

Cnoidal Water Wave Induced Pore Pressure Accumulation in the Seabed

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ABSTRACT

The pore water pressure accumulation in the seabed soil under cnoidal water waves is examined in detail. The cnoidal water wave theory is employed to calculating the water pressure on the double-layer seabed in shallow water region. The modified Terzaghi consolidation equation is the governing equation. A general analytical solution for the consolidation equation is given in an integral form. And a comparative study is carried out to verify the two previous pore pressure generation models: the linear relation and the hyperbola model. Accordingly, a joint model is presented to analysis the pore pressure accumulation in seabed and the soil liquefaction in a practical accident happened in China Shengli Oil Field.

Key words: Cnoidal water wave; Pore pressure accumulation; Hyperbola model; Modified Terzaghi consolidation equation

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INTRODUCTION

When the near-shore water waves travel over the ocean floor, they exert dynamic pressure on the porous seabed, and consequently result in pore pressure and effective stress in the seabed. Generally speaking, there are two mechanisms for the wave induced pore pressure which have been observed in field and laboratory tests. The first is the oscillatory pore pressure and the other is the residual

pore pressure, which is the accumulation of the pore pressure caused by the compression of the soil and pore water under the action of the cyclic loading^[2,12,15,17]. In the past a few decades, numerous researches had been focused on the linear water waves induced transient response in the poroelastic seabed^[6,11,20]. However, the buildup of the pore pressure is not particularly considered. In their studies of approximation for pore water accumulation due to the linear water waves, Cheng *et al.* (2001) applied a source term derived on the basis of the linear relationship between the pore pressure ratio and the cyclic ratio. Jeng *et al.* (2007) investigated in detail the source term and analytical as well as numerical techniques for the pore pressure accumulation within the marine sediment. In view of that the near-shore infrastructures locate mostly in the shallow water areas. And in shallow water, the cnoidal water wave theory is much available. This paper examines the seabed pore water pressure accumulation under the cnoidal water waves.

Mostly, the seabed is layered with loose and pervious surface sediment. Owing to the scour, the fine particles in the surface layer were taken away by water currents. And the permeability of residual is very high for the big void ratio. The wave induced pore water pressure decrease slightly for the geometrical effect in the surface layer. However, the stability of the underlying layer is improved owing to the compression of the surface layer. For simplicity, a double-layered structure for the seabed is assumed in this paper. The pore water pressure does not accumulate in the surface layer. The permeability of the underlying layer is significantly low which may cause the buildup of the pore water. The scheme of the problem stated here is shown in Figure 1.

The residual pore pressure in the seabed is often obtained by solving the modified Terzaghi consolidation equation as followed^[2,6,16].

$$\frac{\partial u(z,t)}{\partial t} = c_v \frac{\partial^2 u(z,t)}{\partial z^2} + f(z,t) \quad (1)$$

where $u(z,t)$ represents the pore water pressure, t is time, z is the vertical co-ordinate. $f(z,t)$ is a source term representing the mean accumulation pore pressure. c_v is the consolidation coefficient.

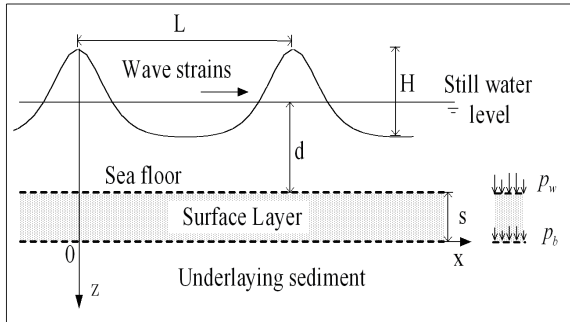


Figure 1
Problem Analyzed

The source term $f(z,t)$ is often come from laboratory test results. There have been many papers devoted to the empirical model about the residual pressure and the cycles of loading under undrained cyclic shear test^[12,15]. For simplicity, the linear expression drawn from Mcdougal *et al.* (1989) is widely used^[2,6]. However, Wu and Liu (2002) argued that hyperbola model is more accurate for the pore pressure ratio and the cyclic ratio. In this paper, a comparison between these two mechanics is carried out and a joint model is proposed to examine the residual pressure.

