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Comparison Chain Method for AHP

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Abstract

Based on the principle of the Analytic Hierarchy Process, under the condition giving the pairwise comparison judgment matrix of the decision maker, a new method to determinate the decision maker's priority vector for the comparison objects called Comparison Chain Method is proposed with a numerical example illustrating its applicability. Finally, the possible strong, weakness and extension of the chain comparison method are discussed, the comparisons with the other methods are made leading to the rationality of CCM, and the possible future research directions for AHP are foreseen.

Key words: Analytic hierarchy process; Pairwise comparison judgment; Decision analysis; Priority aggregation

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INTRODUCTION

Analytic Hierarchy Process (AHP) is a principle and method of multi objective decision analysis based on pairwise comparison judgment of decision goal, decision criterion and decision plan of individual or group decision maker. The two problems of AHP have been the object of scholars ' study. One is pairwise comparison issues, including the comparison of the scale design, the number of comparisons, the problem of incomplete information, the uncertainty in the comparison, and so on; the other is the determination of priorities, that is, how to determine the priority vectors of decision-maker from the results of pairwise comparison judgments. In particular, a number of methods are proposed for the second problem. Such as, Saaty proposed Right Eigenvalue Method (Saaty,1980), Zahedi proposed Mean Transfer Method (Zahedi,1986), Crawford and Williams proposed Row Geometric Mean Method (or Logarithmic Least Square Method)(Crawford, 1987), Column Geometric Mean Method) (Crawford, 1987). Johnson et al. proposed Harmonic Mean Method (or Left Eigenvalue Method)(Johnson), Saaty proposed Simple Row Average Method (Saaty,1980), Chu et al. proposed Ordinary Least Squares Method (Chu, 1986), Kocaoglu proposed Constant Sum Method) (Kocaoglu, 1983), Ra proposed Column-Row Sums Approach (Ra, 1987;1988), etc. Literaturesimulates and compares the existing methods, and the results show that none of them is in the position of advantage or disadvantage in all kinds of situations (Zhou, 1995). Some of these methods are simple to calculate, such as the average method, and some have good mathematical basis, such as eigenvalue method. The author of AHP, Professor Saaty, has pointed out that only eigenvalue method is worth advocating and meaningful in all these methods. However, people have always put forward a variety of simple methods to replace the computational difficulties of eigenvalue method.

At the same time, scholars have also been studying the application of Analytic Hierarchy Process (AHP) in various fields. Such as Seyhan Nisel and Muhlis Özdemir studied the application of AHP in the field of sports (Nisel & Özdemir, 2016) ; *Aylin Çiğdem Köne* and Tayfun Büke studied the application of AHP in the field of ecological efficiency(Köne & Büke, 2017); Shannon Agredo et al. studied the application of AHP in the field of management (Agredo, 2017) and so on.

Based on the pairwise comparison judgment matrix, a new method of determining priority vector is proposed, because it takes into account the various possible cyclic results after pairwise comparison judgment, and uses the method of mathematical statistics, which we call the Comparison Chain Methods (CCM).

1. COMPARISON CHAIN METHODS OF PRIORITY AGGREGATION

Considering a certain decision problem, it is assumed that the decision-maker at a certain decision-making level are confronted with N, respectively, C_1, C_2,..., C_n, and the standard assumption of comparative judgment is known. The specific prerequisites are the same as AHP. By comparing the N objects, the decision maker can get a pairwise comparison judgment matrix A:

$$A=(a_{ij})=\begin{array}{c} \\ C_1 \\ C_2 \\ \cdots \\ C_i \\ \cdots \\ C_n \end{array}\begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & \cdots & C_j & \cdots & C_n \end{array} \\ \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\ \cdots & \cdots & & \cdots & & \\ a_{11} & a_{12} & \cdots & a_{ij} & \cdots & a_{in} \\ \cdots & \cdots & & \cdots & & \\ a_{n1} & a_{n2} & \cdots & a_{nj} & \cdots & a_{nn} \end{pmatrix} \end{array}$$

Among them, a_{ij} is the result of decision-maker to compare the object C_i to the object C_j, that decision maker made the relative importance measure of the object C_i to the object C_j. If we follow the basic axiom of Saaty on AHP, A is the reciprocal matrix, i.e. $a_{ij}=1/a_{ji}$, and $a_{ii}=1$, i , j=1,2,…,n. If we follow the basic principles of Saaty's analytic hierarchy process by using the 9-scale comparison method, a_{ij} can be taken 17 values: 1,2,3,4,5,6,7,8,9,1/2,1 /3,1/4,1/5,1/6,1/7,1/8,1/9. It can also take any other non-negative value, one specific value does not affect the discussion below, and A may not even be an offset matrix. Here we assume that the decision maker can determine the above pairwise comparison judgment matrix A based on one principle and method, and it's required that A is a positive matrix, i.e. each element of A is bigger than 0.

