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Investigation on the Concept of Limit Involving E by Exploring the Secret of Mr. Buffet

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Abstract

This article uses problem-driven teaching methods to explore a mathematical class of important limit in course design. We first start with the story of Buffett's wealth to gain students' interest in the concept. We establish mathematical models based on the financial management issues and guide students to explore them. Then, through the analysis of the compound interest problem, solution with limit expression of the problem and its relationship with the natural constant *e* are obtained. Finally, in response to Buffett's sentiment, we use this limit to analyze and explain the mathematical principles of investment issues and to lead students to think about their views of life and values, resulting in a positive influence on their life planning.

Key words: Advanced mathematics; Course design; Limit; Natural constant

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INTRODUCTION

Calculus is the foundation for science, engineering, and finance majors in higher education institutions and an important basis for future studies. However, students have been having difficulties understanding the concepts of Calculus due to its strong logic and abstraction. As the basis of Calculus, a correct understanding and application of the limit theory is of much significance (Gao and Xiang, 2011; Li and Xie, 2006). This class mainly focuses on the concept, nature, and calculation of $\lim_{x\to\infty}(1+\frac{1}{x})^x = e$. In the previous lessons, we have already covered the concept and basic properties of limit, the Squeeze Theorem, Monotone Convergence Theorem, and common limit calculation methods. However, there is still a kind of limit infinitive needed to be solved; that is, when the limit becomes 1^∞ due to the independent power exponential function. This is not only important as the solution of limit infinitive, but also has a rick practical significance. Therefore, in order to let students understand the essence of Calculus, we adopt problem-driven, heuristic, and cast-study teaching methods (Abeysekera and Dawson, 2015; Zheng, 2008; Lucas, 1974) in the class design, so that students can gradually explore the concept based on their understanding and scientific thinking, reveal its mathematical content, and gain a real understanding of the material and learn the application of such knowledge.

1. COURSE DESIGN

The course begins with the story of Buffett's wealth growth and demonstrates the exponential growth caused by compound interest calculations (Schroeder, 2009). Then, the problem is reduced into a simple investment model, showing the calculation method of compound interest and summarizing the limit problem. Secondly, the limit of the series is inferred and proved through numerical analysis and Monotone Convergence Theorem. On this basis, the natural constant *e* is derived and the definition of the second important limit is given. Subsequently, we revisit the problems mentioned in the beginning of the class, drawing on Buffett's sentiment, and we give a clear analysis and mathematical explanation of the investment problem. Finally, combining teaching with educating people together to has a profound impact on students' learning and research in higher mathematics and related disciplines, as well as their life planning. The course design is shown in Figure 1.

