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Retailer's Optimal Ordering Policies with Two Stage Credit Policies and Imperfect Quality

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Abstract

Two levels of trade credits refers that the supplier provides to his/her retailer a permissible delay period (M) in paying for purchasing items and the retailer also in turn provides a permissible delay period ($N, M > N$) to his/her customer to stimulate his product demand. When lot received by retailer, it may be contain some imperfect quality of goods by the causes of non-ideal production process or other causes. So retailers perform a screening process to find the imperfect items and returned to the supplier immediately. Therefore, an attempt is made in this paper to develop the retailer's optimal ordering policies in supply chain coordination with upstream and downstream trade credits and imperfect quality. The propose paper considers two cases $N \leq M$ and $M \leq N$ that is more near to real world cases. Some numerical examples are used to be show validity of this paper.

Key words: Inventory; Imperfect items; Up-stream and down-stream trade credits and supply chain

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INTRODUCTION

Several research articles have been published in various international journals by taking the assumption of permissible delay in payment in supply chain coordination. It is tacitly assumed that the retailer would pay for the items as soon as the items were received. In general, the supplier offers a grace period term as trade credit to his/her retailer to increase of sale and joined the new customer. In this credit policy, the supplier does not charge any interest if the payment is made before credit period. However, if the payment is paid beyond the predefined period, a high interest is charged. Some of the prominent paper related to trade credit as follows.

Goyal (1985) is believed to be the first who developed the economic order quantity (EOQ) model under the conditions of permissible delay in payments. Chand and Ward (1987) investigated Goyal (1985) model under the assumptions of classical economic order quantity model. Shah et al. (1988) studied the same model incorporating shortages. Mondal and Phaujdar (1989) extended this issue by considering the interest earned from the sales revenue. Shah (1993a, b) also developed EOQ models for perishable items where delay in payment is permissible. Aggarwal and Jaggi (1995) developed Goyal's (1985) model to the case of exponential deterioration, Later Jamal et al. (1997) extend Aggrawal and Jaggi (1995) model when shortage is allowed. Teng (2002) revise Goyal's (1985) model by more realistic assumption that unit selling price is higher than unit cost price.

In above models, the supplier adopts a business strategy of permissible delay period in paying the purchasing cost to attract more customers. The retailer takes the benefit of trade credit and sells the product and earns the interest by putting generated revenue in an interest bearing account. They implicitly assumed that the buyer would pay the supplier as soon as he/she receives the items. As mentioned earlier, supplier only offers a trade credit to the retailer but retailer does not provide any trade credit to his/her customer which means we deal with one level of trade credit. However, in many real-world cases, this condition will not hold. Recently, Huang (2003) modified this assumption by assuming that the retailer will also adopt the trade credit policy to stimulate his/her customer demand to develop retailer replenishment model and this method is called two level of trade credit policy.

In Huang's (2003) model the retailer start to accumulate the revenue from *O* to *T*. But in real world it is not always true. Then Goyal and Teng (2007) developed a model in which the retailer starts to accumulate revenue from *N* to *T+N*, that is more match real life cases and selling price is higher than the cost price.

High quality always translates into high volume and high quality of products produce to customers is key of maintaining in marketing world. But, in sometimes production assembly may be influence by so many causes that result the imperfect items produced. A lot of number interesting papers published in various journals by taking assumption of imperfect items is presented in lots and imperfect items is found by a screening process. Some of the related papers of imperfect items are as follows.

Salameh and Jaber (2000) developed EOQ model with imperfect quality items under random yield. Goyal and Cardenas Barron (2002) presented a practical approach on EOQ model for imperfect items. Chang (2004) added in this area the applications of the fuzzy set theory in EOQ model with imperfect items. Other notable papers in this area are: Papachristos and Konstantarats (2006), Wee et al. (2007), Maddah and Jaber (2008). While Huang (2004), Ouyang et al. (2006) presented integrated vendor buyer model with imperfect items and Chen and Krang (2007, 2010) investigated vendor buyer integrated model with permissible delay in payments.

This paper extends Goyal and Teng (2007) model in the light of imperfect items and lot contains some imperfect items which is found by a screening process and returned to supplier immediately when full screening process ends. In this paper, we developed retailer's optimal ordering policies for imperfect items in supply chain coordination. The basis objective of this paper is cost minimization for retailer. Numerical examples are illustrated to managerial insight to proposed problem.

1. ASSUMPTIONS AND NOTATIONS

For convenience, most Assumption and Notations similar to Goyal and Teng (2007) will be used in the present paper.