The paper is organized in a manner as followed. We first briefly introduce the cnoidal water wave theory and the ocean bottom water pressure formulation. The governing equations and the boundary conditions are considered subsequently. Then a general analytical solution for the consolidation equation is given. Consequently, a practical case happened in China Shengli Oil Field is given and a comparison study between two models for pore pressure build up is carried out. At last, the authors examined the pore pressure accumulation in seabed and the soil liquefaction with a new model for the pore pressure build up.

1. WATER PRESSURE ON THE SEABED INDUCED BY CNOIDAL WAVES

Korteweg and De Vries (1895) presented the theory of cnoidal wave, Keulegan and Patterson (1940), Laitone (1960), Chappellear (1962), Fenton (1979) gave solutions to the theory. Wiegel (1960) analyzed in detail the first-order cnoidal waves and gave graphs for practical use. Isobe (1985) gave expressions for the solution and integral properties of the first-order cnoidal wave theory. Let origin at the mean water level, the free water surface profile of first-order cnoidal wave can be written as^[13]

$$\eta(x,t) = H \left[\frac{1}{m^2} - \frac{E}{m^2 K} - 1 \right] + Hcn^2 \left[2K \left(\frac{x}{L} - \frac{t}{T} \right) \right] \quad (2)$$

in which H is the wave height, L the wave length, T the wave period, x the horizontal coordinate. m the modulus. K and E is the first and second kind of complete elliptic integral respectively and can be expressed as

$$E = \int_0^{\pi/2} \sqrt{1 - m^2 \sin^2 \varphi} d\varphi$$

$$K = \int_0^{\pi/2} \frac{1}{\sqrt{1 - m^2 \sin^2 \varphi}} d\varphi$$

The modulus m is easy to estimate with current computational techniques, and the wavelength L can be determined by

$$L = 4Kmd \sqrt{\frac{d}{3H}}$$

The approximate oscillatory water pressure on the surface of seabed^[18] can be written as

$$p_w = (x,t) = \gamma u(x,t) \quad (3)$$

where γ is the specific weight of the water. Equation (3) is derived with the assumption that the vertical inertia force is negligible as the ratio of water depth to the wave length is a small quantity.

2. GOVERNING EQUATIONS

The governing equation employed in this paper is the same as Equation (1), the consolidation coefficient c_v is given by

$$c_v = \frac{2kG(1-\nu)}{\gamma_w(1-2\nu)} \quad (4)$$

in which k is the hydraulic conductivity of the soil, G is the soil shear modulus, ν is the Poisson's ratio, γ_w is the specific weight of the pore water.

Let $u_g(z,t)$ denote the pore pressure generation due to the cyclic loading, the source term in equation (1) can be written as^[15]

$$f(z,t) = \frac{\partial u_g}{\partial t} \quad (5)$$

The hyperbola model for the pore pressure generation^[19] can be given as followed

$$\frac{u_g}{\partial_0} = \frac{x}{\alpha + \beta x} \quad (6)$$

where α and β are the curve-fitting coefficients. x is the ratio for N/N_L , in which N is real time number of cycles

and N_l is the number leading to liquefaction, σ_0 can be taken as

$$\sigma_0 = \gamma'_2 z \quad (7)$$

γ'_2 is the specific weight of the underlying sediment as shown in Figure 1. The number to liquefaction N_l should be determined by laboratory tests.

The linear model of pore pressure builds up was first proposed by Seed *et al.* (1975), and it is commonly used since then. This mechanism can be written as

$$\frac{u_g}{\sigma_0} = \frac{N}{N_l} \quad (8)$$

A simple formula for N_l is assumed as^[12]

$$N_l = \left(\frac{\tau_0}{\kappa \sigma_0} \right)^{1/\lambda}$$

Here τ_0 is the maximum cyclic shear stress in sediment, κ and λ are curve-fitting coefficients.