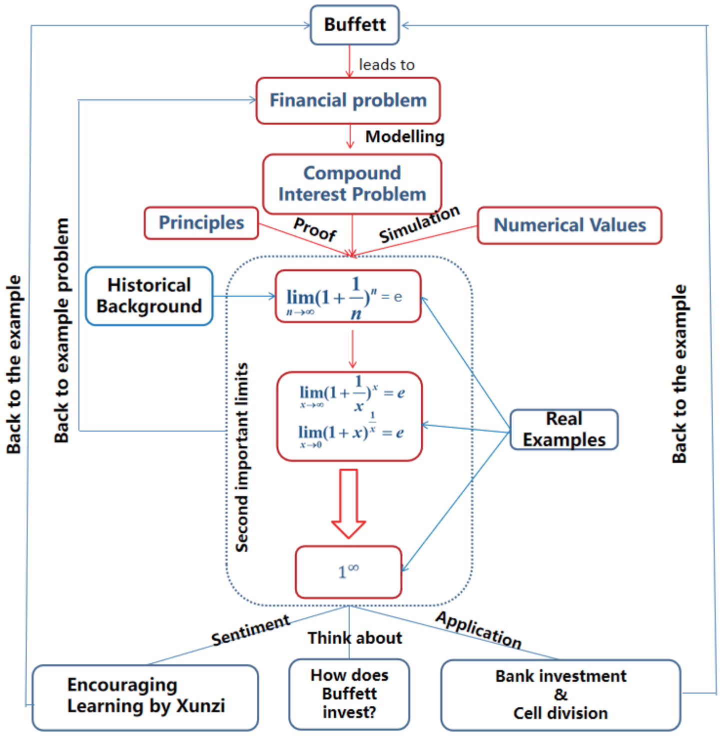

Figure 1
Course Design

2. INSTRUCTIONAL PROCESS

2.1 Propose the Problem

First, we show Buffett's wealth growth curve (Fig. 2), let students intuitively observe the phenomenon of the "explosive growth" of Buffett's wealth (Schroeder, 2009), and lead students to think about the secret of Buffett's wealth 'explosion' after the age of 60 and the mathematical principles behind such phenomenon.

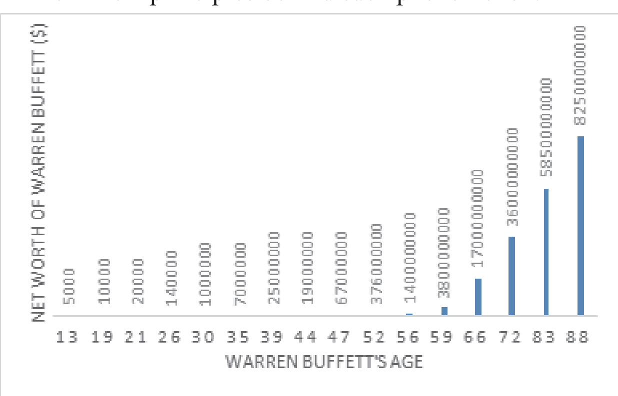

Figure 2
Histogram of Buffett's Wealth as a Function of His Age (curve)

In order to reveal the secret of Buffett's wealth, let's start with a simple investment game. Let's assume the investment capital is \$1, and the annual interest is 100%. If the interest is calculated on an annual basis, then the total capital after a year is 1+1=\$2. What if the interest is calculated on a six-month basis? That means the interest is calculated every six months. The interest rate for six months is 50%. Therefore, after six months, the total capital is sum of the original capital \$1 and \$0.5 interest, which is \$1.5. Such amount is used as the capital when calculating the second six-month interest, which is \$0.75. Thus, the sum of these two terms, $(1+\frac{1}{2})^2$, is \$2.25.

Evidently, the total profit increases by \$0.25 when the number of interest cycle shifted from an annual basis (once) to a six-month basis (twice). If we increase the number of interest cycle to four times per year, then the total capital at the end of year is $(1+\frac{1}{4})^4$, resulting in additional \$0.44. Similarly, with 12 interest cycles per year, the total additional capital is \$0.61, and \$0.71 with 365 interest cycles per year (Table 1). It is obvious that as the number of interest cycle increases, the total amount of capital will also increase at the end of one year. That is, with a limited number of interest cycle, the more cycles per year, the higher the profit. Let the number of interest cycle be a natural number n, then the total capital is $(1+\frac{1}{n})^n$. At this point, students will naturally think about the next step: will the total capital approaches infinity if n approaches positive infinity? This is, will $\lim_{n\to\infty}(1+\frac{1}{n})^n$ converge?

Table 1
Total Capital and Additional Profit Calculated Using Different Numbers of Interest Cycle With $1 of Investing Capital and 100% Annual Interest Rate

Interest period	Number of interest cycle	Total capital after a year	Additional interest
Annual	1	1+1=2	
6 months	2	$(1+\frac{1}{2})^2 = 2.25$	0.25
quarterly	4	$(1+\frac{1}{4})^4 \approx 2.44$	0.44
monthly	12	$(1+\frac{1}{12})^{12} \approx 2.61$	0.61
daily	365	$(1+\frac{1}{2})^2 \approx 2.71$	0.71
Any time	n	$(1+\frac{1}{n})^n$	?

