CSCanada
Advances in Petroleum Exploration and Development
Vol. 10, No. 2, 2015, pp. 83-87
DOI:10.3968/7703

ISSN 1925-542X [Print]
ISSN 1925-5438 [Online]
www.cscanada.net
www.cscanada.org

Damage Constitutive Model of Mudstone Creep Based on the Theory of Fractional Calculus

ZENG Jia[a],*; ZHANG Jun[a]

[a] Northeast Petroleum University Petroleum Engineering Institute, Daqing, China.
*Corresponding author.

Received 18 October 2015; accepted 2 December 2015
Published online 31 December 2015

Abstract
When swelling mudstone, its various mechanical parameters will change significantly. Establish mudstone's fractional calculus creep damage model structure diagram, which describe the extent of damage in rock damage variable. Assumed that the stress level of rock exceeded its the long-term strength and under the condition of mudstone accelerated strain rate loading, nonlinear damage model based on fractional calculus theory was established. Therefore mudstone creep constitutive model considering the accelerated strain rate loading conditions was obtained. The results of constitutive model showed that there appears to be an exponential function between strain and time in the accelerated creep stage. The creep test data were used to verify the proposed model and the results suggested that the fractional order b was the intrinsic parameters of rock mass which could reflect its hardness. During the accelerated creep stage, fractional creep damage constitutive model could describe the stress-strain relationship well.

Key words: Fractional calculus; Stress-strain relationship; Damage mechanics; Mudstone accelerated creep

Zeng, J., & Zhang, J. (2015). Damage constitutive model of mudstone creep based on the theory of fractional calculus. *Advances in Petroleum Exploration and Development*, *10*(2), 83-87. Available from: URL: http://www.cscanada.net/index.php/aped/article/view/7703
DOI: http://dx.doi.org/10.3968/7703

INTRODUCTION

Rock creep is one of the important mechanical properties of rock and also is one of the important reasons for engineering wall rock deformation instability. Be established to fully reflect the accelerated creep characteristics of mudstone rock creep model is an important subject of current research of rock creep properties. Rock creep characteristics showed three distinct phases in the long term loads. Attenuation creep stage, the creep rate decreases, showing a significant non-linear phenomena; when entering the steady-state phase, creep rate remained at a constant value, showing a nearly linear features; when the creep into the acceleration phase, the deformation rate began to increase, showing a tendency to increase the nonlinear acceleration, showing a more significant nonlinear characteristics. In recent years, many scholars have done a lot of research on mudstone creep. But during the study, the main consideration is the first creep stage and second creep stage. In fact, in the process of rock creep, the third stage of creep properties has more significance[1]. Krang et al. (1977) through test results, the volume of rock was inelastic strain reaches a critical value, accelerating creep phase began and led to the final destruction of the specimen, failure time associated with stress. Genevois et al. (1979) through the experiment data of information, get a graph which is reflecting the relationship between stress ratio and accelerating creep time; Dragon et al. (1979) proposed a viscoplastic model of rock, and assumed crack density parameter reaches a critical value, creep rate increases rapidly lead to rock failure; Okubo et al. (1911) pointed out at accelerating creep stage, the relationship between strain rate and rock is inversely proportional to the increase[2]. Deng et al[3]. introduced a nonlinear viscous dampers, and used viscous damper to established comprehensive rheological mechanics model, this model can describe three kinds of creep deformation at the same time. Chen et al.[4] put forward two kinds of nonlinear element to set up composite rheological mechanics model, and this model can well describe the characteristics of soft rock accelerating creep stage. Cao et al.[5] changed viscous coefficient to nonlinear, in viscosity model, and improved

Nishihara Masao model. This model can reflect the non-attenuation creep properties of rocks. Wang et al.[6] based on the improved Nishihara model established parametric nonlinear creep model, which could reflect the rock specimens of the three stages of creep process, especially in nonlinear accelerating creep deformation stage. Yin et al.[7] used the theory of fractional order calculus, put forward a kind of software component, which is used to simulate the geotechnical materials between ideal solid and fluid. Although the combined model which is made up of software component and classical linear mechanical components, can describe the nonlinear behavior of the attenuation of the rock creep and steady creep of the rock creep, can't depict the acceleration of rock creep properties. Based on the above research, establish mudstone's fractional calculus creep damage model structure diagram, which describe the extent of damage in rock damage variable. Assumed that the stress level of rock exceeded its the long-term strength and under the condition of mudstone accelerated strain rate loading, nonlinear damage model based on fractional calculus theory was established. Therefore mudstone creep constitutive model considering the accelerated strain rate loading conditions was obtained. The results of constitutive model showed that there appears to be an exponential function between strain and time in the accelerated creep stage. The creep test data were used to verify the proposed model.