Using Equations (5), (6) and (8), the source terms corresponding to the linear and hyperbola pore pressure generating models may be expressed as

$$f_l(z, t) = \frac{\sigma_0}{T N_l} \quad (9)$$

$$f_h(z, t) = \frac{\alpha T \sigma_0 N_l}{(\alpha T N_l + \beta t)^2} \quad (10)$$

in which T is the wave period, f_l and f_h are the source terms corresponding to the linear and hyperbola models.

The initial and boundary conditions for equation (1) are required as follows

$$u(0, t) = 0 \quad (11a)$$

$$u(z, 0) = 0 \quad (11b)$$

$$\frac{\partial u}{\partial z}(\infty, t) = 0 \quad (11c)$$

Equation (11a) implies that there is no pore water pressure accumulation in the surface. Equation (11b) states that the initial excess pore water pressure everywhere is zero.

3. ANALYTICAL SOLUTION

It can be seen from Equation (9) that the cyclic stress ratio which is classically defined as the maximum within a period of wave of ratio of the shear stress to the initial normal effective stress is essential to the calculation of the number of liquefaction. In this paper, the analytical

solution given by Dormieux and Delage (1988) is employed. The maximum of the wave induced oscillatory shear stress can be written as

$$\bar{\tau}(x, z, t) = \frac{1}{\pi} \int_{-\infty}^{\infty} p_b(\zeta, t) \frac{z[(x - \zeta)^2 - z^2]}{[(x - \zeta)^2 + z^2]^2} d\zeta \quad (12)$$

in which p_b is the water pressure at the bottom of the superficial layer. It is very difficult to give a precise value of p_b in this case. In view of that the thickness of the surface layer is relatively thin, the seabed is assumed isotropic and an estimation based on the geometric damping of the oscillatory pore pressure is taken, then

$$p_b(x, t) = \frac{1}{\pi} \int_{-\infty}^{\infty} p_a(\zeta, t) \frac{s}{[(x - \zeta)^2 + s^2]^2} d\zeta \quad (13)$$

The analytical solution to equation (1) can be obtained by superimposing the fundamental solution for the diffusion equation and using the method of images to satisfy the boundary conditions. Thus,

$$u(z, t) = \int_0^t d\tau \int_0^{\infty} G(z, t; \xi, \tau) f(\xi, \tau) d\xi \quad (14)$$

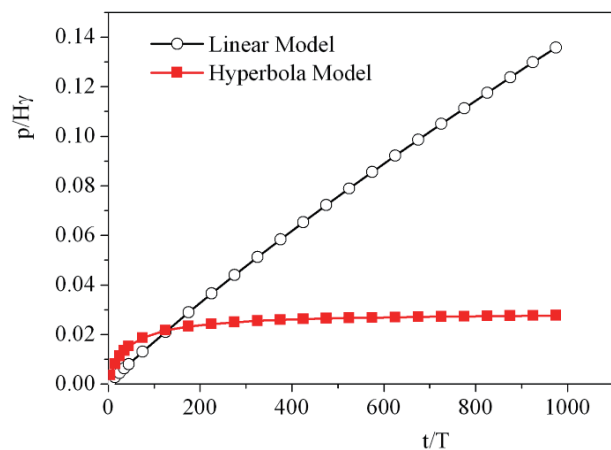
where $G(z, t; \xi, \tau) = \Gamma(z - \xi, t - \tau) - \Gamma(z + \xi, t - \tau)$, and

$$\Gamma(x, t) = \frac{1}{2\sqrt{c_v \pi t}} e^{-\frac{x^2}{4c_v t}} \quad (15)$$

Invoking the preceding derivation, it is in principle possible to solve the pore pressure accumulation in the seabed induced by cnoidal water waves. As a practical case in China Shengli Oil Field, the collapse of No. 3 platform on Sep. 2010 has attracted much attention domestic and abroad. In the following section, the authors examine the pore pressure accumulation in seabed and the soil liquefaction under the heavy waves induced by the typhoon "Malou" in the Bohai Sea.