Before we start a rigorous theoretical analysis, let's do a numerical simulation and estimation of the limit. First, let's express the relation between the number of interest cycle n and the total capital using $f(n) = (1+\frac{1}{n})^n$. The numerical results are listed in Table 2 and Fig. 3. It is not hard to draw the conclusion based on observation that as $n \to \infty$, $f(n)$ converges to a value between 2.7 and 2.8. Then, the monotonicity and boundedness of the limit can be analyzed using the Binomial Theorem and Monotone Convergence Theorem. Finally, the convergence of the limit can be proved, and the converging value is recorded as a special constant e. Such conclusion embodies the hard work and talents of many great mathematicians in 17th and 18th centuries, including John Napier, Gottfried Wilhelm Leibniz, Jakob Bernoulli, Leonhard Euler, etc.

Table 2
Values of $f(n)$ as a Function of n

n	1	2	4	12	365	1000	10000	100000	…
$f(n)$	2	2.25	2.44	2.613	2.714567	2.71692	2.71815	2.71827	…

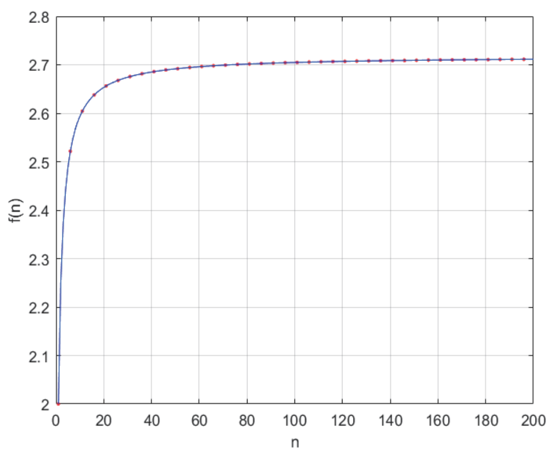

Figure 3
The Total Capital $f(n)$ as a Function of the Number of Interest Cycle n

2.2 Importance of Limit

We have proved the convergence of $\lim_{n\to\infty}(1+\frac{1}{n})^n$ and its limit value. As the number of interest cycle approaches infinity, the total capital converges and approaches *e*. Next, how to relate this problem to power exponential function from a mathematical aspect? That is, as $x\to\infty$, will $f(x)=(1+\frac{1}{x})^x$ converge and approach *e*, as well? This question requires students to keep thinking about the continuous compound interest problem. Because the limit of number sequence is a special case of the limit of function, we can lead students to use the relationship between the two limits as the starting point and conclude that $\lim_{x\to\infty}(1+\frac{1}{x})^x$ converges and approaches *e* by analogy and induction. Later, we will demonstrate the rigorous mathematical proof that leads to such conclusion. So far, we have started with a real-life problem, and the background and mathematical principles of this important limit are elaborated and displayed through the in-depth analysis. It is considered to be a very important limit, not only because it establishes the relationship between rational numbers and irrational numbers *e*, but also because it provides ideas and methods for solving a class of similar extreme problems.

Apparently, this important limit has a certain degree of particularity in its form. That is, when the power exponential function $f(x)$ has a base number of 1 plus the inverse of variable *x* and the exponential term is the variable *x* itself, then as $x\to\infty$, $f(x)$ approaches *e*. Such understanding is not enough, we should encourage students to discover its true nature and reveal the connotation and extension of such limit. Then, let's begin with the extreme cases of the reciprocal term and let $t=\frac{1}{x}$. As $x\to\infty$, $t\to 0$. By direct substitution, we get $\lim_{t\to 0}(1+t)^{\frac{1}{t}}$ approaches *e*, as well. At this point, we have extended the important limit from its original form $\lim_{n\to\infty}(1+\frac{1}{n})^n$ to the standard form of a limit function, $\lim_{x\to\infty}(1+\frac{1}{x})^x$ and $\lim_{x\to 0}(1+x)^{\frac{1}{x}}$. Looking at the later two limits, it is not hard to find that the base number approaches 1 and the exponential term approaches infinity in both limit functions $f(x)$ and $f(t)$ as $x\to\infty$ and $t\to 0$ despite the small difference in forms. These two limits both belong to the class of limit infinitive 1^∞. On the other hand, this indicates that when the limit problem has the form of 1^∞, students could apply the concept about important limit we just learned. Different types of examples can be interspersed during the course to help students understand the full application of important limit in solving similar problems.

