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The Coordination of Group Preference Based on Compatible Relation

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Abstract

A number of theoretical approaches to preference relations for multiple attribute group decision making (MAGDM) problem. Consistency among the preference relations is very important to the result of the final decision. This paper is proposed procedure which based on compatible preference relation sets to solve multiple attribute group decision making problems. These sets are satisfied all experts additive consistency and determine weight of expert on every attribution. Moreover, we can adjust assessment of expert though the sets. Finally a numerical example is used to test the proposed approach and the results illustrate that the method is simple, effective, and practical.

Key word: Multiple attribute group decision making; Compatible preference relation; Opinion coordination mechanism; Weight distribution

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INTRODUCTION

There are many situations in our daily life and in the workplace which need us making a decision problem. When more than one person is interested in the same problem, it then becomes a group decision making problem. However, no single alternative works is best for all performance attributes, and the assessment of each alternative given by different decision makers diverge considerably. Preference relations are comparisons between two alternatives for a particular attribute. A higher preference relation means that there is a higher degree of preference for one alternative over another. After experts have provided their assessment of the preference relation, the appropriateness of the comparison from each expert must be tested. Consistency is one of the important properties for verifying the appropriateness of choices (Herrera-Viedma, Herrera, Chiclana, & Luque, 2004).

Over the last decade there has a lot of studies on multiple attribute group decision making consensus reached. Herrera-Viedma, Herrera-Viedma, and Chiclana (2001) used the method of measuring distance to solve the group decision-making problem of consistent heterogeneous favor. Xu and Chen (2008) use entropy method for group decision-making in the weight vector of each attribute, then put forward a matrix constituted by each attribute weights and group preference to obtain comprehensive evaluation each scheme, which can range for the optimal solution. J. X.Wang and W. J. Wang (2007) proposed a method to rank and select alternatives with evaluation indicators, which include indexes of similarity and concentration. Ensuring the consistency of result, Sun and Tian (2008) transformed the complex decision-making problem into multistage decision problem. Xu and Xu (2008) proposed a comprehensive evaluation method of multi-attribute group decision-making, which through several rounds of interaction to approach satisfaction solution.

However, heterogeneity among experts should also be considered (Olcer & Odabas, 2005). For example, if the expert who assigns the greatest weight to a preference relation makes different choices evaluations of the other experts who assign lower weights, then the group decision procedure can be distorted and imperfect. Moreover, the

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determination of attribute weight is also an important issue (Boz´oki, 2008). Some attributes are considered to be more important in the experts' professional judgment. But for these important attributes, the preference relation provided by experts may be dissimilarity. Previous research hasn't addressed all of the issues simultaneously.

This paper proposed the compatible concept of evaluated opinions, then use the experts' preference relation of evaluated compatibility to revise the expert's opinion and make the final decision consistently. At the same time, this study assigns the expert weight on the basis of attribute score in compatible set to ensure experts are satisfied with this score, which will improve the consensus of group decision.

The remainder of this paper is organized as follows: In Section 1 Introduce the normal model of Multiple Attribute Group Decision-Making, and definite its parameters. Then we proposed compatibility relation concept in Section 2, and build the compatibility relation model. In Section 3 we describe the procedure of group preference aggregation, which aim at reaching consensus in the final decision by revise assessment of expert through the compatibility relation sets. Then the model is tested and examined with an example. Finally summarizes conclusions.

1. MULTIPLE ATTRIBUTE GROUP DECISION MAKING MODEL DEFINITION

In general, experts are asked to evaluate all pairs of alternatives and then construct a preference matrix with full information. Assuming there are t ($t \geq 2$) experts in the group decision-making $E = \{e_1, e_2, \cdots, e_t\}$, and the schemes for experts to choose are limited $A = \{a_1, a_2, \cdots, a_m\}$, $m \geq 2$. It respect to have an particular criteria on evaluating the schemes by some attributes, the scheme of attribute set, C $= \{c_1, c_2, \cdots, c_n\}$, $n \geq 2$. Each expert gives the assessment by score. These scores are constituted an evaluation matrix $X = \{(v_{i,j}^k)_{m\times n}\}$, it represents the expert e_k ($0 \leq k \leq t$) makes evaluation about attribute c_j ($0 \leq j \leq n$) of scheme a_i ($0 \leq i \leq m$). Experts weight set is $W = \{w_1, w_2, \cdots, w_t\}$, $t \geq 2$, and $0 \leq w_k \leq 1$, $\Sigma_{k=1}^{t} w_k = 1$.

