A Strategy Considering Both Magnitude and Duration of Drawdown

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Abstract
Drawdown is one of the most important measures to evaluate the risk of a portfolio. However, when it comes to the drawdown measure, the scholar’s concern more about the magnitude than the duration of the drawdown. For examples, there are various indicators of drawdown measures including the maximum drawdown, Conditional Drawdown-at-Risk (CDaR), etc. Most of them evaluate the magnitude of drawdown. In this paper, a strategy is built considering both the magnitude and duration of drawdown. Using the historical average maximum drawdown duration as the condition of filtering assets, this paper improves the strategy without considering the duration of the drawdown by leading an empirical analysis.

Key words: Drawdown magnitude; CDaR; Drawdown duration

INTRODUCTION
Optimal portfolio allocation is always a heated topic in both practical portfolio and academic research since Markowitz proposed the mean-variance model. From a standpoint of fund managers, keeping the existing accounts and to attract the new ones in order to increase their revenues is the most important things they should concern. That is to say, losing the accounts is equivalent to the death of their business. Practically, if the drawdown of the portfolio is large or last a long time, the investors will be likely to redeem their portions. Therefore, drawdown is one of the most essential standards on portfolio management.

Several studies discussed portfolio optimization with drawdown constraints. Grossman and Zhou obtained an exact analytical solution to portfolio optimization with constraint on the maximal drawdown. They study the drawdown of the portfolio with one riskless asset and one risk asset in continuous time. Based on their research, Cvitanic and Karatzas applied the model to the multiple risky assets and a riskless assets. However, in order to characterize portfolio risk for the whole investment period, drawdown should be translated into a single number. Chekhlov proposed a drawdown measure called Conditional Drawdown-at-Risk (CDaR) in both a single sample-path of portfolio return and extended it into multiple one. Also Chekhlov formulated the optimization problem and reduced it into a linear programming with Rockafellar’s formular for Conditional Value-at-Risk (CVaR). However, few scholars concern the indicator about drawdown duration.

To consider the drawdown measure in another perspective, we put forward a strategy that considers the maximum drawdown duration. We use the average historical maximum drawdown duration as the condition of filtering stocks in order to improve the portfolio performance. The rest of paper is organized as follows. Section 1 introduces the Conditional Drawdown-at-Risk (CDaR) and proposes the definition maximum drawdown duration. Then a strategy considering both the magnitude and duration of drawdown is proposed. Section 2 verifies the strategy with 28 constituent stocks of Chinese stock market. Section 3 summarizes the whole work of this paper.
1. A STRATEGY BASED ON DRAWDOWN DURATION AND MAGNITUDE

1.1 Conditional Drawdown-at-Risk

CDaR is a concept first put forward by Chekhlov. Define the notion of absolute drawdown as the absolute value that decline from a historical peak in cumulative profit series. Specifically, it is applied to a sample path of the uncompounded portfolio rate of return. Assume that there are \( n \) stocks, the weight of portfolio is \( w \), return of stocks at moment \( t \) is \( r_t = (r_{tx}, r_{tx-1}, \ldots, r_{tx-n}) \), then return of the portfolio at moment \( t \) is \( w^T r_t \). Let \( \alpha \) be the lower bound and upper bound of the weight of stocks. The uncompounded cumulative portfolio rate of return at time \( t \) is:

\[
W_t = \begin{cases} 0, & t = 0 \\ \sum_{j=1}^{\alpha} w^T r_j, & t > 0. \end{cases}
\]

Then the absolute drawdown can be defined as follow:

\[
AD(W) = \xi = (\xi_1, \ldots, \xi_n), \quad \xi_i = \max\{W_j - W_{j-t} \mid 0 \leq j \leq t\}.
\]

The conditional drawdown at risk, like the CVaR, is defined as the mean of tail drawdown exceeding DaR, which is the maximum possible drawdown during the time if we exclude worse outcomes whose probability is less than \( \alpha \). According to paper proposed by Chekhlov (2005) the Conditional Drawdown-at-Risk (CDaR) is defined as:

\[
\text{CDaR}_\alpha(\xi) = \left(\frac{\pi_\xi(\zeta(\alpha) - \alpha)}{1 - \alpha}\right) \zeta(\alpha) + \frac{1}{(1 - \alpha)N} \sum_{\xi_j > \zeta(\alpha)} \xi_j.
\]

According to Chekhlov’s study, CDaR can be transformed as:

\[
\text{CDaR}_\alpha(\xi) = \min_y h(y),
\]

where \( h(y) = y + \frac{1}{(1 - \alpha)N} \sum_{i=1}^{N} z_i, \quad z_i = [\xi_i - y] \). The portfolio optimization of minimizing CDaR, which is proposed by Chekhlov:

\[
\min y + \frac{1}{(1 - \alpha)N} \sum_{i=1}^{N} z_i
\]

s.t. \( w^T r \geq \mu \)

\[
\begin{align*}
\xi_j &\geq u_j - y \\
u_j &\geq u_{j-1} - r_w \\
u_j &\geq 0, u_0 = 0, z_i \geq 0 \\
w^T e &= 1
\end{align*}
\]

Where \( \mu \) is the expected return of the portfolio. \( e \) denotes the vector of all ones. \( w \) and \( \overline{w} \) denotes the lower bound and upper bound of the weight of stocks respectively.

1.2 Constructing the Strategy

Like drawdown, which is the measure of the decline from a historical peak in cumulative profit series, drawdown duration is the time last since the last peak of the cumulative profit series. Then the maximum drawdown duration is the largest one of the drawdown duration. It is difficult to consider the drawdown duration as a variable in the programming problem of portfolio optimization. Here, we set the drawdown duration as a condition of filtering the stocks. We make a hypothesis that the stocks with large maximum drawdown duration in past may cause large maximum drawdown duration in future. According to this hypothesis, we construct the strategy as follows.