4. COMPARISON ANALYSIS

(a) $z = 0.1$ d



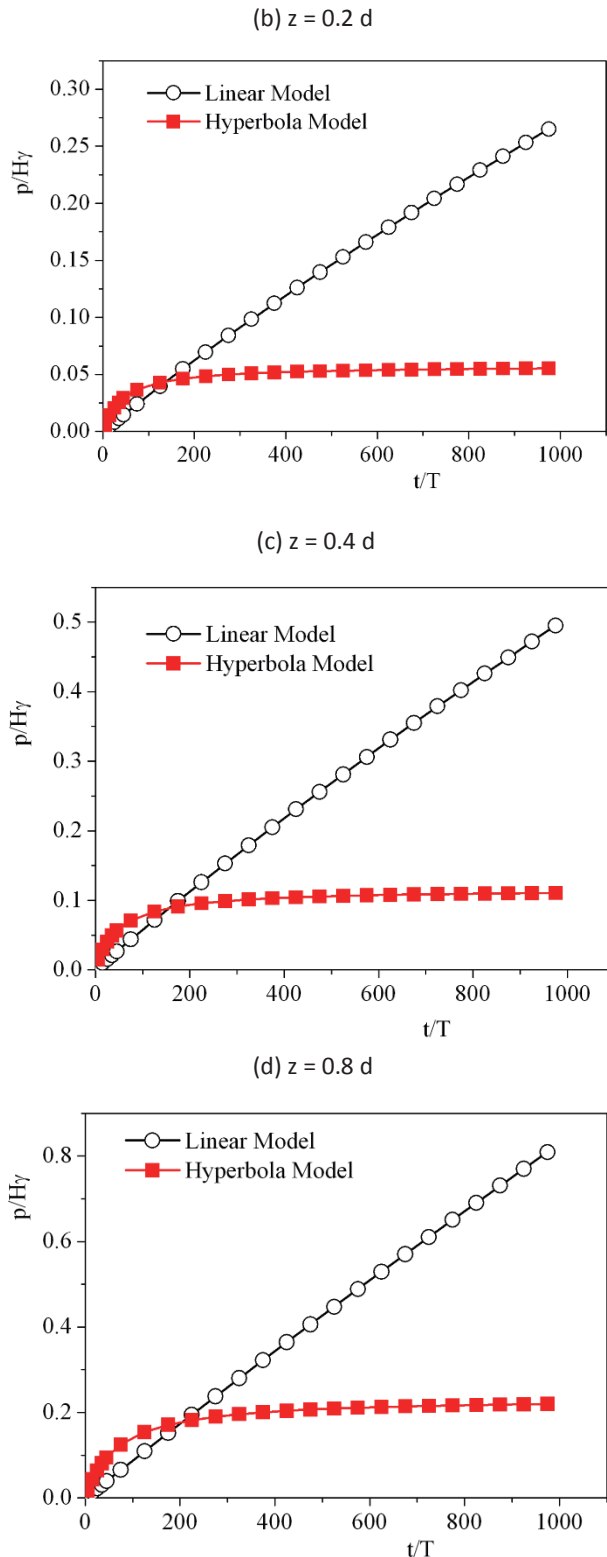


Figure 2
Variations of Pore Water Pressure Accumulation with Time in Different Depths

The Bohai Sea is an enclosed sea in the northeast of China. The Shengli offshore oil field locates at the submerged delta of the Yellow River, southwest of the

sea. The main formation in the seabed is silty soil with a coarse sand layer in surface. The mean water depth near the platform is 7.5 m and the typhoon induced wave period is 10.0 s, the average wave height is 3.0 m. Thus the associated profile of cnoidal wave can be determined.