Define1 Expert score value of evaluation scheme $X = \{v_{i,j}^k\}$ $v_{i,j}^k$ is integer and range from 1 to 10 Average value $\tau_{i,j}$ is all experts initial average score about attribute c_j of a_i scheme. It calculates as follow:

$$\tau_{i,j} = \frac{1}{n}\sum_{k=1}^{n} v_{i,j}^k \qquad (1)$$

Expert average value calculation is in order to negotiate consensus in the process initially, it wants to retain experts' original score.

Define2 Difference between experts score about attribute of scheme a_i is

$$d(e_p, e_q) = |v_{i,j}^p - v_{i,j}^q| , \qquad (2)$$

is a key point for expert consensus reached, it a judgment for us to find out which expert score on the basis of compatibility need to revise in the process.

2. THE COMPATIBLE PREFERENCE RELATION MODEL

2.1 Compatible Preference Relation

Compatible preference relation is the relation own reflexivity and symmetry on the given numerical area, M is a given set. An arbitrary subset R of is called Binary relation, $M\times M$ is all field relations.

(1) If $\forall x \in A$, and $\langle x, x\rangle \in R$,, then R is reflexive;

(2) $\forall x, y \in A$, if $\langle x, y\rangle \in R$, and $\langle y,x\rangle \in R$, then R is symmetrical;

(3) If the relation has reflexivity and symmetry at the same time, we say it has a compatible preference relation.

Define3 If any two elements conform to the compatible preference relation, we define it compatible classes, and these elements construct compatible sets.

Let R is compatible preference relation of compatible sets M. $\forall x \in A$, the compatible classes of x about M is

$$T_R(x) = \{y | y \in M \cap \langle x, y\rangle \in R\} . \qquad (3)$$

2.2 Expert Group Compatible Classes

For each comparison between a pair of alternatives, the assessment of the preference relations provided by experts would be different. Arrow's impossibility theorem had proved that it is not reasonable for decision-making group to achieve complete consistency assessment on a scheme (Enrique, Herrera, & Chiclana, 2002). Actually, the goal of group decision making problems is to find an optimal solution lets all experts are satisfied with final decision. In the process of evaluation, as long as the score differences between experts within a certain threshold, the scheme evaluation is acceptable.

Define 4 Setting a threshold μ If the evaluation value of expert e_p and experton e_q the attribute c_j of scheme a_i is satisfied with the following conditions, they can be classified as compatible classes T_{ij}:

If $p = q$, then $\langle e_p, e_q\rangle \in T_{i,j}$;

$$\text{If } p \neq q, \text{ when } d(e_p,e_q) = |v_{i,j}^p - v_{i,j}^q| \leq \mu \quad \text{then } \langle e_p,e_q\rangle \in T_{i,j} . \qquad (4)$$

3. GROUP PREFERENCE AGGREGATION

3.1 Group Consensus Judgment

When identifying whether decision-making group achieve consensus, we need to establish a compatible classes $T_{i,j}$, $i = 1,2,\cdots m$; $j = 1,2,\cdots n$ between experts in firstly. Then $T_{i,j} = M\times M$, it means all experts reach a consensus on evaluation of attribute c_j of scheme a_i; If $\forall T_{i,j} = M\times M$, $i = 1,2,\cdots m$; $j = 1,2,\cdots n$, that all evaluation of schemes reach consensus.

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3.2 Expert Weight Distribution Based on the Compatible Classes

When all the experts are satisfied $\forall T_{i,j} = M\times M$, in this case each expert weight is equal, the expert weight $w_k = \frac{1}{t}$ $(0 \le k \le t)$; If $\exists T_{i,j} \ne M\times M$, we need to calculate expert weight depend on the number of expert opinion proportion in compatible classes. Let $N_k(T_{i,j})$ as the number of expert e_k whose opinion is contained in compatible classes, and its weight is

$$w_k = \frac{N_k(T_{i,j})}{\sum_{k=1}^{t} N_k(T_{i,j})} . \quad (5)$$

3.3 Evaluate Adjustment

Experts need to modify the corresponding evaluation information when the assessment didn't reach a consensus. If expert doesn't want to adjust his score of evaluation value, we have to set his score to average value τ_{ij}, as the average value has considered all experts' initial opinion on attribute.