1. NONLINEAR MODELING

1.1 The Basic Components of Fractional Calculus

Fractional order calculus is expanded the order of calculus into areas of fraction or even negative number. The software components contains fractional order calculus is regarded as a component model. The model is between the ideals of solid and fluid and can well reflect the viscoelastic characteristics of geotechnical materials. This article uses the Riemann-Liouville fractional calculus operator theory, for β order integral of function $f(t)$, is defined as:

$$\frac{\mathrm{d}^{-\beta} f(t)}{\mathrm{d}t^{-\beta}} = {}_{t0}D_t^{\beta} f(t) = \frac{\mathrm{d}^n}{\mathrm{d}t^n}\left[{}_{t0}D_t^{-(\mathrm{n}-\beta)} f(t)\right]. \quad (1)$$

Fractional differential is defined as:

$$\frac{\mathrm{d}^{\beta} f(t)}{\mathrm{d}t^{\beta}} = {}_{t0}D_t^{\beta} f(t) = \frac{\mathrm{d}^n}{\mathrm{d}t^n}\left[{}_{t0}D_t^{-(\mathrm{n}-\beta)} f(t)\right]. \quad (2)$$

In Equations (1) & (2): $\beta > 0$, moreover $n - 1 < \beta \le n$ (n is a positive integer).

$\Gamma(\beta)$ is the Gamma function, is defined as：

$$\Gamma(\beta) = \int_0^{\infty} e^{-t} t^{\beta-1} \mathrm{d}t \left(\mathrm{Re}(\beta) > 0\right). \quad (3)$$

The Laplace transform formula of fractional order calculus is：

$$L\left[{}_0D_t^{-\beta} f(t), \mathrm{p}\right] = p^{-\beta}\bar{f}(p),$$
$$L\left[{}_0D_t^{\beta} f(t), \mathrm{p}\right] = p^{\beta}\bar{f}(p). \quad (4)$$

($f(t)$ in the vicinity of t=0 can be integral, $0 \le \beta \le 1$)

In Equation $\bar{f}(p)$ is a Laplace transform of $f(t)$.

According to the classical theory of solid mechanics and fluid mechanics, the constitutive relations of ideal solid should meet the Hooke's law $\sigma(t)$ - $\varepsilon(t)$, ideal fluid should satisfy Newton law of viscosity $\sigma(t)$ - $\mathrm{d}^1\varepsilon(t)/\mathrm{d}t^1$. Fractional order calculus is a mathematical problem, which is study any order differential, integral operator features and applications. If change the $\sigma(t)$ - $\varepsilon(\mathrm{t})$ to $\sigma(t)$ - $\mathrm{d}^0\varepsilon(\mathrm{t})/\mathrm{d}t^0$, then, the geotechnical materials which is between ideal solid and fluid, its fractional order differential form of stress-strain relation is

$$\sigma(t) = \xi \frac{\mathrm{d}^{\beta}\varepsilon(t)}{\mathrm{d}t^{\beta}}. \quad (5)$$

When $0 < \beta < 1$, Equation (5) describe the state of matter that is between ideal solid and fluid; when $\beta > 1$, Equation (5) describe the accelerating rheological state of matter, this paper mainly studies accelerate rheological material status. According to this narrative, we call it the software components when $\beta > 1$ (as shown in Figure 1), ξ is viscoelastic coefficient, similar to the elastic modulus of Hooke's law, the ξ dimension is [stress time β].

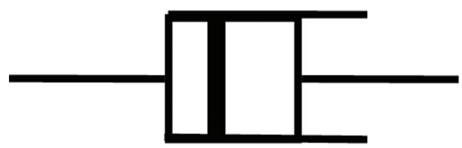

Figure 1
Fractional Order Calculus of the Viscoelastic Element

When $\sigma(t)$ = const, that is stress is a constant value, to Equation (5) on the basis of Riemann-Liouville fractional order operator theory to carry on the integral operation is

$$\varepsilon(t) = \frac{\sigma}{\xi}\frac{t^{\beta}}{\Gamma(1+\beta)}. \quad (6)$$

In the case of constant stress when β take different values, analysis of the relationship between strain and time and their change trend. Creep curve as shown when $\beta > 1$, based on the fractional order creep model, the curves of strain with time as shown in Figure 2.