1.1 Assumptions

1) The demand rate and defective rate are constant over time.

2) Lead time zero and Shortages are not allowed.

3) Replenishment rate is infinite.

4) Imperfect items are treated as a single batch and returned to the supplier immediately when the 100%screening process ends. The supplier will discount all defective items for sale to customers.

5) The delay period is larger than the screening time.

6) To avoid shortages for the retailer, the expected quantity of perfect items should be greater than or equal to the demand during the screening time.

7) Trade credit between supplier and retailer as follows:

o If the retailer pays by *M*, then supplier does not charge the retailer any interest.

o If the retailer pays after *M* but before *N*, he keeps his/her profit and sells revenue is utilized to earn interest with annual rate *Ie*. Then the supplier charges the retailer an interest rate of Ic_1 on the balance amount.

o If the retailer pays after second permissible delay period *N*, then supplier charges the retailer an interest rate of Ic_2 on the balance amount, with $Ic_2 > Ic_1$.

1.2 Notation

1) *D* The demand rate per year

2) *h* Unit stock- holding cost per year excluding interest charge

3) *P* The selling price per year

4) *c* The unit purchasing cost, with $c < P$

5) *A* The ordering cost per order

6) *T* The replenishment cycle

7) *M* The retailer's trade period offered by supplier in years

8) *N* The customer's trade period offered by retailer in years $M > N$

9) *Ie* The interest earned per $ in stock per year

10) *Ic* The interest charged per $ in stocks per year by supplier in years.

11) *Q* The order quantity

12) T^* The optimal cycle time of $TC(T)$

13) χ The screening rate

14) *y* The random variable representing the percentage of defective items

15) *Y* The expected value of *y* i.e. $Y=E(y)$

16) *v* The unit warranty cost for defective items including the penalty cost for the supplier

17) *K* The required time for screening the defective items, $K=\frac{Q}{\chi}$

18) *TC* The total cost of an inventory system/unit time

2. MATHEMATICAL FORMULATION

The total relevant cost consists following elements:

Expected annual ordering cost per unit time is $=\frac{A}{T}$ (1)

Expected screening cost per unit time is $dQ/dT= dD/(1-Y)$

$$= h\left(\frac{Q(1-Y)T}{2} + \frac{YQ^2}{x}\right)/T \quad (2)$$

Expected holding per unit time is

$$= h\left(\frac{D}{2} + \frac{YD^2}{x(1-Y)^2}\right)T \quad (3)$$

Expected interest earned and additional sales for defective items per unit time is

$$= \frac{(v-v)QY}{T} + \frac{vIeYQ(M-k)}{T}$$

$$= \frac{(v-v)DY}{(1-Y)} + \frac{vIeYD(M-k)}{(1-Y)} \quad (4)$$

Case 1 $N \le M$

Sub-case 1-1 $M \le T + N$

In this paper, the sales revenue is utilized to earn interest during the period of (M , $T + N$) when the account is settled, the items which are still in inventory have to be financed with annual rate. Therefore, the annual interest payable is as follows in figure 1.

The interest charged per year is $= \frac{cIcD}{2T}[T+N-M]^2$ (5)

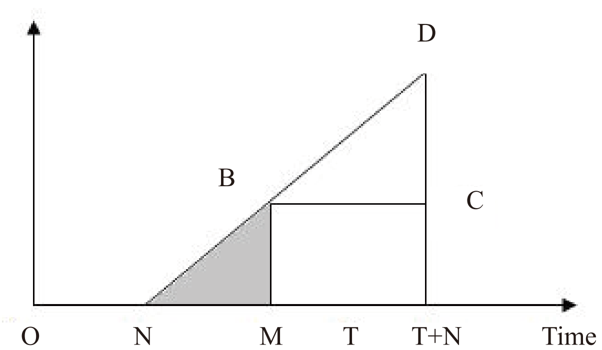

Figure 1
$N \le M$ and $M \le T + N$

Retailers sell the product at time 0 but he actually received the payment at time N. Therefore, sale revenue is accumulated from (N to M) and interest earned in shaded region is pIe. The annual interest earned is as follows,

The interest earned per year is $= \frac{pIeD(M-N)^2}{2T}$ (6)

The annual total relevant cost for the retailer as

$$TC_1(T) = \frac{A}{T} + \frac{dD}{(1-Y)} + h\left(\frac{D}{2} + \frac{YD^2}{x(1-Y)^2}\right)T - \frac{vIeYD(M-k)}{(1-Y)} + \frac{cIcD}{2T}[T+N-M]^2 - \frac{pIeD(M-N)^2}{2T} \quad (7)$$

