5, x_6, x_7

Table 12
Operating Results With Consideration of Interactions

Parameter	Objective function value	Selected projects
α=0	5.6316	x_1, x_2, x_4, x_5, x_7
α=0.001	5.6319	x_1, x_2, x_4, x_5, x_7
α=0.002	5.6321	x_1, x_2, x_4, x_5, x_7
⋮	⋮	⋮
α=0.5132	7.0184	x_1, x_2, x_4, x_5, x_7
α=0.5133	7.0187	x_1, x_4, x_5, x_6, x_7
α=0.5134	7.0190	x_1, x_4, x_5, x_6, x_7
α=0.5135	7.0194	x_1, x_4, x_5, x_6, x_7
⋮	⋮	⋮
α=1	8.5060	x_1, x_4, x_5, x_6, x_7

According to Table 11 and 12, some conclusions can be obtained.

a) When bigger α is and the more optimistic investors are, the higher level of selected projects’ utility is. Because optimistic investors prefer the larger value closer to the right side when the objective functions are whitened. This is consistent with the actual situation.

b) With the changes of parameter α, investors’ degree of optimism–pessimism changes, selected projects also will change. According to Table 11, when $0 \le \alpha \le 0.6667$, the result of selected projects is x_1, x_2, x_4, x_6, x_7. When $0.6667 \le \alpha \le 1$, the result of selected projects is x_1, x_4, x_5, x_6, x_7. According to Table 12, when $0 \le \alpha \le 0.5132$, the result of selected projects is x_1, x_2, x_4, x_5, x_7. When $0.5132 \le \alpha \le 1$, the result of selected projects is x_1, x_4, x_5, x_6, x_7.

c) By comparing Table 11 with Table 12, when parameters α are same, investors’ degrees of optimism–pessimism are same. The utility level of consideration of interactions is higher than that without consideration. In the portfolio selection process, consideration of interactions will help to select projects with synergistic effect, and avoid selecting projects with competitive relationship. Therefore, the objective function with consideration of interactions is higher.

Therefore, in the portfolio selection process, investors should set appropriate parameters according to their actual situation, at the same time, when selecting or excluding certain projects, investors should consider the interactions among projects.

CONCLUSION

In conditions of capital constraints, a single-objective nonlinear 0-1 integer programming model is proposed based on grey theory, in which the objective functions are multi-attribute utility function. Experts’ reviews that defined as clear numbers are inaccurate because of uncertainty of human judgment, so we define values of every attribute weights and scores of each project on different attributes as grey numbers. Meanwhile, in the portfolio selection process, selecting and excluding one or more projects will result in significant changes of benefits, risks and feasibility in multi-project portfolio, which is different from investment in single project, and the negligence of projects’ interactions will result in an undesirable outcome. Therefore, it is very necessary to consider interactions of different projects’ attributes. For comparison purposes, the paper made two calculations of models considering interactions and without interactions in different attributes. Finally, a numerical example illustrates effectiveness and practicality of the proposed model.

However, this study also has some shortcomings. For example, the paper only considers interactions between two projects, because interactions among three or more projects will increase difficulty of calculation. At the same time, constraints given in this paper are only capital constraint, but in real economic activity, investment constraints are not just the case. Therefore, future research could extend in two aspects. On one hand, the research can try to consider interactions among three or more projects. On the other hand, increasing the number of constraints will improve the usefulness of the model.

REFERENCES

Baker, N., & Freeland, J. (1975). Recent advances in R&D benefit measurement and project selection methods. *Management Science, 21,* 1164-1175.

Carlsson, C., & Fullér, R. (2001). On possibilistic mean value and variance of fuzzy numbers. *Fuzzy Sets and Systems, 122*(2), 315-326.

Carlsson, C., & Fuller, R. (1995). Multiple criteria decision making: the case for interdependence. *Computers & Operations Research, 22,* 251-260.

Deng, J. L. (1989). Introduction to grey system theory. *Journal of Grey System, 1*(1), 1-24.

Fox, G. E., Baker, N. R., & Bryant, J. L. (1984). Economic models for R&D Project selection in the presence of project interactions. *Management Science, 30*(7), 890- 902.

Hassanzadeh, F., Nemati, H., & Sun, M. (2014). Robust optimization for interactive multiobjective programming with imprecise information applied to R&D project portfolio selection. *European Journal of Operation Research, 238(1),* 41-53.

Liu, S. F., Dang, Y. G., & Fang, Z. G. (2004). *Grey system theory and its applications.* Science Press.

Rupak, B. (2015). A grey theory based multiple attribute approach for R&D project portfolio selection. *Fuzzy Information and Engineering, 7,* 211-225.

Shou, Y. Y., Wang, C. C., Li, Y., & Yao, W. J. (2014). Doubledecision method of project portfolio selection and scheduling. *Journal of Systems Management, 23*(4), 489-494.

Wong, H. L. (2015). Time series forecasting with stochastic Markov models based on fuzzy set and grey theory. *Applied Mechanics & Materials, 764,* 975- 978.

Yu, L., Wang, S., & Wen, F. (2012). Genetic algorithm-based multi-criteria project portfolio selection. *Annals of Operations Research, 197*(1), 71-86.

Zhang, W. G., Mei, Q., & Lu, Q., et al. (2011). Evaluating methods of investment project and optimizing models of the portfolio selection in the fuzzy uncertainty. *Computers & Industrial Engineering, 61*(3), 721-728.