2.3 In-depth Understanding

The final value of such important limit does not approach infinity; thus, the continuous compound interest does not reproduce the miracle of Buffett's wealth within a year of investment period. How did Buffett accumulate such wealth then? In fact, as he said, "Life is like a snowball. The important thing is finding wet snow and a really long hill." (Schroeder, 2009) Here, 'wet snow' and 'long hill' imply high return rate plus long-term persistence. The result will be immeasurable! Let us revisit the example at the very beginning and calculate the total capital *V* at *k*th year with the same investing capital and a daily interest cycle:

$$V=\lim_{n\to\infty}A_0(1+\frac{r}{365})^{365k}=A_0e^{rk}$$

where *r* is the annual interest rate, A_0 is the investing capital, and *k* is the number of years. The value of V as a function of *r* and *k* are shown in Table 3 assuming $A_0=1$.

Based on the results, when the annual interest rate is fixed at 4%, the total capital almost doubled after an investing period of 20 years. Even with continuous compound interest, the short-term total capital has not increased a lot under a fixed annual interest rate. When the investing period has been extended by 10 times, that is, after two centuries, the total capital will increase by nearly 3000 times. Now, let us look at the results with a fixed annual interest rate of 20%, comparing with the results based on an annual interest rate of 4%, though the annual interest rate only increased by 5 times, as time goes by, the total capital will be 10,000,000 times more than the total capital with an annual interest rate of 4% after a century. Although, the success of Buffett is a result of multiple factors, with this important limit function, it is not hard to find that a rational choice of 'wet snow' and insisting on 'a really long hill' must be the two implacable elements of his success.

Table 3
The Total Capital as a Function of Annual Interest Rates and Numbers of Investing Periods

	4%	7%	10%	20%
1year	1.04	1.07	1.10	1.22
10 years	1.49	2.01	2.72	7.39
20 years	2.18	4.05	7.39	54.54
50 years	7.39	33.10	148.31	21966.22
100 years	54.59	1095.90	21996.32	482514991.51
200 years	2979.65	1200991.12	483838036.95	2.33×10^{17}

2.4 Sentiment

Here, we start from the compound interest problem, explore and discover a class of important limit, and understand the origin of e; through the discussion of the expression of this important limit, we learn the essence of this kind of limit and its important application in solving problems involving infinitive limit. We use these mathematical languages to further reveal the secret of Buffett's success. Although we may not be able to copy his legend, as early as the 3rd century BC, the famous Chinese ancient thinker Xunzi once said that "Step after step the ladder is ascended." This concludes what we have learned today. It reminds us that in daily life, as long as you are making a little progress every day, you will finally reach the peak of your life!

CONCLUSION

This article introduces and explains to students a very important limit in the theory of higher mathematics that is closely related to the natural constant e. Using the combination of problem-driven and heuristic teaching methods, using the secret of Buffett's wealth growth as a case study, the students will be guided to experience the explosive growth brought by compound interest. This example will stimulate students' interest in learning, and lead to the course material. Then, in response to abstract mathematical problems, students are inspired to observe, think, communicate, and discuss the essence behind the phenomenon in the problem, the expression of important limit and its nature. Finally, with the help of important limit, using the language of mathematics to interpret Buffett's famous words, through the example of investment and financial management, the key factors in the compound interest problem are deeply analyzed, then the effects of investing period and interest rate on the total capital of compound interest are studied, and the theory of Buffett's wealth growth is revealed. The questions raised in the opening paragraph are answered, and the famous saying of Xunzi echoed this conclusion. This course design not only cultivates the students' ability in applying mathematics knowledge to solve practical problems, but also guides students to experience the beauty and the ubiquitous charm of mathematics. It has a positive influences on students' views of life and value.

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