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Unit Root Tests for Long Memory Series in the Presence of Structural Breaks in Variance

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Abstract

This paper extends the unit root tests to long memory observations in the existence of variance breaks. Given for the case of non-constant variance, the asymptotic properties of commonly used unit root tests are derived under the null hypothesis. It is shown that the non-constant variance can both inflate and deflate the rejection frequency, thus the statistic tests are not robust. The simulation results also indicate the extent of size distortion is heavily sensitive to the location and magnitude of change points, long memory index and sample size.

Key words: Unit root tests; Long memory; Variance breaks; Asymptotic properties

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INTRODUCTION

Unit root tests have been one of the most popular statistical methodologies for analyzing economic and financial time series which are mostly characterized by non-stationary components. Testing for unit roots has attracted a great deal of attention since the influential works of Dickey and Fuller (1979). Various research direction is also to widen the applicability of the unit root tests under broader correlation structures for the innovations. Phillips (1987) and Perron (1988) developed tests that are applicable under weakly dependent innovation. Furthermore, a great patience has paid to the effects of breaks in variance and any other on the unit root tests. Clemente, Montane and Reyse (1998) testing for a unit root in variables with a double change in the mean. Cavaliere and Ttaylor (2007) consider a completely different approach to the problem and develop unit root tests which are robust to a very general class of volatility changes. Sen (2009) shows that unit root tests in the presence of an innovation variance break that has power against the mean break stationary alternative. Harvey, Leybourne and Taylor (2014) proposed on infimum Dickey–Fuller unit root tests allowing for a trend break under the null. oh and Kim (2015) propose Lagrangian multiplier type tests based on the residual's marginal likelihood unit root tests when a time series has multiple shifts in its level and the corresponding volatility. For more details, we refer the reader to Hamori and Tokihisa (1997) and Kim et al. (2002) and Giuseppe Cavaliere (2004) among others.

However, the above literature is constrained on short memory, a few authors have analyzed the unit root tests under long memory series. For example, Sowell (1990) generalized the unit root distribution to fractionally integrated errors. It is shown that the limiting distributions of fractionally integrated series are radically different than for series integrated of order zero or one. Glaura, Reisen and Paula (2004) using semi-parametric estimators of the long-memory parameter on the unit root tests. Guglielmo and Gil-Alana (2004) further examine unit root by using parametric and semiparametric techniques for modelling long memory. Alexander and Lajos (2011) gives an account of some of the recent work on structural breaks in time series models and cover how one can disentangle structural breaks (in the mean and/or the variance) on one hand and long memory or unit roots on the other. Smallwood (2014) analyzed the consequences of using long memory methods to test for unit roots when the "truth" derives from regime switching, structural breaks, or other types of mean reverting nonlinearity.

In this paper, the goal of the article is to extend the unit root tests to long memory observations in the existence of variance breaks. The following work to the limit distribution of statistics is obtained and analyzes the influence of the statistics for the break points. We can conclude that the variance breaks not only depends on location, but also depend on long memory index and any other.

The paper is organized as follows. Section 2 introduces the main results, the model and limit distribution. Some Monte Carlo experiments are included in Section 3. Section 4 draws the conclusion.

1. MAIN RESULT

This section exports the limiting distribution of test statistics when change points are taken into consideration. Before presenting our main results, we should introduce the long memory series. In the last two decades, we have witnessed a rapid development for statistical inference of long range dependent (or long memory) time series; see Beran (1994), Robinson (2003) among others for book-length treatments of this topic. Let

$$(1-L)^d z_t=\varepsilon_t,\ t\in Z,$$

where L is the backward shift operator and $\{\varepsilon_t\}$ is a mean zero covariance stationary dependent process. We say that the process z_t possesses long memory if $d\in(0,0.5)$ since it exhibits long-range dependence in the sense that $\sum_{j=-\infty}^{\infty}\gamma(j)=\infty$ where $\gamma(j)$ is the autocovariance function at lag j. and short memory if $d\in(-0.5, 0)$.

We focus on the stationary process, and require it obeying a functional central limit theorem, as stated in the lemma below. Let $\xrightarrow{L}$ stand for the weak convergence.

The following lemmas are the cornerstone of our paper, which have been proved by Sowell (1990) and Tsay (1999).