The CCM is mainly based on the consideration that: for n comparison objects, decision-maker can determine the priority of each object by providing a comparison chain about comparison objects and complete (n-1) times pairwise comparison or judgment. The comparative chain of comparison objects refers to a sort of n comparison objects, which are then compared to the pairwise important levels based on this sort, and the times of comparisons is (n-1). For example, suppose the decision-maker gives a comparison chain $C_nC_{n-1}\ldots C_1$ about n comparison objects $C_1,C_2,\ldots,C_n$, the decision-maker can determine the relative decision priority of the comparison object $C_1,C_2,\ldots,C_n$ as long as the relative importance of C_{i+1} and C_i are compared with i=1,2,…,n-1. In fact, if the importance of C_{i+1} relative to C_i is judged as d_i, the absolute importance is D_1 of any given object C_1, and the absolute importance of $C_1,C_2,\ldots,C_n$ can be determined once, in order: D_1, $D_2=D_1*d_1$, $D_3=D_2*d_2$, …, $D_n=D_{n-1}*d_{n-1}$. Then make it normalized, we can determine the relative importance priority vector $w=(w_1,w_2,\ldots,w_n)$ of $C_1,C_2,\ldots,C_n$, i.e. $w_i=D_i/(D_1+D_2+\ldots+D_n)$. The question here is how to determine the decision-maker's comparative chain about comparing objects. In fact, the pairwise judgment matrix gives the decision-maker more information than the comparison chain. With the help of the pairwise comparison matrix, we can consider the results of the various comparison chains of decision-maker, then obtain an average result by method of statistical averaging, and then measures the decision-maker's consistency problem about pairwise compared with standard deviation. In this way, a Comparison Chain Method based on pairwise comparison judgment matrix is established to determine the priority of decision.

The operation steps of the Comparison Chain Method are as follows:

1.1 The First Step: Decision-Maker Provides Pairwise Comparison Judgment Matrix A

Suppose the comparison object is N: $C_1,C_2,\ldots C_n$. The pairwise comparison judgment matrix given by decision-maker is $A=(a_{ij})$, in which a_{ij} is the important measure of C_i relative to C_j. The specific determination process is the same as general analytic hierarchy method. We skip it here.

1.2 The Second Step: Determining the Relative Priority Vector of the Comparison Chain Based on Information Extraction

If the comparison chain is $C_1C_2\ldots C_n$, a partial information about the comparison chain is extracted from the pairwise comparison judgment matrix A, that is, the importance of C_1 relative to C_2 measures a_{n-1n}, the importance of C_2 relative to C_3 measures a_{23},..., the importance of C_{n-1} relative to C_n measures a_{n-1n}. Then make the absolute importance of Cn is any value, respectively, determine the absolute importance of each comparison object, through normalization, determine the relative importance of each comparison object, that is, the decision priority of $C_1C_2\ldots C_n$ based on the comparative chain.

For different comparison chains, the decision priority based on the chain can be determined according to this method. If the comparison objects have n, then the different comparison chains have a total of n!, so we can get the n! decision priority groups. In general, the value of n is 3~5. Such as n=4, then, the different comparison chains altogether have n!=24.

The decision priority vectors obtained from different comparison chains can be recorded as $P^1,P^2,\ldots,P^{n!}$.

1.3 The Third Step: Statistical Average Processing

In the second step, a group of decision priority based on different chain are processed, and then the decision-maker's decision priority P about the comparison object can be obtained. The average can be either an arithmetic average or a geometric average, which can generally take an arithmetic average.

$$P_j = \frac{1}{n!}\sum_{i=1}^{n!} P_j^i, \ldots\ldots, j = 1,2,\ldots,n$$

1.4 The Fourth Step: Consistency Analysis

The validity and acceptability of the relative priority identified above can be analyzed by introducing evaluation indicators. This article uses the standard deviation of decision priority based on the comparison chain to define the inconsistency of relative priority. The indicator of inconsistency is I_c.