Combined with the characteristics of typical rock creep deformation. According to derivate and change the fractional theory model, we could use to simulate creep curve. $\beta > 1$ (Software element) describe the viscoplastic deformation that is accelerating rheological material status.

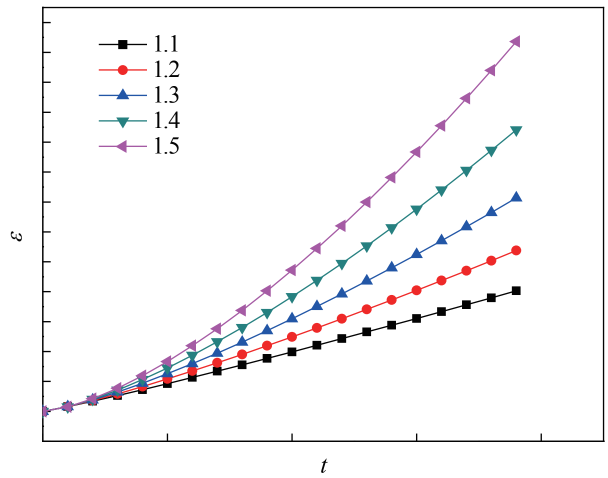

Figure 2
β > 1 Curve Diagram

When $\varepsilon(t)$ = const, through the fractional order calculus deduce the relaxation equation is

$$\sigma(t)=\xi\,\varepsilon_0\frac{t^{-\beta}}{\Gamma(1-\beta)}. \tag{7}$$

ε_0 is the initial stress of the initial strain. For different materials, adjust the parameters β and ξ of components to change the creep curve or relaxation curve, thus accurately fitting material test results.

1.2 In the Process of Rock Creep Damage

Rock creep damage stress threshold is the long-term strength of rock, when the load stress is greater than the long-term strength of rock mass itself, with the increase of time rock begins to creep damage. Based on damage mechanics theory, on account of the internal micro cracks of rock mass expanding, encryption, increase under the action of external loading, the ability to resist damage and distortion of rock mass decreases causing damage, and led to the decrease of the mechanical parameters of rock mass, so the rock damage degree related to the size of the load stress and time. According to the definition of damage variable method, this paper discussed damage variable of rock mass damage is defined by adopting the method of elastic modulus

$$D(\sigma,t)=1-\frac{E(\sigma,t)}{E_0}. \tag{8}$$

In Equation (8), E_0 is the initial elastic modulus of rock mass; $D(\sigma,t)$ is the damage variable of rock mass in t time; $E(\sigma,t)$ is the elastic modulus of the rock mass in t time, mainly related to Outside the load stress level at this moment. After damage due to creep, rock mass has not be capable of bearing, according to Liu Baoguo mudstone creep damage experiment research[8], definition of $E(\sigma,t)$ is define

$$E(\sigma,t) = E_0 \exp[-\langle\sigma - \sigma_\infty\rangle t/b]. \tag{9}$$

In Equation (9), σ_∞ is the long-term strength of rock material which can be determined by tests; b is rock material constant; $\langle\sigma - \sigma_\infty\rangle$ is step function, is defined as:

$$\langle\sigma-\sigma_\infty\rangle=\begin{cases}0 & (\sigma\le\sigma_\infty)\\ \sigma-\sigma_\infty & (\sigma>\sigma_\infty)\end{cases}. \tag{10}$$

Take the Equation (9) into the Equation (8)

$$D(\sigma,t) = 1 - \exp[-\langle\sigma - \sigma_\infty\rangle t/b]. \tag{11}$$

When the external load stress is greater than long-term strength of rock mass by (11), that is $\sigma > \sigma_\infty$, $t\rightarrow\infty$, $D = 1$, rock mass materials completely damage. With the increase of the time t and stress σ, material elastic modulus decay gradually, damage variable was gradually increased.

According to the Equation (11) damage variable D define effective stress $\mathring{\sigma}$ and nominal stress σmeet relations

$$\mathring{\sigma}=\frac{\sigma}{1-D(\sigma,t)}. \tag{12}$$

In Equation (12), $\mathring{\sigma}$ is effective stress, σ is nominal stress. Take the Equation (7) into the Equation (8)

$$\mathring{\sigma} = \sigma \exp[\langle\sigma - \sigma_\infty\rangle t/b]. \tag{13}$$

In Equation (13), σ_∞ is the long-term strength of rock material, could be determined by tests; b is rock material constant; $\langle\sigma - \sigma_\infty\rangle$ is step function.