Lemma 1: If $(1-L)^d\varepsilon_t=u_t\sim IID(0,\sigma_u^2), -\frac{1}{2}<d<\frac{1}{2}$ and $S_T=\sum_{t=1}^{T}\varepsilon_t$, then

$$\mathrm{Var}(S_T)=\frac{\sigma_u^2\Gamma(1-2d)}{(1+2d)\Gamma(1+d)\Gamma(1-d)}\times\left[\frac{\Gamma(1+d+T)}{\Gamma(-d+T)}-\frac{\Gamma(1+d)}{\Gamma(-d)}\right]$$

and

$$\sigma_\varepsilon^2=\lim_{T\to\infty}\mathrm{Var}(S_T)/T^{1+2d}=\frac{\sigma_u^2\Gamma(1-2d)}{(1+2d)\Gamma(1+d)\Gamma(1-d)},$$

where σ_ε^2 is the long-run variance.

Lemma 2: If $(1-L)^d\varepsilon_t=u_t\sim IID(0,\sigma_u^2)$, $-\frac{1}{2}<d<\frac{1}{2}$, and $E|u_t|^r<\infty$ for $r\ge\max[4, -8d/(1+2d)]$, then

$$T^{-(\frac{1}{2}+d)}S_{[Tr]}\xrightarrow{L}\sigma_\varepsilon B_{\varepsilon,d}(r),$$

where σ_ε is defined in Lemma 1 and $B_{\varepsilon,d}(r)$ is fractionally Brownian Motions. Marinucci and Robinson (1999) has given the fractionally Brownian Motions as follows:

$$B_{\varepsilon,d}(\tau)\equiv\frac{1}{\Gamma(1+d)}\{\int_0^\tau(\tau-s)^d W(s)+\int_{-\infty}^0[(\tau-s)^d-(-s)^d]\mathrm{d}W(s)\},$$

where $\Gamma(\cdot)$ is the Gamma function and $W(s)$ is a standard Brownian motion.

Now, we suppose ε_t and η_t are stationary long memory series with long memory index $d\in(0,0.5)$ and satisfy the Lemma 1 and 2. The model is considered in this paper as follows:

$$y_t=y_{t-1}+z_t,\ t=1,2,...,T,$$

where

$$z_t=\varepsilon_t+\eta_t DU_t,$$

$$DU_t=\begin{cases}1, & t>T_B,\ (1<T_B<T)\\ 0, & t\le T_B,\end{cases},$$

and ε_t and η_s are independent for all t and s. Here, there is a variance break at T_B, where the location of change point T_B is assumed to be unknown.

Next, we estimate the model using the following regression equation:

$$y_t=\hat{\rho}y_{t-1}+\hat{z}_t,$$

where

$$\hat{\rho}=\frac{\sum_{t=1}^{T}y_{t-1}y_t}{\sum_{t=1}^{T}y_{t-1}^2},$$

and $\hat{z}_t$ is the residual from the regression y_t on intercept. Based on $y_t=\hat{\rho}y_{t-1}+\hat{z}_t$, we consider following two types of the statistic for the unit root tests:

$$\hat{\rho}-1=\frac{\sum_{t=1}^{T}y_{t-1}z_t}{\sum_{t=1}^{T}y_{t-1}^2},$$

$$t_{\hat{\rho}}=\frac{\hat{\rho}-1}{\hat{\sigma}_\rho},$$

where

$$\hat{\sigma}_\rho=\sqrt{\frac{s^2}{\sum_{t=1}^{T}y_{t-1}^2}},s^2=\frac{\sum_{t=1}^{T}(y_t-\hat{\rho}y_{t-1})^2}{T-1}.$$

By simplification, the test statistics can be expressed as follows:

$$t_{\hat{\rho}} = \frac{\sum_{t=1}^{T} y_{t-1} z_t}{\sqrt{\sum_{t=1}^{T} y_{t-1}^2} \cdot s} .$$

Taking into account breaks in variance, we derive the limiting distribution of the test statistic as follows:

First,

$$T^{-(\frac{1}{2}+d)} \sum_{t=1}^{[Tr]} \varepsilon_t \xrightarrow{L} \sigma_\varepsilon B_{\varepsilon,d}(r),$$

$$T^{-(\frac{1}{2}+d)} \sum_{t=1}^{[Tr]} \eta_t \xrightarrow{L} \sigma_\eta B_{\eta,d}(r),$$

hold true, where $B_{\varepsilon,d}(\cdot)$ and $B_{\eta,d}(\cdot)$ are independent Brownian Motions. σ^2_ε and σ^2_η are the long-run variance defined in Lemma 1 and 2.