In the case $T_{i,j} \ne M\times M$, adjustment as follows:

Step 1: Using formula (1) to calculate the average value τ_{ij}, which represents all experts assessment score on attributes c_j of scheme a_i;

Step 2: $\forall e_k \in M$, establish the compatible classes $T_{i,j}(e_k)$ of each expert according to Formula (3)

$$T_{i,j}(e_k) = \{e_v \mid e_v \in M \cap \langle e_k, e_v \rangle \in T_{i,j}\} .$$

We can find out which experts didn't reach agreement through analyzing the compatible classes, and which expert have a poor degree of consensus with other experts.

If $es \notin Ti,j(ek)$, expert es and expert ek didn't reach consensus; when $|T_{i,j}(e_p)| = \min\{|T_{i,j}(e_p)| \mid e_k \in M\}$, it means expert e_p the minimum compatible classes, indicating there are few experts come to agreement with expert.

Step3: Let M' represents the minimum expert set of compatible classes. If there is $\forall e_p \in M'$, we calculated$(v_{i,j}^p, \tau_{ij})$

Step4: Let σ represents the minimum score.

If $d(v_{i,j}^p, \tau_{i,j}) \ge \mu$, let $v_{i,j}^{p'} = v_{i,j}^p \pm \mu$ (be near to average value);if

$$d(v_{i,j}^p, \tau_{i,j}) < \mu \begin{cases} d(v_{i,j}^p, \tau_{i,j}) \ge \sigma, & v_{i,j}^{p'} = v_{i,j}^p \pm \sigma \\ d(v_{i,j}^p, \tau_{i,j}) < \sigma, & v_{i,j}^{p'} = v_{i,j}^p \end{cases} .$$

In addition, expert is unwilling to modify his opinions confirmedly, we shall set $v_{i,j}^{p'} = \tau_{i,j}$.

3.4 Methods for Reaching Consensus

Based on the above analysis, the consensus reaching method of multiple attribute group decision-making as follows:

Step1: Setting evaluate consistency degree threshold μ.

Step2: According to the definition4, establishing experts' compatible classes $T_{i,j}$, $i = 1,2,\cdots m$; $j = 1,2,\cdots n$ Observation expert set of compatible classes, if there is $\forall T_{i,j} = M\times M$, $i = 1,2,\cdots m$; $j = 1,2$ it indicates that all experts reach a consensus.

Step3: According to expert evaluation compatible classes $T_{i,j}$ and Formula (5), calculate the weight w_k of each expert.

Step4: If $T_{i,j} = M\times M$, based on the evaluate adjustment described in part 4.3, modify the evaluate value of expert who isn't concluded in compatible classes $T_{i,j}$; otherwise, turn into step 5.

Step5: According to the modified evaluation value, use each expert weight and his corresponding score matrix to calculate comprehensive score matrix V_A of each scheme. Attributes score in Matrix of each scheme v_j^{ai} is

$$v_j^{ai} = \sum_{k=1}^{i} wk \times v_{i,j}^{k'} . \quad (6)$$

Then calculate comprehensive score value V_{ai} of each scheme based on comprehensive score matrix, the highest Score scheme will be selected to be the optimal decision scheme.

$$V_{ai} = \sum_{j=1}^{n} v_j^{ai} . \quad (7)$$

Step6: Over.

