CSCanada
Canadian Social Science
Vol. 11, No. 11, 2015, pp. 103-106
DOI:10.3968/7900

ISSN 1712-8056[Print]
ISSN 1923-6697[Online]
www.cscanada.net
www.cscanada.org

Extending Retirement Age and Increasing Consumption: A Quantitative Assessment

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Received 30 August 2015; accepted 15 October 2015
Published online 26 November 2015

Abstract
The quantitative exercises of this paper determine the quantitative expression between consumption and retirement age and are certain that there is positive relation between extending retirement age and increasing consumption.

Key word: Retirement age; Consumption; Overlapping generation model

Cao, Z. (2015). Extending Retirement Age and Increasing Consumption: A Quantitative Assessment. *Canadian Social Science, 11*(11), 103-106. Available from: http://www.cscanada.net/index.php/css/article/view/7900
DOI: http://dx.doi.org/10.3968/7900

INTRODUCTION

This paper has sought to present how a particular employment practice—the use of extending retirement age as a means of enhancing output per person through increasing consumption. Extending retirement age means increasing labor supply then enhancing income. Moreover, consumption explains more than 70% of incomes. So the issue is to determine the relation between income and consumption.

The relation between extending retirement age and consumption is closely related to the permanent-income theory of consumption (Friendman, 1957) and the life-cycle theory of consumption and income (Modigliani & Brumberg, 1954). Rational forward-looking consumption is expected to spend at older age and to save at younger age so as to maintain a constant utility level during the period of the life-cycle. There are two doubts raised in empirical and theoretical analyses against the consumption smoothing hypothesis. Firstly, it was known that consumption expenditures and income fell for retirement in United Kingdom (Books et al, 1998; Smith, 2006), the USA (Berhein et al., 2001), in Japan (Wakabayashi, 2008). Secondly, Hamermesh (1984) proposed that saving of consumers are not enough to maintain the level of consumption constant after retirement.

From the view of model study, there are two kinds of models. The first model is conceptually the simplest. Competitive firms hire labor and rent capital to produce and sell output, and a fixed number of infinitely lived households hold capital, supply labor, saves and consumes. This model, which was developed by Ramsey (1928), Cass (1965), and Koopmans (1965), debars all market imperfections and all issues brought up by heterogeneous households and links among generations.

The second model is the overlapping-generations model developed by Diamond (1965). The key difference between the Ramsey-Cass-Koopmans model and the Diamond model is that the diamond model presumes that there is continual entry of new households into the economy.

However, studies above typically find changes in income can have significant impacts on the consumption. In contrasts, there is little work on how an increase in the retirement age affects consumption. This paper will offer a direct relation between retirement age and consumption and propose a quantitative expression between retirement age and consumption

1. MODEL

This paper begins by analyzing the economy's firm sector. This permits to describe the engine of endogenous growth, which relies on an interfirm externality, in the spirit of Romer (1989) and Saint-Paul (1992). The latter also induces to a constant (real) interest rate, a result that simplifies the deriving of the macroeconomic equilibrium.

1.1 Firms

This paper discusses a closed economy, supposes a standard neoclassical production function, and assumes a Cobb-Douglas production with exogenous technological progress to carry out quantitative exercises relevant to a growing economy. The production function is given by

$$Y(t)=F[K(t), A(t)N(t)]=K(t)^{\alpha}[A(t)N(t)]^{1-\alpha} . \quad (1)$$

Here $0<\alpha<1$, and $Y(t)$, $K(t)$, $A(t)$ and $N(t)$ represent, respectively, output, capital input, technological level and labor input at time t. Technological progress is represented by

$$A(t)=A(0)e^{gt} . \quad (2)$$

Here g is the rate of labor-augmenting technological progress. Capital accumulates

$$d[K(t)]/dt=Y(t)-C(t)-\delta K(t) . \quad (3)$$

Here δ is the depreciation rate and $c(t)$ is consumption at time t.