The symbol $\xrightarrow{L}$ signifies convergence in distribution. Therefore, the following equation is derived that

$$T^{-(\frac{1}{2}+d)} \sum_{t=1}^{[Tr]} z_t = T^{-(\frac{1}{2}+d)} \sum_{t=1}^{[Tr]} \varepsilon_t + I_{\{t>T\lambda\}} \sum_{t=T\lambda+1}^{[Tr]} \eta_t$$

$$\xrightarrow{L} \sigma_\varepsilon B_{\varepsilon,\mathrm{d}}(r) + \sigma_\eta [B_{\eta,d}(r) - B_{\eta,d}(\lambda)] du(r)$$

where

$$\mathrm{d}u(r) = \begin{cases} 1, & r > \lambda \equiv T_B / T \\ 0, & r \le \lambda \end{cases},$$

and the coefficient λ is the ratio of pre-break sample to total sample size. Because $B_{\eta,d}(r)$-$B_{\eta,d}(\lambda)$ has the same distribution as $B_{\eta,d}(r\text{-}\lambda)$, we obtain

$$\mathrm{Var}\,(\sigma_\varepsilon B_{\varepsilon,\mathrm{d}}(r) + \sigma_\eta [B_{\eta,d}(r) - B_{\eta,d}(\lambda)] \mathrm{d}u(r))$$

$$= \sigma_\varepsilon^2 r^{1+2d} + \sigma_\eta^2 (r-\lambda)^{1+2d} \,\mathrm{d}u(r)$$

$$= r^{1+2d} (\sigma_\varepsilon^2 + \sigma_\eta^2 (1 - \frac{\lambda}{r})^{1+2d} \,\mathrm{d}u(r)) \equiv r^{1+2d} \sigma_z^2 .$$

where

$$\sigma_z^2 = \sigma_\varepsilon^2 + \sigma_\eta^2 (1 - \frac{\lambda}{r})^{1+2d} \,\mathrm{d}u(r) .$$

In order to more clearly express the effects of the breaks in variance, we perform some transformations. The new fractionally Brownian Motion is defined as

$$B_{z,d}(r) = \frac{\sigma_\varepsilon B_{\varepsilon,\mathrm{d}}(r) + \sigma_\eta [B_{\eta,d}(r) - B_{\eta,d}(\lambda)] \mathrm{d}u(r)}{\sigma_z} .$$

In view of Lemma 1 and 2, thus we obtain

$$T^{-(\frac{1}{2}+d)} \sum_{t=1}^{[Tr]} z_t \xrightarrow{L} \sigma_z B_{z,d}(r) .$$

Notice that,

$$\sum_{t=1}^{T} y_t^2 = \sum_{t=1}^{T} (y_{t-1} + z_t)^2 = \sum_{t=1}^{T} y_{t-1}^2 + 2\sum_{t=1}^{T} y_{t-1} z_t + \sum_{t=1}^{T} z_t^2 ,$$

then

$$\sum_{t=1}^{T} \frac{y_{t-1} z_t}{T^{1+2d}} = \frac{y_T^2 - \sum_{t=1}^{T} z_t^2}{2T^{1+2d}} .$$

Due to the ergodicity of these stationary process, we have

$$T^{-1} \sum_{t=1}^{T} z_t^2 = T^{-1} \sum_{t=1}^{T} \varepsilon_t^2 + T^{-1} \sum_{t=[T\lambda]}^{T} \eta_t^2 \xrightarrow{P} E\varepsilon_t^2 + (1-\lambda) E\eta_t^2$$

$$= \gamma_\varepsilon(0) + (1-\lambda)\gamma_\eta(0)$$

$$= \frac{\Gamma(1-2d)}{\Gamma(1-d)\Gamma(1-d)} \sigma_u^2 + (1-\lambda) \frac{\Gamma(1-2d)}{\Gamma(1-d)\Gamma(1-d)} \sigma_v^2 .$$