Table 1
Input Parameters

Soil characteristics		
	The surface layer	The underlying ediment
Depth	1.5 m	-
Void ratio	2.0	0.8
Seepage coefficient	-	$2.0 \times 10^{-7} \text{ m/s}$
Inherent friction Angle	-	25°
Wave characteristics		
Modulus of the wave	0.98685	
The first complete elliptic integral (K)	3.223	
The second complete elliptic integral (E)	1.036	
The wave length (L)	87.1 m	
Parameters used in the calculation		
Hyperbola parameter (α)	0.252	
Hyperbola parameter (β)	0.98	
Liquefaction parameter (κ)	0.246	
Liquefaction parameter (λ)	-0.165	
Poisson's ratio (ν)	0.38	
Shear modulus (G)	$4.71 \times 10^5 \text{ N/m}^2$	
Consolidation coefficient (C_v)	9.73×10^{-6}	

For the vertical section as shown in Figure 1, the buildups of the pore water pressure in different depths are shown in Figure 2. In view of that the response of pore pressure is very small in the deep sediment. The authors only considered the pore pressure within the depth of d , namely the water depth. It can be seen that in the superficial zone, the accumulation of the pore water pressure is not notable owing to the short infiltration length. It is notable that the pore pressures build up with a higher rate in the hyperbola model compared to the linear model at the initial stage (Figure 2 a, b). However, when the depth of the observing point gets deeper, it will take longer time for the pore pressure accumulated in the linear model turns bigger than the hyperbola model (Figure 2 c, d).

Comparisons of the pore water pressures build up in the vertical section in the linear model and the hyperbola model are show in Figure 3. It is clear that differences between the two models are small in the beginning time (e.g. Figure 3 a, b), however, when the time is much long, pressures built up in the linear model get much bigger than the hyperbola model (e.g. Figure 3 c, d) in the shallow seabed, and in the deep soils, pore pressure generating in

the hyperbola model grows faster than that of the linear model in a long time period (Figure 3 d).

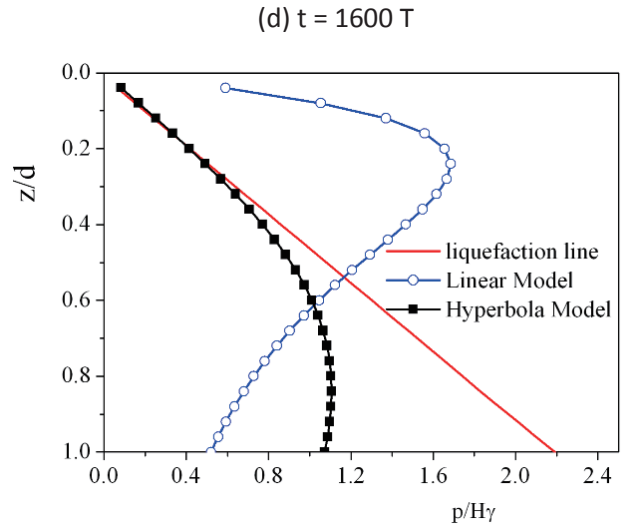
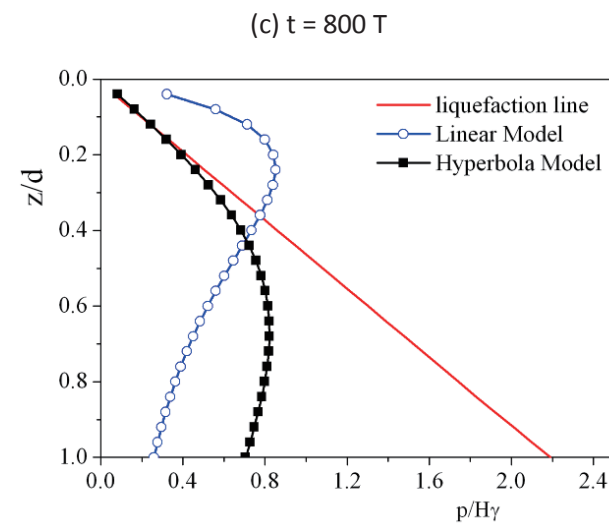
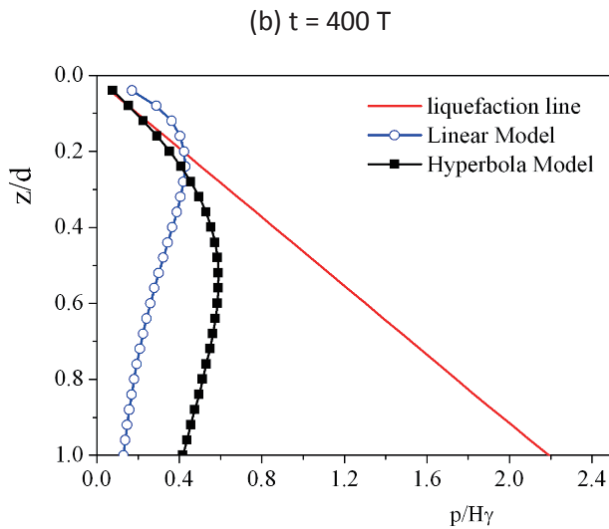
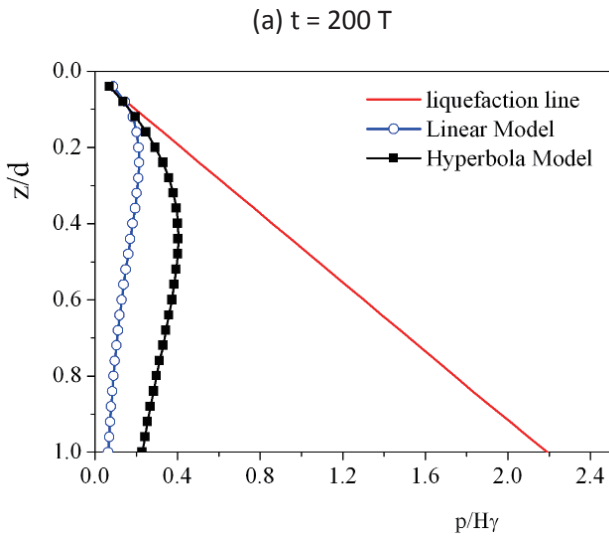