The standard deviation of the corresponding component can be obtained by calculating variance for each component:

$$\sigma_{P,j} = \sqrt{\frac{1}{n!-1}\sum_{i=1}^{n!}(P_j^{\,i} - P_j)^2}\ , j=1,2,\ldots n$$

Define an inconsistency indicator I_c as:

$$I_c = \frac{1}{n}\sum_{j=1}^{n}\sigma_{P,j}$$

Generally, if $I_c<0.10$, we can think that the relative priority is acceptable. The second to fourth step above may require some computational software to help, because when n is large, n! becomes like an astronomical number. However, in general, just like the AHP, the decision-maker can make pairwise comparisons of the object that n will not be made larger than 5. In this way, it is entirely feasible to implement by computer.

2. NUMERICAL EXAMPLES

Consider a decision problem with only four comparison objects: a, b, c, d. It is assumed that the decision-maker obtains the following pairwise comparison matrix A (reciprocal) through some kind of pairwise comparison:

$$A = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 3/2 & 1/3 & 1/4 \\ 2/3 & 1 & 1 & 3/5 \\ 3 & 1 & 1 & 3/4 \\ 4 & 5/3 & 4/3 & 1 \end{bmatrix} \end{array}$$

Consider the comparison chain abcd. a is 3/2 more important than B, b is 1 more important than c, c is 3/4 more important than d, then the absolute importance of the measure is d--1, c--3/4, b--3/4, a--9/8, and the decision priority of comparison chain abcd is: a--0.3103, b--0.2069, c--0.2069, d--0.2759. For different comparison chains can be handled similarly. Table 1 below gives the standardized decision priorities of the various comparison chains, as well as the final decision priority and consistency index.

Table 1
Statistical Preference Priority

Various chains	Standardized statistical preference priority			
	P_a	P_b	P_c	P_d
1. abcd	0.3103	0.2069	0.2069	0.2759
2. abdc	0.2769	0.1846	0.2308	0.3077
3. acbd	0.0833	0.2500	0.2500	0.4167
4. acdb	0.0962	0.2308	0.2884	0.3846
5. adbc	0.1021	0.2449	0.2449	0.4081
6. adcb	0.0909	0.2727	0.2727	0.3637
7. bacd	0.1154	0.0769	0.3462	0.4615
8. badc	0.1154	0.0769	0.3462	0.4615
9. bcad	0.0909	0.2727	0.2727	0.3637
10.bcda	0.0909	0.2727	0.2727	0.3637
11.bdca	0.0962	0.2308	0.2884	0.3846
12.bdac	0.0962	0.2308	0.2884	0.3846
13.cabd	0.1731	0.1154	0.5192	0.1923
14.cadb	0.0962	0.2308	0.2884	0.3846
15.cbad	0.1579	0.1052	0.1052	0.6317
16.cbda	0.1020	0.2449	0.2449	0.4082
17.cdab	0.1154	0.0770	0.3461	0.4615
18.cdba	0.2769	0.1846	0.2308	0.3077
19.dacb	0.0909	0.2727	0.2727	0.3637
20.dabc	0.1579	0.1052	0.1052	0.6317
21.dbac	0.1731	0.1154	0.5192	0.1923
22.dbca	0.833	0.2500	0.2500	0.4167
23.dcab	0.1154	0.0769	0.3462	0.4615
24.dcba	0.3103	0.2069	0.2069	0.2759
Mean Value	0.1424	0.1890	0.2810	0.3876
Standard Deviation	0.0744	0.0735	0.0963	0.1063

2.1 The Fifth Step: Inconsistency Analysis

The inconsistency index is I_c=0.0876<0.10, so we can think that the priority is valid, that is, the inconsistency of pairwise preference judgment of decision-maker is acceptable.

The above calculation results are derived from an Excel template designed by the author. For different judgment matrices, different templates can be designed conveniently so as to realize the computerization of the comparison chain method designed in this article. As long as the user input the corresponding judgment matrix into the corresponding template data lattice, the calculation result can be obtained immediately.