Also since

$$\frac{y_T^2}{T^{1+2d}} \xrightarrow{L} \{\sigma_z B_{z,d}(1)\}^2 ,$$

and

$$\frac{\sum_{t=1}^{T} y_t^2}{T^{2+2d}} \xrightarrow{L} \int_0^1 \{\sigma_z B_{z,d}(r)\}^2 \,\mathrm{d}r ,$$

it directly shows that

$$T^{-1}(\hat{\rho} - 1) = \frac{T^{-(1+2d)} \sum_{t=1}^{T} y_{t-1} z_t}{T^{-(2+2d)} \sum_{t=1}^{T} y_{t-1}^2} \xrightarrow{L} \frac{\{\sigma_z B_{z,d}(1)\}^2}{2\int_0^1 \{\sigma_z B_{z,d}(r)\}^2 \,\mathrm{d}r} .$$

On the other hand,

$$s^2 = T^{-1} \sum_{t=1}^{T} (y_t - y_{t-1} - (\hat{\rho}-1) y_{t-1})^2$$

$$= T^{-1} \sum_{t=1}^{T} (z_t - (\hat{\rho}-1) y_{t-1})^2$$

$$= T^{-1} \sum_{t=1}^{T} z_t^2 - 2(\hat{\rho}-1) T^{-1} \sum_{t=1}^{T} y_{t-1} z_t + (\hat{\rho}-1)^2 T^{-1} \sum_{t=1}^{T} y_{t-1}^2 .$$

As in the proof of the above, the first term converges in probability to $\gamma_\varepsilon(0) + (1\text{-}\lambda)\gamma_\eta(0)$ and the other two terms both converge to zero. This is because

$$2(\hat{\rho}-1) T^{-1} \sum_{t=1}^{T} y_{t-1} z_t = \frac{O_p(T^{2d}) \cdot O_p(T^{1+2d})}{O_p(T^{2+2d}) - O_p(T^{1+2d}) - O_p(T)} = o_p(1)$$

and

$$(\hat{\rho}-1)^2 T^{-1} \sum_{t=1}^{T} y_{t-1}^2 = \frac{(O_p(T^{2d}))^2}{O_p(T^{2+2d}) - O_p(T^{1+2d}) - O_p(T)} = o_p(1) .$$

Combine these results we can obtain

$$s^2 = T^{-1}\sum_{t=1}^{T} z_t^2 \xrightarrow{L} \gamma_\varepsilon(0) + (1-\lambda)\gamma_\eta(0) ,$$

and

$$t_{\hat{\rho}} \xrightarrow{L} \frac{\{\sigma_z B_{z,d}(1)\}^2}{2\sqrt{(\int_0^1 \{\sigma_z B_{z,d}(r)\}^2 dr)\cdot(\gamma_\varepsilon(0)+(1-\lambda)\gamma_\eta(0))}} .$$

2. MONTE CARLO STUDY

In this section we conduct a simulation study to have the effects of variance changes on the size of the test statistics. We consider the data generating process (DGP) as follows,

$(1\text{-}L)^d \varepsilon_t = u_t$

$(1\text{-}L)^d \eta_t = v_t$

$$y_t = y_{t-1} + z_t, \quad \begin{cases} \mathrm{var}(z_t) = \sigma_u^2 & , t \le [\lambda T] \\ \mathrm{var}(z_t) = \sigma_u^2 + \sigma_v^2 & , t > [\lambda T] \end{cases} .$$

We analyzed each experiment under the following settings and all results are in 5% confidence level:

T=100, 250

$$\frac{\sigma_v}{\sigma_u} = 0, 1, 4, 10 .$$

λ=0.1, 0.3, 0.5, 0.7, 0.9

d=0, 0.2, 0.4

ρ=0.8, 0.9, 0.95

y_0=0

Note that coefficient σ_1 is normalized to be one. When σ_v / σ_uthere is no structural break in variance. In each experiment, the number of replications was 20,000.