Define a variable per unit of effective labor as the variable divided by AN, and refer it in lower case letter {e.g. $y(t)=Y(t)/[A(t)N(t)]$}. With this information, the production function in intensive form is given by

$$y(t)=F\{K(t)/[A(t)N(t)],1\}\equiv f[k(t)]=k(t)^{\alpha}. \quad (4)$$

Furthermore, (2)-(4) and (9) lead to

$$d[k(t)]/dt=k(t)^{\alpha}-c(t)-(\delta+g+n)k(t). \quad (5)$$

1.2 Households

This paper also assumes that the economy consists of agents with different birth dates. Represent the probability that an individual survives to the age x at least by the survival function $p(x)$, where $x\in[0,\Omega]$, Ω is the maximum age, and $p(0)$ is normalized to 1. Since most overlapping-generations models suppose that the adult stage starts at age 20, in the following analysis x is interpreted as the adult age, which is defined as actual age minus 20. So the survival probability in terms of adult age according to $p(x)=p_{\text{actual}}(x+20)/p_{\text{actual}}(20)$, here $p_{\text{actual}}(.)$ is the survival probability based on actual age.

The instantaneous mortality rate at age x, $\mu(x)$, is related to $p(x)$ by

$$\mu(x)=-(1/p(x))(dp(x)/dx). \quad (6)$$

In the following analysis, suppose that each individual has some fixed lifespan, T, and works up to an age T_r which is strictly less than Ω and then retire. Thus, the labor supply in the model can be represented by

$$m(s,v)=\begin{cases}1, if\ 0\le v-s\le t_r\\ 0, \text{if } t_r\langle v-s\le\Omega\end{cases} . \quad (7)$$

Here $m(s,v)$ is the labor supply of a cohort s individual at time v, and T_r is the retirement age. Further, the number of people born per unit time must grow at rate n for the overall population to be growing at rate n and the age distribution to be well behaved. That is,

$$B(t)=B(0)e^{nt}. \quad (8)$$

Where n is the constant growth rate of the number of births, and $B(t)$ is the number of births at time t.

Based on the death and birth histories of different cohorts, the population at time t is given by

$$N(t)=\int_{t-\Omega}^{t}B(s)p(t-s)ds=\int_{t-\Omega}^{t}B(t)e^{-n(t-s)}p(t-s)ds$$

$$=B(t)\int_{0}^{\Omega}e^{-nx}p(x)dx. \quad (9)$$

The term $\int_0^\Omega e^{-nx}$ in (9) expresses the sum of the number of survival persons (when $p(0)$ is normalized to 1) at various ages, adjusted by n (growth rate of births). It is clear that the population grows at the same rate as that of the number of births because this term is independent of time t.

Discussing individual's consumption decision. An individual born at time s selects $\{C(s,v)\}_t^{s+\Omega}$ at time t (here $s\le t\le s+\Omega$) to maximize

$$\int_{t}^{s+\Omega}e^{-\rho(v-t)}[p(v-s)/p(t-s)]\{[c(s,v)^{1-1/\sigma}-1]/[1-1/\sigma]\}dv. \quad (10)$$

Subject to the flow budget constraint

$$dZ(s,v)/dv=[r(v)+\mu(v-s)]Z(s,v)+w(v)m(s,v)-C(s,v) , \quad (11)$$

where σ is the intertemporal elasticity of substitution, ρis the discount rate, $C(s,v)$ is consumption of a cohort s individual at time v, $r(v)$ is the (real) interest rate at v, $Z(s,v)$ is the financial wealth of a cohort s individual at time v, and $w(v)$ is the (real) wage rate at v. Individuals are born without liabilities or financial assets, and face a terminal condition of non-negative financial wealth. The two boundary conditions for a particular cohort are given by

$$Z(s, s)=0, \quad Z(s, s+\Omega)\ge 0. \quad (12)$$

To focus on the saving-for-retirement motive, this paper follows D'Albis (2007), and Sau-Him (2009) to suppose that individuals have no bequest motive. Note that the objective function in (10) is weighted by the conditional probability of survival at different ages, $p(v-s)/p(t-s)$. The budget constraint (11) presumes the presence of an actuarially fair annuity(Yaari,1965), in which an individual of age x will accept an extra amount equal to $\mu(x)Z(s, s+x)$ if he is still alive, but will give up all the amount $Z(s, s+x)$ to the insurance company if death occurs.