Figure 3
Variations of Pore Water Pressure Accumulation in the Vertical Section in Different Times

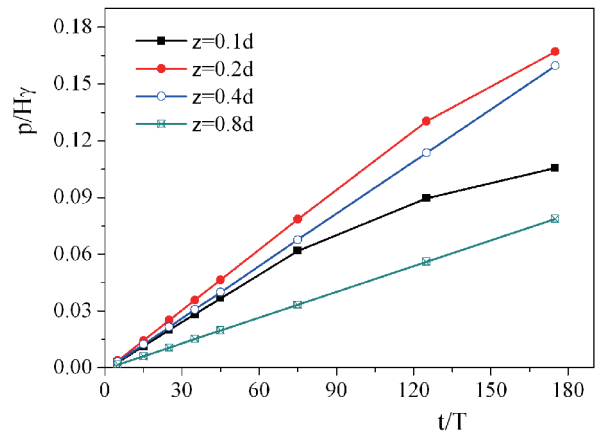
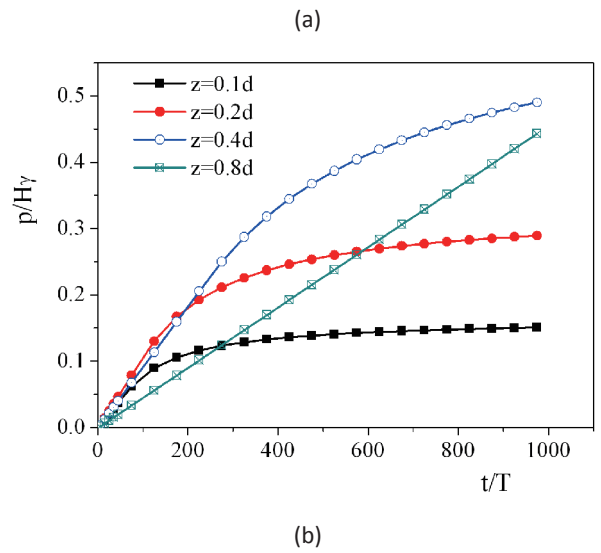


Figure 4
Pore Pressures Build up in Different Depths
Notations: Represents in the Depth of 0.1 d, 0.2 d, 0.4 d and 0.8 d Respectively

5. ANALYSIS WITH A JOINT MODEL

Note that in the linear model, the pore pressure increases proportional to the time (Figure 2), this is unreasonable when the time is very long. However, in the hyperbola model the pore pressure builds up at a relative high rate in the early time, it enables liquefaction occurs at the very beginning (Figure 3), which is unrealistic. Thus a joint model is presented by the authors

$$u_s = \begin{cases} u_{gl} & ; f_i(z,t) < f_h(z,t) \\ u_{gh} & ; f_i(z,t) > f_h(z,t) \end{cases} \quad (16)$$

in which u_{gl} represents the pore pressure generating in the linear model and u_{gh} , the hyperbola model.