Table 1
Empirical Size :$t_{\hat{\rho}}$ Statistic

		T=100					*T*=250				
		λ									
	σ_v/σ_u	0.1	0.3	0.5	0.7	0.9	0.1	0.3	0.5	0.7	0.9
d=0	0	5.095	5.040	5.365	5.290	5.500	5.015	5.405	5.145	5.410	5.330
	1	5.640	6.745	6.735	6.350	5.510	5.930	6.815	7.335	6.760	5.590
	4	6.470	8.770	11.490	13.395	11.390	6.520	8.775	11.770	13.835	11.540
	10	6.375	9.225	13.675	20.015	22.990	6.545	9.460	13.970	20.680	24.780
d=0.2	0	5.495	5.330	5.445	5.430	5.800	5.505	4.945	5.035	5.240	5.230
	1	5.915	6.390	6.410	6.315	5.760	5.980	6.370	6.320	6.485	5.380
	4	5.920	8.570	11.050	12.490	9.940	6.050	8.165	10.460	11.870	9.760
	10	6.340	9.350	13.325	18.430	19.105	5.760	8.965	12.805	17.670	19.210
d=0.4	0	5.805	4.725	4.425	4.750	5.725	5.615	4.965	4.630	4.745	5.120
	1	5.460	5.790	5.135	4.895	6.020	5.545	6.075	5.345	5.130	5.715
	4	5.600	7.265	8.545	7.830	7.780	5.755	7.055	7.615	7.730	7.825
	10	5.525	7.400	9.865	11.525	11.765	5.525	7.160	8.920	10.300	11.790

Table 2
Empirical Power ρ=0.8:$t_{\hat{\rho}}$ Statistic

		T=100					*T*=250				
		λ									
	σ_v/σ_u	0.1	0.3	0.5	0.7	0.9	0.1	0.3	0.5	0.7	0.9
d=0	0	99.125	99.070	99.230	99.175	99.170	100.000	99.995	100.000	100.000	100.000
	1	98.860	98.095	96.705	94.200	90.420	100.000	100.000	99.985	99.895	99.505
	4	98.715	97.430	95.105	89.380	73.515	100.000	99.990	99.905	99.200	93.090
	10	98.695	97.530	94.490	87.900	69.955	100.000	99.995	99.935	99.065	89.440
d=0.2	0	99.105	99.120	99.035	98.955	99.220	100.000	100.000	100.000	99.995	100.000
	1	98.795	97.850	96.250	93.340	89.815	100.000	100.000	99.940	99.820	99.195
	4	98.735	97.225	94.480	88.125	73.135	100.000	99.975	99.880	98.950	91.090
	10	98.700	97.050	93.885	87.105	67.540	100.000	99.970	99.840	98.585	86.725
d=0.4	0	90.840	90.470	90.365	90.745	93.755	99.090	99.100	98.965	99.050	99.135
	1	88.375	86.285	81.855	76.200	72.250	98.610	97.730	95.685	93.065	87.115
	4	88.255	83.985	77.050	67.350	51.690	98.515	96.940	93.630	86.230	67.930
	10	88.095	83.040	76.505	65.850	46.170	98.445	96.965	93.395	84.785	60.970

Table 3
Empirical Power ρ=0.9: $t_{\hat{\rho}}$ Statistic

		T=100					T=250				
		λ									
	σ_v/σ_u	0.1	0.3	0.5	0.7	0.9	0.1	0.3	0.5	0.7	0.9
d=0	0	75.480	75.635	74.590	74.750	75.210	98.905	98.920	98.790	99.015	98.995
	1	74.495	72.050	69.475	67.055	61.860	98.775	97.880	96.550	93.960	89.800
	4	73.640	70.490	66.745	61.930	53.800	98.630	97.040	94.875	88.805	74.925
	10	73.545	69.700	66.360	60.955	53.500	98.575	97.255	94.530	87.760	70.905
d=0.2	0	75.395	75.410	75.045	74.560	76.775	98.575	98.560	98.525	98.295	98.675
	1	73.135	71.430	68.645	65.020	62.360	98.245	97.310	95.220	91.795	87.500
	4	72.970	69.880	65.920	60.925	52.170	98.015	96.185	92.805	85.035	69.990
	10	72.585	69.435	65.445	59.675	51.400	98.210	96.320	92.585	84.295	65.690
d=0.4	0	59.585	60.360	58.310	57.275	65.440	84.550	84.033	83.980	83.655	87.595
	1	57.260	54.915	50.580	45.750	47.220	82.075	78.960	73.370	66.905	62.915
	4	56.445	52.280	47.900	41.210	34.540	80.915	75.830	68.760	57.825	43.495
	10	56.015	52.550	47.090	39.795	32.495	81.220	75.770	68.880	56.640	38.915