Keynes-Ramsey rule for this intertemporal consumption problem is that for $v\ge t$,

$$dC(s,v)/dv=\sigma[r(v)-\rho]C(s,v). \quad (13)$$

Since labor supply is exogenous, it can be shown from (8) that

$$N(t) = \int_{t-\Omega}^{t} B(s)p(t-s)\,ds = B(t)\int_0^{T_r} e^{-nx}p(x)\,dx. \quad (14)$$

Thus, aggregate labor supply increases at the same rate n as the population does.

1.3 Households and Firms in the Steady-State Equilibrium

Now, the paper discusses the steady-state equilibrium of the economy and express a variable at the steady-state equilibrium with a^*. The steady-state equilibrium defined by $dk(t)/dt=0$ in (5); that is

$$(k^*)^{\alpha}-c^*-(\delta+g+n)\,k^*=0. \quad (15)$$

Furthermore, the wage rate and the competitively determined interest rate at the steady-state equilibrium are given by

$$r^*(t)=f'(k^*)-\delta=\alpha(k^*)^{\alpha-1}-\delta\equiv r^*, \quad (16)$$

and

$$w^*(t)=A(t)[f(k^*)-k^*f'(k^*)]=A(t)(1-\alpha)(k^*)^{\alpha}. \quad (17)$$

Integrating (13) along the steady-state equilibrium gives

$$C^*(s,v)=eg^*_c(v-s)C^*(s,s). \quad (18)$$

Here $C^*(s,s)$ is the steady-state consumption of a cohort s individual at the beginning life, and

$$g^*_c=\sigma(r^*-\rho), \quad (19)$$

is the steady-state growth rate of individual consumption. Substituting (2), (7), (12) and (16)-(19) into a cohort s individual's lifetime budget at the beginning of adult life, then simplifying, yields

$$C^*(s,s) = e^{gs}w^*A(0)\Big[\int_0^{T_r} e^{-(r^*-g)x}p(x)\,dx\Big]\Big/\Big[\int_0^{\Omega} e^{-(r^*-g_c^*)x}p(x)\,dx\Big]. \quad (20)$$

It is easy to conclude that starting consumption levels of different cohorts are growing at g, the rate of technological progress because all terms on the right-hand side, except e^{gs}, of (20) are independent of s. Furthermore, aggregate consumption at the steady-state equilibrium is associated with the starting consumption level of a particular cohort, $C^*(0,0)$, according to

$$\begin{aligned} C^*(t) &= \int_{t-\Omega}^{t} B(s)p(t-s)C^*(s,t)\,dx \\ &= \int_{t-\Omega}^{t} B(t)e^{-n(t-s)}p(t-s)e^{g_c^*(t-s)}C^*(s,s)\,ds \\ &= B(t)e^{gt}C^*(0,0)\int_0^{\Omega} e^{-(g+n-g_c^*)x}p(x)\,dx. \end{aligned} \quad (21)$$

Substituting the equation, $C^*(0,0)=w^*A(0)\Big[\int_0^{T_r} e^{-(r^*-g)x}p(x)\,dx\Big]\Big/\Big[\int_0^{\Omega} e^{-(r^*-g_c^*)x}p(x)\,dx\Big]$, into equation (21) gives us

$$\begin{aligned} C^*(t) &= \Big\{B(t)e^{gt}w^*A(0)\Big[\int_0^{T_r} e^{-(r^*-g)x}p(x)\,dx\Big]\Big/\Big[\int_0^{\Omega} e^{-(r^*-g_c^*)x}p(x)\,dx\Big]\Big\}\int_0^{\Omega} e^{-(g+n-g_c^*)}p(x)\,dx \\ &= M(k^*)^{\alpha}\int_0^{T_r} e^{-(r^*-g)x}p(x)\,dx. \end{aligned} \quad (22)$$

Where $M=B(t)e^{gt}(1-\alpha)A(0)\int_0^{\Omega} e^{-(g+n-r^*)x}p(x)\,dx > 0$.