Sub-case1-2 $M > T + N$

In this sub-case, the retailer has enough money to pay full the supplier. So, there is no interest charge. While retailer receives the total amount at time $T+N$ and can pay full at time M. Therefore, interest earned in shaded region is as follows

The annual interest earned is $= \frac{PIeDT^2}{2T} + \frac{PIeDT(M-T-N)}{T}$ (8)

$$= PIeD(M-N) - \frac{PIeDT}{2}$$

Thus, the annual total relevant cost is

$$TC_2(T) = \frac{A}{T} + \frac{dD}{(1-Y)} + h\left(\frac{D}{2} + \frac{YD^2}{x(1-Y)^2}\right)T - PIeD(M-N) + \frac{PIeDT}{2} - \frac{vIeYD(M-k)}{(1-Y)} \quad (9)$$

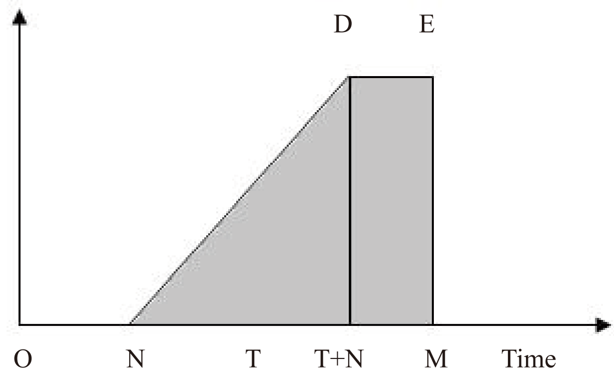

Figure 2
$N \le M$ and $T + N \le M$

Case 1 $N \ge M$

In this case, there is no interest earned by retailer. So retailer will take finance to pay all items to be paid at time M with interest rate Ic and pay off the finance amount after N. Thus interest charged per cycle in shaded region with the interval (M, $T+N$) as follows

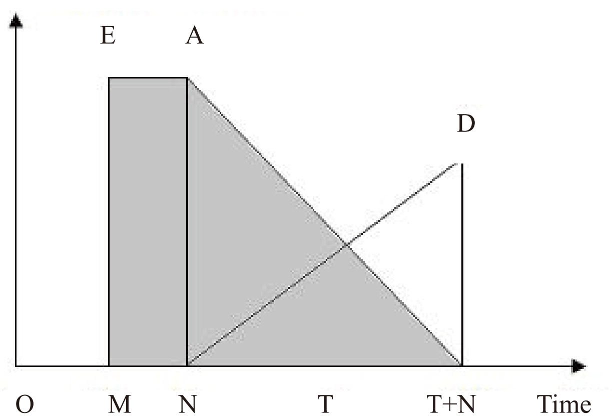

Figure 3
$N \ge M$

The interest charged per year is $= \frac{cIcD}{2}[2(N-M)+T]$ (10)

Thus, the annual total relevant cost is

$$TC_3 = \frac{A}{T} + \frac{dD}{(1-Y)} + h\left(\frac{D}{2} + \frac{YD^2}{x(1-Y)^2}\right)T + \frac{cIcD}{2}\{2(N-M)+T\} \quad (11)$$

3. DETERMINATION OF THE OPTIMAL CYCLE TIME T^*

In this section, we determine the optimal value of T which minimizes $TRC(T)$. The necessary and sufficient condition for the optimality is $TRC'(T) = 0$ and $TRC''(T) < 0$. Taking the first order and second order derivatives of $TRC_1(T)TRC_2(T)$ and $TRC_3(T)$ with respect to T, we obtain

$$\frac{dTC_1}{dT} = -\frac{A}{T^2} + h\left(\frac{D}{2} + \frac{YD^2}{x(1-Y)^2}\right) + \frac{cIcD}{2} - \frac{cIcD(M^2+N^2)}{2T^2} + \frac{cIcDMN}{T^2} + \frac{PIeD(M-N)^2}{2T^2} + \frac{vIeYD^2}{(1-Y)^2x} \quad (12)$$

$$\frac{d^2TC_1}{dT^2} = \frac{2A}{T^3} + \frac{cIcD(M^2+N^2)}{T^3} - \frac{2cIcDMN}{T^3} - \frac{PIeD(M-N)^2}{T^3} \quad (13)$$