As the basic principle of AHP assumes, in this case we assume that the pairwise judgment matrix given by the decision-maker is reciprocal.

What is particularly noteworthy here is that if the matrix is not reciprocal, our approach is still valid. Interested readers may wish to try.

3. COMPARISON OF COMPARISON CHAIN METHOD AND OTHER METHODS

Compared with the other methods described in the introduction, the Chain Comparison Method given in the previous section is more computational than the others, especially in terms of ranks and methods. But from the result analysis, the Chain Comparison Method fully utilizes the decision-maker's various preference information, thus obtains the result to be more realistic. The following list shows the relative priority of the decision-maker using different methods (accurate to two decimal places) with the example of the third section.

Table 2
Comparative Results of Various Technologies

Specific method name	Relative priority vector P
(1) Comparison Chain Method	(0.14, 0.19, 0.28, 0.39)
(2) Constant Sum Method	(0.14, 0.20, 0.27, 0.39)
(3) Column-Row Sums Approach	(0.15, 0.19, 0.28, 0.38)
(4) Simple Row Average Method	(0.15, 0.16, 0.29, 0.40)
(5) Ordinary Least Squares Method	(0.14, 0.20, 0.27, 0.39)
(6) Right Eigenvalue Method	(0.14, 0.19, 0.28, 0.39)
(7) Mean Transformation Method	(0.14, 0.19, 0.27, 0.40)
(8) Row Geometric Mean Method	(0.14 0.18, 0.28, 0.40)
(9) Column Geometric Mean Method	(0.14, 0.18, 0.28, 0.40)
(10) Harmonic Mean Method	(0.14, 0.19, 0.28, 0.39)

The above comparisons are also done through an Excel module created by the author. As long as the corresponding pairwise comparison matrix is entered, the calculation result of the corresponding technique can be obtained. There are other techniques or methods to determine the priority of relative decisions, and no further analysis is made here. From the results, the relative priority determined by most methods is (0.14, 0.19, 0.28, 0.39), it's the same as that determined by the chain comparison method in this article, so we can say that the comparison chain method is a reasonable and feasible method. In other papers, the author will discuss the theoretical comparison between comparison chain method and various methods.

CONCLUSION

In this article, a new method of priority synthesis is proposed, which is called the Comparison Chain Method, based on the pairwise comparison judgment matrix given by decision-maker. The comparison chain method provides statistical treatment and analysis of the pairwise comparison judgment matrix, and then determines the relative priority evaluation and consistency evaluation of decision-maker. The comparison chain method can be easily implemented with extended table software, such as Microsoft Excel. Another advantage of the comparison chain method is that it can be used to deal with the problem of the priority synthesis of the pairwise comparison judgment matrices given by the decision-maker, which is not considered by the existing methods such as AHP.

As for the comparison chain method and related AHP, the following content is still worthy of further study:

(a) Comparative study of comparison chain method and other methods. Computer simulation technology can be used to compare and analyze various priorities synthesis methods, including synthetic effect, synthetic efficiency, consistency analysis and so on.

(b) In terms of AHP, there are many questions worthy studying, such as hierarchical modeling, incomplete comparison, consistency analysis, the relationship between AHP and utility theory, uncertainty in AHP, sensitivity analysis, comparison of various priorities determination methods, group judgment and consistency analysis, and so on.

(c) Multi-objective decision making problem. Although the AHP has considered the multi-objective problem, if in the pairwise comparison, the comparison standard can be more specific, decision-maker is easier to master the comparison of the scale, so as to make more accurate pairwise comparison judgment, and give a more reasonable pairwise comparison matrix. At this point, the decision-maker gives multiple pairwise comparison judgment matrices, and multiple targets still need to be integrated. We can use AHP to determine the priority of each comparison standard, and finally to integrate priorities.

(d) Group decision problem. The comparison chain method only considers the scenario of a decision-maker. If we consider the problem of group decision making, it is still worth studying how to use the comparison chain method to integrate the group priorities, which has multiple decision-makers and pairwise comparison judgments.

(e) In addition, if the decision-maker gives pairwise comparison judgment result is uncertainty situation, like fuzzy, probability, interval, gray, rough and so on, how to carry on the priorities synthesis analysis is also a very meaningful research topic.

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