Definition: $F(t)=C^*(t)*(k^*)^{\alpha}= M\int_0^{T_r} e^{-(r^*-g)x}p(x)\,dx$. (23)

Since all terms on the right-hand side, except $\int_0^{T_r} e^{-(r^*-g)x}p(x)dx$, of (23) are independent of T_r, one takes the T_r derivative of both sides of (23), yields:

$$dF(t)/dT_r=M^{e-(r^*-g)Tr}p(T_r)>0. \quad (24)$$

Yet

$$dF(t)/dT_r=[(k^*)^{\alpha}dC^*(t)/dT_r-\alpha(k^*)^{\alpha-1}C^*(t)dk^*/dT_r]/(k^*)^{2\alpha}. \quad (25)$$

One takes the T_r derivative of both sides of Equation (15), then yields

$$\alpha(k^*)^{\alpha-1}dk^*/dT_r - dC^*(t)/dT_r - (\delta+g+n)dk^*/dT_r = 0 \quad (26)$$

Substituting the Equation (26) into Equation (25) gives us

$$\begin{aligned} dC^*(t)/dT_r &= \alpha(k^*)^{\alpha-1}dk^*/dT - (\delta+g+n)dk^*/dT_r \\ &= [\alpha(k^*)^{\alpha-1} - (\delta+g+n)]dk^*/dT_r. \end{aligned} \quad (27)$$

Substituting the Equation (15) into Equation (27) gives us

$$\begin{aligned} dF(t)/dT_r &= (k^*)^{\alpha-1}dk^*/dT_r[\alpha(\delta+g+n)k^* - k^*(\delta+g+n)]/(k^*)^{2\alpha} \\ &= (k^*)^{\alpha-1}dk^*/dT_s(\alpha-1)(\delta+g+n)/(k^*)^{2\alpha} > 0. \end{aligned}$$

And $(k^*)^{\alpha-1}(\alpha-1)(\delta+g+n)/(k^*)^{2\alpha}<0$, yields

$$dk^*/dT_r < 0. \quad (28)$$

Definition $w(t)=\alpha(k^*)^{\alpha-1}-(\delta+g+n)$

$$dw(t)/dk^*=\alpha(\alpha-1)(k^*)^{2-\alpha}<0$$

$$w(0)=-(\delta+g+n)<0, \text{ yields}$$

$$w(k^*)<0. \quad (29)$$

Substituting the Equations (28) and (29) into the Equation (26) yields

$$dC^{*}(t)/dTr=[\alpha(k^{*})^{\alpha-1}-(\delta+g+n)dk^{*}/dT_{r}] =w(k^{*})dk^{*}/dT_{r}>0\ . \quad (30)$$

That is, $C^{*}(t)$ will be enhanced when T_r is extended.

2. RESULT

Summarizing the main model predictions:

Two main implications emerge from the analysis in the section

(a) The quantitative expression between consumption and retirement age is entirely determined by the Equation (22).

(b) There is positive relation between extending retirement age and increasing consumption, Equation (30).

(c) Aggregate consumption at the steady-state equilibrium will be growing at g, the rate of technological progress because starting consumption levels of different cohorts are growing at g and because aggregate consumption at the steady-state equilibrium is associated with the starting consumption level of a particular cohort, $C^{*}(0,0)$.

To summarize, this paper considers version of an overlapping-generations model. In sharp contrast to the relation between the population growth rate and capital accumulation in an overlapping-generation model proposed by D'Albis (2007) and Sau-Him (2009), the relation between consumption and retirement age is given in this paper. Therefore, while Sau-Ham (2009) got the result related to the relation between population growth on capital accumulation in the overlapping-generation, this paper reaches the conclusion related to quantitative relation between extending retirement age and increasing consumption.

CONCLUSION

The permanent income hypothesis forecasts that the marginal propensity to consume out of permanent income is large and that the marginal propensity to consume out of transitory income is very small. Modern theories of consumption presume that individuals want to maintain relatively smooth consumption over their lifetimes. Their consumption behavior is contained in their long-term consumption opportunities—permanent income plus wealth. Therefore, current income is only one of the determinants of consumption spending. Wealth and expected income play a role too.

This paper builds a quantitative relation between consumption and retirement age. The model results show that the quantitative expression between consumption and retirement age is entirely determined by the Equation (22), that there is positive relation between extending retirement age and increasing consumption, and that aggregate consumption at the steady-state equilibrium will be growing at g, the rate of technologic progress

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