4. APPLICATION

Cadres selection is a multiple index comprehensive evaluation problem. Assuming there are four evaluation indexes c_j (j=1,2,3,4) for a department to select its cadre. The indexes are work ability (c_1), Ideological and moral (c_2), educational level (c_3) and work style (c_4). In order to evaluate candidates comprehensively, it invites 3 aspects of expert group ek (k = 1,2,3) to grade candidates of each index respectively. The expert groups including masses (e_1), experts (e_2) and scholars (e_3). After statistical processing, it elected 3 candidates. The question is to pick out the most suitable one among them served as cadres. Experts score each candidate on each evaluation index, values as follows:

$$V^1 = \begin{bmatrix} 9 & 8 & 7 & 7 \\ 6 & 7 & 5 & 9 \\ 5 & 6 & 5 & 9 \end{bmatrix} \quad V^2 = \begin{bmatrix} 6 & 5 & 8 & 7 \\ 5 & 9 & 4 & 6 \\ 6 & 6 & 7 & 8 \end{bmatrix} \quad V^3 = \begin{bmatrix} 5 & 7 & 9 & 6 \\ 4 & 8 & 6 & 6 \\ 8 & 9 & 6 & 7 \end{bmatrix} .$$

V^1 is the score value matrix of e_1, V^2 is score value matrix of e_2, V^3 is score value matrix of e_3.

The evaluation process:

Step1: Setting evaluation threshold μ=2 of consistency, it means that the gap of expert' score on each index must Less than this value.

Step2：Combined with the formula (3) and formula (4), establish the compatible relation of each expert and build compatible classes $T_{i,j}$, ($i = 1,2,\cdots m$; $j = 1,2,\cdots n$).

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We find that the compatible classes of group are $T_{1,3} = T_{1,4} = T_{2,1} = T_{2,2} = T_{2,3} = T_{3,3} = T_{3,4} = M\times M$, it indicates that all experts reach a consensus of evaluated value on each index c_j of each scheme a_i The compatible classes of the experts reached a consensus on some scheme index as follows:

$$T_{1,1} = \{\langle e_2,e_3\rangle,\langle e_3,e_2\rangle\},$$
$$T_{1,2}=\{\langle e_1,e_3\rangle,\langle e_2,e_3\rangle,\langle e_3,e_2\rangle,\langle e_2,e_3\rangle\},$$
$$T_{2,4} = \{\langle e_2,e_3\rangle,\langle e_3,e_2\rangle\},$$
$$T_{3,1} = \{\langle e_1,e_2\rangle,\langle e_2,e_1\rangle,\langle e_2,e_3\rangle,\langle e_3,e_2\rangle\},$$
$$T_{3,2} = \{\langle e_1,e_2\rangle,\langle e_2,e_1\rangle\}.$$

Step3: According to expert evaluation compatible classes, we calculate the weight w_k of each expert. As the situation $\exists T_{i,j} \neq M\times M$ is existed, then we need to count the number of each expert in the compatible classes.$T_{i,j} \neq M\times M$ The results are

$$N_1(T_{i,j}) = 2,\ N_2(T_{i,j}) = 5, N_3(T_{i,j}) = 4 .$$

Then according to Formula (5) we can get the experts weight respectively:

$$w_1 = 0.182, w_2 = 0.454, w_3 = 0.364 .$$

Step4: As there is $T_{i,j} \neq M\times M$, using Formula (1) $\tau_{i,j} = \frac{1}{n}\sum_{k=1}^{n} v_{i,j}^{k}$ to calculate each index evaluation mean value:

$$\begin{array}{lll} \tau_{1,1} = 6.7 & \tau_{1,2} = 6.7 & \tau_{1,3} = 8 \\ \tau_{1,4} = 6.7 & \tau_{2,1} = 5 & \tau_{2,2} = 8 \\ \tau_{2,3} = 5 & \tau_{2,4} = 7 & \tau_{3,1} = 5.3 \\ \tau_{3,2} = 7 & \tau_{3,3} = 6, & \tau_{3,4} = 8 \end{array}$$

Based on the evaluation adjustment process described in part 4.3, modify expert's evaluation value which isn't reached consensus on attributeof each scheme. Due to the scale of evaluation score is 1, so we set.