This new model enables that the pore pressures build up in a linear model (Figure 4 b) at the primary stage and a hyperbola model (Figure 4 a) later.

Figure 4 shows that, when the depth of the soil approaches the depth of the still water depth, much more time is needed for the accumulation of the pore water, which will be verified impossible for the liquefaction of the superficial soil. It indicates that the seabed liquefaction induced by the waves occurs only in the superficial zone.

Figure 5 shows that there is an inflexion with respect to the depth about the residual pressure. This agrees well with the hypothesis of the paper.

6. DISCUSSIONS

When water waves propagate over the porous seabed, they exert a negative pressure under the troughs on the fluid-bed interface. Meanwhile, when the vertical net force on the granular particles of soil is zero, momentary liquefaction could occur^[14,21]. This implies that the criterion for the local soil fluidization can be written as

$$u - p_{wt} \geq \gamma_1' s + \gamma_2' z \quad (16)$$

in which p_{wt} is the bottom water pressure on the surface of the surface layer as shown in Figure 1 when the wave troughs pass over, and γ_1 the specific buoyant weight of the soil in the surface layer.

The relationship between the pore water pressure and the depth is plotted in Figure 5. Herein, we can see that when nearly 3600 periods passed, local soil liquefaction occurs, this agrees well with the practical case. Actually, the platform collapsed after it had been suffering the waves for about 10 hours.

The double-layered seabed model came from the engineering practice of the authors. In the near shore area of China Bohai Sea, the extensive scouring by the current and waves every year has made this seabed structure very common. Maybe it is not a general model. But it can represent some near-shore seabed, especially the seabed under scouring.

The joint model for the pore pressure generation in this paper is a simple combination of the previous models, and maybe some other non-linear model is more applicable. In fact, the authors are considering a full non-linear model for the pore pressure relation regarding the double-layered seabed structure. The authors take σ_0 in this paper as Equation (7) which is different from the original intention of Seed *et al.* (1975). If taken σ_0 as the gravitational load, in the hyperbola model, the calculated N_f is very big in the very shallow depth of the sediment owing to the non-linear effect. However, the linear model can be a good choice under this condition.

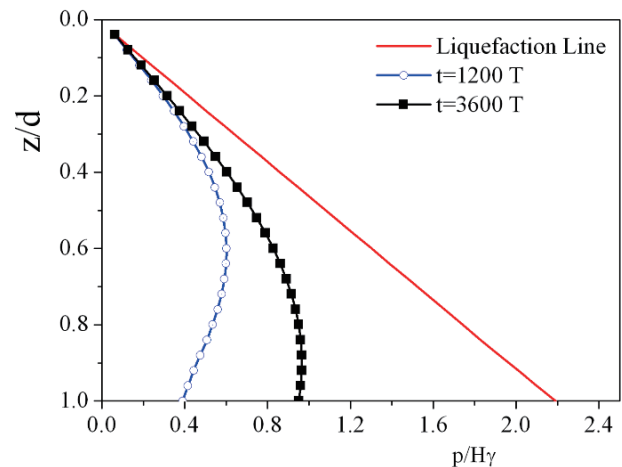


Figure 5 Pore Pressure Accumulation in the Vertical Section

CONCLUSIONS

In this paper, a two-layer seabed structure is introduced. It can represent some seabed structure especially the seabed under extensive scour.

A comparative study is carried out to verify the utility of the previous two pore pressure generation models, and a joint model is presented to determine the accumulation of the pore water pressure under the waves. Result from this study conforms well to the practical case.

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