**Figure 1**

The Flow Chart of the Strategy
Here the left part of the flow chart is the strategy we built while the right part of the flow chart is the original model of Chekhlov, which does not consider the drawdown duration.

2. EXPERIMENTS

In this section, we consider a numerical experiment to illustrate the effectiveness of the strategy. We select the 28 stocks that are the constituent stocks of SSE 50 in Chinese stock market from July 2013 to July 2016. To verify the filtering condition with maximum drawdown duration can improve the strategy. We use the average maximum drawdown duration every 20 market days in the past 400 market day to get a factor called average historical maximum drawdown duration. The stocks are divided into three groups by the rank of the factor. That is to say, Group one contains the stocks with the lowest average historical maximum drawdown duration.

We use two ways to verify the condition of average maximum drawdown duration. First, we use the method of equal-weighted to construct the portfolio. Second we use the CDaR model to construct the portfolio ($\alpha=0.95$, $\mu=0.0007$). To make the result more reasonable, we do the experiment every 20 market day. Then we can get 28 experiment groups and the results are as follows:

### Table 1
The Performance of Different Groups (Equal-Weighted Edition)

<table>
<thead>
<tr>
<th></th>
<th>Group one</th>
<th>Group two</th>
<th>Group three</th>
<th>Rate of winning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit</td>
<td>1.20%</td>
<td>1.07%</td>
<td>0.54%</td>
<td>53.57%</td>
</tr>
<tr>
<td>Mdd</td>
<td>5.62%</td>
<td>5.58%</td>
<td>7.49%</td>
<td>78.57%</td>
</tr>
<tr>
<td>Mdd duration</td>
<td>6.6786</td>
<td>7.6071</td>
<td>7.7143</td>
<td>67.86%</td>
</tr>
</tbody>
</table>

### Table 2
The Performance of Different Groups (CDaR Model Edition)

<table>
<thead>
<tr>
<th></th>
<th>Group one</th>
<th>Group two</th>
<th>Group three</th>
<th>Rate of winning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit</td>
<td>2.17%</td>
<td>1.42%</td>
<td>1.47%</td>
<td>57.14%</td>
</tr>
<tr>
<td>Mdd</td>
<td>4.88%</td>
<td>5.70%</td>
<td>6.64%</td>
<td>85.71%</td>
</tr>
<tr>
<td>Mdd duration</td>
<td>6.5714</td>
<td>7.6071</td>
<td>7.7857</td>
<td>75.00%</td>
</tr>
</tbody>
</table>

Specifically, the profit, Mdd (maximum drawdown) and Mdd duration (maximum drawdown duration) is the average value of the 28 experiment groups’ profit Mdd and Mdd duration respectively. The rate of winning here represents the percentage of times that the group one performs better than the group three in total 28 times. In the result, we find that the performance of group one is always better than the one of group three in terms of the rate of winning and the average three evaluating indicators. The three evaluating indicator also shows roughly monotonicity, which proves the effectiveness of maximum drawdown duration as the condition of selecting stocks. In addition, when comparing the two different edition of portfolio by constructing by two methods, we find the one with CDaR model performs better than the one with equal-weighted. The result indicates that it is essential to concern both the magnitude and duration of the drawdown.

After that, we should verify the strategy by getting rid of the stocks with the largest average historical maximum drawdown duration. We consider three strategy here. The stocks can be selected is constrained in just the stocks in group one in strategy one (S1) while constrained in the stocks in both group one and group two in strategy two (S2). Strategy three (S3) does not get rid of any stock. Then we use the CDaR model to get the optimal portfolio and we get the following result.

### Table 3
The Winning Rate Among Different Strategies

<table>
<thead>
<tr>
<th></th>
<th>Profit</th>
<th>Mdd</th>
<th>Mdd duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1 to S3</td>
<td>60.71%</td>
<td>78.57%</td>
<td>71.43%</td>
</tr>
<tr>
<td>S2 to S3</td>
<td>46.43%</td>
<td>85.71%</td>
<td>71.43%</td>
</tr>
</tbody>
</table>

As the result showed in Table 3, we can easily find that the strategies that get rid of the stocks with the largest average historical maximum drawdown duration perform better the one without any disposition of the stocks in terms of the maximum drawdown and maximum drawdown duration. In some extent, the strategy built by selecting stocks without largest average historical maximum drawdown duration can improve the risk control ability of the portfolio.

### CONCLUSION

In this paper, considering that the scholars rarely model with the drawdown duration, we use the average historical maximum drawdown duration as a condition to filter the stocks. In order to prove the condition of considering the drawdown duration, we divide the stocks into three groups according to their average historical maximum drawdown duration. Then we find that the group with the lowest average historical maximum drawdown has
a better performance than the group with the largest one. In particular, the paper proves it in two ways of constructing the portfolio, including the equal-weighted one and the CDaR model. When comparing the two different ways, we find the one use CDaR model performs better. The result is obvious, because in fact, the fund manager will not concern only the duration of the drawdown. That is to say, when we concern about the drawdown measure, we should consider both the magnitude and duration.

Proving the historical average maximum drawdown duration has the discriminative power of the profit of the future. Then we check the performance of the strategy by getting rid of the stock with the largest average historical maximum drawdown duration. Comparing the original strategy, which does not get rid of any stock of the CDaR model, the strategy built-in this paper has the higher rate of winning in maximum drawdown and the maximum drawdown duration.

REFERENCES