Table 4
Empirical Power ρ=0.95: $t_{\hat{\rho}}$ Statistic

		T=100					T=250				
		λ									
	σ_v/σ_u	0.1	0.3	0.5	0.7	0.9	0.1	0.3	0.5	0.7	0.9
d=0	0	32.710	32.410	33.220	32.465	32.725	73.745	73.205	73.145	73.665	73.535
	1	33.330	33.720	34.875	33.365	30.660	72.775	70.300	68.360	65.610	60.980
	4	33.755	35.785	37.495	39.370	36.535	71.725	69.580	65.270	61.505	54.365
	10	33.490	35.330	37.800	40.815	42.550	72.265	69.140	65.320	60.900	53.550
d=0.2	0	34.115	33.660	33.500	33.415	35.065	74.125	73.685	72.555	76.005	74.640
	1	33.375	34.390	34.475	33.320	30.795	71.255	69.180	66.375	63.045	60.485
	4	33.215	34.990	37.185	37.860	35.180	71.020	67.695	63.040	58.495	50.930
	10	33.300	34.595	36.980	39.935	39.800	71.135	67.345	63.220	57.235	49.070
d=0.4	0	30.165	29.060	26.385	26.170	32.570	54.340	54.020	52.915	53.225	61.400
	1	28.270	28.095	25.780	22.905	25.985	51.100	48.980	44.335	39.690	42.730
	4	28.335	27.555	26.575	25.260	23.215	50.945	46.700	41.080	34.780	30.140
	10	28.055	27.340	27.135	25.190	24.620	50.675	46.380	40.700	34.170	26.865

The figures in Table 1 show the percentage of rejections of the null hypothesis H_0, in the presence of variance break. They indicate that the empirical size tends to significant level of 5% when the magnitude of change points is zero. However, there is little over-sizing with larger sample size. For example, as λ=0.3 and σ_v/σ_u=10, the rejection frequency is 9.225% and 9.46% for T=100,250.

It is also worth mentioning that as the magnitude σ_v/σ_u increases, the impact of broken variance is more pronounced, leading to over-sized test. The rejection rate in the case of d=0.2 and T=100, λ=0.5 are 6.41% for σ_v/σ_u=1 and 11.05% for σ_v/σ_u=4. Furthermore, with the increase of long memory index, the rejection frequency is gradually decreased. For example, if σ_v/σ_u=4, T=250 and λ=0.7 are 13.835% for d=0 and 11.87% for d=0.2. This indicates that the extent of size distortions is heavily sensitive to the degree of changes and long memory index. Final with the change of the position of the change point, the rejection rate also changes. if d=0, T=100, σ_v/σ_u=10, λ=0.1,0.9, the rejection frequency is 6.375% and 22.99% respectively.

Tables 2-4 show the power of the test. From these three tables, we can draw the following conclusions. First, the impact is more obvious in small samples, but in the large sample not have some impacts. For example, when T=100 λ=0.7, σ_v/σ_u=4, the rejection frequency is 88.125% and 67.35% for d=0.2,0.4. However, when ρ=0.8, T=250, λ=0.7, σ_v/σ_u=4, the rejection frequency is 98.95% and 86.23% for d=0.2,0.4. Second, as the magnitude of change point increases, the rejection of the alternative hypothesis decreases gradually. For example, if ρ=0.8, T=100, λ=0.5, d=0.4, the rejection frequency is 81.855% and 77.05% for σ_v/σ_u=1, σ_v/σ_u=4. Third, with the change of the position of the change point, the rejection rate also changes. If ρ=0.8, d=0.4, T=100, σ_v/σ_u=10, λ=0.1 is 88.095% and λ=0.9 is 46.17%. Final, with the increase of long memory index, the rejection frequency are gradually decreased. if λ=0.9, T=100, σ_v/σ_u=1, λ=0.1, d=0.2 is 73.135% and d=0.4 is 57.26%.

CONCLUSION

The present study examined the unit root tests to long memory observations in the existence of variance breaks. The limiting distribution of the test statistics was derived, and Monte Carlo evidence on finite samples was provided. It is shown that the non-constant variance can both inflate and deflate the rejection frequency, thus the statistic tests are not robust. The simulation results also indicate the extent of size distortion is heavily sensitive to the location and magnitude of change points, long memory index and sample size. The experimental results are consistent with our theoretical results.

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