$$\frac{dTC_2}{dT} = -\frac{A}{T^2} + h\left(\frac{D}{2} + \frac{YD^2}{x(1-Y)^2}\right) + \frac{PIeD}{2} + \frac{vIeYD^2}{(1-Y)^2x} \quad (14)$$

$$\frac{d^2TC_2}{dT^2} = \frac{2A}{T^3} > 0 \quad (15)$$

$$\frac{dTC_3}{dT} = -\frac{A}{T^2} + h\left(\frac{D}{2} + \frac{YD^2}{x(1-Y)^2}\right) + \frac{cIcD}{2} \quad (16)$$

$$\frac{d^2TC_3}{dT^2} = \frac{2A}{T^3} > 0 \quad (17)$$

When $N \le M$, it is clear from that $TRC_2(T)$ is a strictly convex function in T, consequently, we obtain the corresponding unique cycle time T_2^* as

The first-order conditions $TRC_2(T)$ in Eqs. (14) is $\frac{dTRC_2}{dT} = 0$. Thus, the values of for sub-case 2 is

$$T_2^* = \sqrt{\frac{A}{\frac{D(h+PIe)}{2} + \frac{(h+vIe)D^2Y}{x(1-Y)^2}}} \quad (18)$$

T_2^* will satisfy the condition $T+N \le M$ provided if and only if

$$\Delta_1 = 2A - (M-N)^2\left\{D(h+PIe) + \frac{2(h+vIe)D^2Y}{x(1-Y)^2}\right\} \le 0 \quad (19)$$

Therefore, the optimal order quantity $Q_2^* = DT_2^*$ (20)

Consequently, we know that $TRC_2(T)$ is also strictly concave function in T. Likewise, we can easily obtain the unique optimal replenishment cycle time T_1^* as

$$T_1^* = \sqrt{\frac{2A + D(M-N)^2\{cIc - pIe\}}{\{h+cIc\}D + \frac{2\{h+vIe\}D^2Y}{x(1-Y)^2}}} \quad (21)$$

T_1^* will satisfy the condition $M \le T+N$ provided, if and only if

$$\Delta_1 = 2A - (M-N)^2\left\{D(h+PIe) + \frac{2(h+vIe)D^2Y}{x(1-Y)^2}\right\} \ge 0$$

Hence, the optimal order quantity Q_1^* is $Q_1^* = DT_1^*$ (22)

From the above argument, we obtain the following results.

Theorem 1

(A) if $\Delta_1 \ge 0$, then $T^* = T_1^*$ and $Q^* = Q_1^*$.
(B) if $\Delta_1 \le 0$, then $T^* = T_2^*$ and $Q^* = Q_2^*$.
(C) if $\Delta_1 = 0$, then $T^* = T_1^* = T_2^*$ and $Q^* = Q_1^* = Q_2^*$.

In classical economic order quantity model assumed that the retailer's and the customer would be pay for the items as soon as the items received by him. Therefore, M=N=0, the classical optimal EOQ is

$$Q_4^* = \sqrt{\frac{2AD}{(h+cIc)}} \quad (23)$$

As a result, we can easily obtain the following theoretical result.

Theorem 2

When $N \le M$

(A) if $pIe < cIc$, then both Q_1^* and Q_2^* are larger than Q_4^*.

(B)if $pIe >$ cIc, then both Q_1^* and Q_2^* are larger than Q_4^*.

(C)if $pIe = cIc$, then $Q_1^* = Q_2^* = Q_4^*$.

Next, let us discussed the case in which $N \ge M$, when $N \ge M$, we know that from (17) that $TRC_3(T)$ is a strictly convex function in T. Consequently we obtain the unique optimal cycle time T_3^* as

$$T_3 = \sqrt{\frac{A}{\frac{D(h+cIc)}{2} + \frac{hD^2Y}{x(1-Y)^2}}} \quad (24)$$

Therefore, the optimal order quantity Q_3^* is

Q_3^*=DT_3^* (25)

As a result, if $N \ge M$, then the retailer's optimal order quantity is exact the same as the classical EOQ.