Then set up equivalence class for each expert：

(1) $T_{1,1}(e_1) = \{e_1\}$, $T_{1,1}(e_2) = \{e_2,e_3\}$, $T_{1,1}(e_3) = \{e_2,e_3\}$, We easily find out the minimum expert set of compatible classes $M' = \{e_1\}$, so expert e_1 needs to adjust his evaluate value $v_{1,1}^1$. Due to $d(v_{1,1}^1,\tau_{1,1}) = 2.3 \geq \mu$ that calculate by Formula (2), the evaluation value of expert adjust as $v_{1,1}^1=7$

(2) $T_{1,2}(e_1) = \{e_1,e_2\}$, $T_{1,2}(e_2) = M$, $T_{1,2}(e_3) = \{e_2,e_3\}$, The minimum expert set of compatible classes $M' = \{e_1,e_3\}$, Due to $d(v_{1,2}^1,\tau_{1,2}) = 1.3 \leq \mu$, the evaluation value of expert e_1 is adjust as $v_{1,2}^1 = 7$; $d(v_{1,2}^3,\tau_{1,2}) = 0.3 \leq \mu$, and $0.3<\sigma$, the evaluation value of expert e_3 is adjust as $v_{1,2}^3=7$.

(3) $T_{2,4}(e_1) = \{e_1\}$, $T_{2,4}(e_2,e_3)$, $T_{2,4}(e_3) = \{e_2,e_3\}$, The minimum expert set of compatible classes $M' = \{e_1\}$, as $d(v_{2,4}^1,\tau_{2,4}) = 2 = \mu$, the evaluation value of expert e_1 is adjust as $v_{2,4}^1=7$.

(4) $T_{3,1}(e_1) = \{e_1,e_2\}$, $T_{3,1}(e_2) = M$, $T_{3,1}(e_3) = \{e_2,e_3\}$, the minimum expert set of compatible classes $M' = \{e_1,e_3\}$, $d(v_{3,1}^1,\tau_{3,1}) = 0.3 \leq \mu$, and $0.3<\sigma$ the evaluation value of expert e_1 is adjust as $v_{3,1}^1 = 5$; and $d(v_{3,1}^3,\tau_{3,1}) = 2.7 \geq \mu$, the evaluation value of expert e_3 is adjust as $v_{3,1}^3=6$.

(5) $T_{3,2}(e_1) = \{e_1,e_2\}$, $T_{3,2}(e_2) = \{e_1,e_2\}$, $T_{3,2}(e_3) = \{e_3\}$,The minimum expert set of compatible classes $M' = \{e_3\}$, $d(v_{3,2}^3,\tau_{3,2}) = 2 = \mu$, the evaluation value of expert adjust as $v_{3,2}^3=7$.

After modified the evaluate value of experts, we get a new evaluation matrix of each expert:

$$V^{1'} = \begin{bmatrix} 7 & 7 & 7 & 7 \\ 6 & 7 & 5 & 7 \\ 5 & 6 & 5 & 9 \end{bmatrix} \quad V^{2'} = \begin{bmatrix} 6 & 5 & 8 & 7 \\ 5 & 9 & 4 & 6 \\ 6 & 6 & 7 & 8 \end{bmatrix} \quad V^{3'} = \begin{bmatrix} 5 & 7 & 9 & 6 \\ 4 & 8 & 6 & 6 \\ 6 & 7 & 6 & 7 \end{bmatrix}$$

At this point, we can easily find that there has $\forall T_{i,j} = M\times M$, it indicates experts have reached a consensus evaluation on each scheme of each index . So we can enter next step.

Step5: Using the Formula (6) combined with the expert weight to calculate the comprehensive score values of each candidate V_{ai}.

$$V_A = \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} \begin{bmatrix} 5.818 & 5.364 & 8.182 & 6.636 \\ 4.818 & 8.272 & 4.910 & 6.182 \\ 5.818 & 6.364 & 6.272 & 7.818 \end{bmatrix}$$

Finally, use Formula (7) to calculate comprehensive evaluation value of each scheme. The values are $V_{a1} = 26$, $V_{a2} = 24.182$ $V_{a3} = 26.272$. As V_{a3} is the high score, we choose No. 3 candidate to be the department cadres.

CONCLUSION

This paper proposes a procedure for solving multiple attribute group decision making problems. It proposes compatible preference relation on the basis of expert evaluation to classify expert, and adjusted minority experts' opinion. It greatly retained the expert's original opinion both in the classification process and modification process. At the same time, in order to improve the consensus of group decision, we put forward an innovative method for experts weight distribution to make sure the agreement of expert index scores. Finally, we illustrate the feasibility and effectiveness of the group opinion gathering method by a case analysis. But we don't strictly define input language for expert evaluation, we will explore different input models in the next study, which will make the method is more convenient and operation in the concrete practice.

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