4. NUMERICAL EXAMPLE

(1) D = 4200, h = \$4, A=\$150, x = \$175, d = \$0.4, Ie= \$0.09, N= 60/365, M = 90/365, Ic = \$0.12, Y = 0.02, c=\$20, P = \$40, ν = \$30

$\Delta_1 < 0$, $T_2 = 0.07068$, $Q_1 = 296.86$, $k = 1.70$, $TC_1 =$ \$4651.65

(2) Ie = \$0.15 and other parameters is same as in example.1

$\Delta_1 > 0$, $T_1 = 0.10594$, $Q_1 = 444.95$, $k = 2.54$, $TC_1 =$ \$7116.39

(3) M = 100/365 and other parameters is same as in example.1

$\Delta_1 < 0$, $T_2 = 0.07068$, $Q_1 = 296.86$, $k = 1.70$, $TC_2 =$ \$4237.42

(4) Y = 0.03 and other parameter is same as in example.1

$\Delta_1 < 0$, $T_1 = 0.06325$, $Q_1 = 265.65$, $k = 1.52$, $TC_2 =$ \$4858.93

CONCLUSION

This paper extends Goyal and Teng (2007) model by assuming imperfect items is presented in lots and imperfect items found by a screening process. In this paper, the retailer accumulated revenue from N to T+N that is more matches to real world problem and selling price is significantly higher than the unit cost. There are two cases $M > N$ and $N > M$ discussed with $Ic > Ie$ and variety of examples are investigated for validity of model.

REFERENCES

Aggarwal, S.P., & Jaggi, C.K. (1995). Ordering Policies of Deteriorating Items under Permissible Delay in Payments. *Journal of the Operational Research Society, 46*, 658-662.

Chand, S., & Ward, J. (1987). A Note on Economic Order Quantity under Conditions of Permissible Delay in Payments. *Journal of Operational Research Society, 38*, 83-84

Chang, H.C. (2004). An Application of Fuzzy Sets Theory to the EOQ Model with Imperfect Items. *Computer and Operations Research, 31*(12), 2079-2092.

Chen, L.H., & Kang, F.S. (2010). Coordination between Vendor and Buyer Considering Trade Credit and Items of Imperfect Quality. *International Journal of Production Economics, 123*, 52-61.

Chen, L.H., & Kang, F.S. (2007). Integrated Vendor-buyer Cooperative Models with Variant Permissible Delay in Payments. *European Journal of Operational Research, 170*(2), 481-495.

Goyal, S.K. (1985). Economic Order Quantity under Conditions of Permissible Delay in Payments. *Journal of Operational Research Society, 36*, 335-338.

Goyal, S.K., & Cardenas-Barron, L.E. (2002). Note on: Economic Production Quantity Model for Items with Imperfect Quality-a Practical Approach. *International Journal of Production Economics, 77*(1), 85-87.

Huang, C.K. (2004). An Optimal Policy for a Single-buyer Integrated Production-inventory Problem with Process Unreliability Consideration. *International Journal of Production Economics, 91*, 91-98.

Huyang, Y.F. (2003). Optimal Retailer'S Ordering Policies in the EOQ Model under Trade Credit Financing. *Journal of the Operational Research Society, 54*, 1011-1015.

Jamal, A.M.M., Sarker, B.R., & Wang, S. (1997). An Ordering Policy for Deteriorating Items with Allowable Shortage and Permissible Delay in Payment. *Journal of the Operational Research Society, 48*, 826-833.

Maddah, B., & Jaber, M.Y. (2008). Economic Order Quantity for the Items with Imperfect Quality: Revisited International *Journal of Production Economics, 112*, 28-8-815.

Mondal, B.N., & Phaujdar, S. (1989). Some EOQ Models under Permissible Delay in Payments. *International Journal of Management Science, 5*, 99-108.

Papachristos, S., & Konstantaras, I. (2006). Economic Order Quantity Model for the Items with Imperfect Quality. *International Journal of Production Economics, 100*(1), 148-154.

Salameh, M.K., & Jaber, M.K. (2000). Economic Production Quantity Model for the Items with Imperfect Quantity. *International Journal of Production Economics, 64*(13), 59-64.

Shah, N.H. (1993a). Probabilistic Time-scheduling Model for an Exponentially Decaying Inventory when Delay in Payments is Permissible. *International Journal of Production Economics, 32*, 77-82.

Shah, N.H. (1993b). A lot Size Model for Exponentially Decaying Inventory when Delay in Payment is Permissible. *Cahiers du CERO, 35*, 115-123

Shah, V.R., Patel, N.C., & Shah, D.K. (1988). Economic Ordering Quantity when Delay in Payments of Order and Shortages are Permitted. *Gujarat Statistical Review 15*(2), 52-56.

Teng, J.T. (2002). On the Economic Order Quantity under Conditions of Permissible Delay in Payments. *Journal of the Operational Research Society, 53*, 915-918.

Wee, H.M., Yu, J., & Chen, M.C. (2007). Optimal Inventory Model for Items with Imperfect Quality and Shortage Backordering. *Omega, 35*(1), 7-11.