1.3 Mudstone Fractional Order Damage Creep Model

The initial load, rock mass materials exists elastic deformation. Accelerated creep of rock mass characteristics is based on time and external load stress change gradually, there is damage threshold, so by reference fractional order damage components which could reflect the creep damage process to describe the viscoplastic accelerating deformation process of rock mass, model structure as shown in Figure 3.

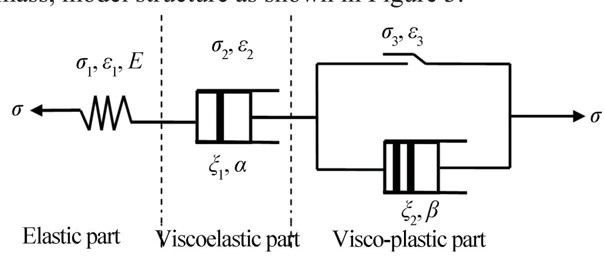

Figure 3
Fractional Order Damage Creep Model

When the external load stress is greater than long-term strength of rock mass, that is mudstone accelerated creep, according to the model diagram available

$$\sigma = \sigma_1 = \sigma_2 = \sigma_3, \quad \varepsilon = \varepsilon_1 + \varepsilon_2 + \varepsilon_3. \tag{14}$$

Among them

$$\begin{cases}\sigma_1=E\varepsilon_1\\ \sigma_2=\xi_1\dfrac{\mathrm{d}^\alpha\varepsilon_2}{\mathrm{d}t^\alpha}\\ \sigma_3=\sigma_\infty+\xi_2\dfrac{\mathrm{d}^\beta\varepsilon_3}{\mathrm{d}t^\beta}\end{cases}. \tag{15}$$

In Equation (15), when the load stress is less than long-term strength of rock mass conditions, that is Creep stage Ⅰ and Creep stage Ⅱ mudstone viscoelastic deformation, $0 < \alpha < 1$. Laplace transform and Laplace inverse transform of the Equation (15) and take into the Equation (14), for the resulting nonlinear creep constitutive equation when the load stress greater than long-term strength of rock mass is that:

$$\varepsilon = \frac{\sigma}{E_0} + {}_0D_t^{-\alpha}\left(\frac{\sigma}{\xi_1}\right) + {}_0D_t^{-\beta}\left(\frac{\sigma - \sigma_\infty}{\xi_2}\right). \quad (16)$$

Because of the load stress greater than long-term strength of rock mass, that is $\sigma \geq \sigma_\infty$, rock mass will enter the stage of creep damage. The Equation (16) can be rewritten as

$$\varepsilon = \frac{\sigma}{E_0} + {}_0D_t^{-\alpha}\left(\frac{\sigma}{\xi_1}\right) + {}_0D_t^{-\beta}\left(\frac{\sigma\%\text{–}\sigma_\infty}{\xi_2}\right). \quad (17)$$

Taking the initial conditions $t = 0$ and $\sigma = \sigma_0$ into Equation (17) and according to the Riemann-Liouville fractional order calculus, acquire the nonlinear creep constitutive equation

$$\varepsilon = \frac{\sigma_0}{E_0} + \frac{\sigma_0}{\xi_1}\frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{\sigma\%\text{–}\sigma_\infty}{\xi_2}\frac{t^\beta}{\Gamma(1+\beta)}. \quad (18)$$

Taking the Equation (13) into (18) acquire rock creep damage constitutive model which is based on the theory of fractional order calculus, that is

$$\varepsilon = \frac{\sigma_0}{E_0} + \frac{\sigma_0}{\xi_1}\frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{\sigma_0 \exp\left[\langle \sigma - \sigma_\infty \rangle t/b\right] - \sigma_\infty}{\xi_2}\frac{t^\beta}{\Gamma(1+\beta)}. \quad (19)$$

2. ACCELERATE THE STRAIN RATE LOAD THE MUDSTONE CONSTITUTIVE MODEL

Known to accelerate creep stage based on the theory of fractional calculus of Rock creep damage constitutive model is

$$\varepsilon = \frac{\sigma}{E_0} + \frac{\sigma}{\xi_1}\frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{\sigma \exp\left[\langle \sigma - \sigma_\infty \rangle t/b\right] - \sigma_\infty}{\xi_2}\frac{t^\beta}{\Gamma(1+\beta)}.$$

According to the actual situation, assuming that $\varepsilon = ae^t$ and taking into Equation (19), can be calculated under the condition of accelerate the strain rate load. The stress-strain relationship of mudstone creep is

$$\varepsilon = \frac{\sigma}{E_0} + \frac{\sigma}{\xi_1}\frac{(\ln\varepsilon - \ln a)^\alpha}{\Gamma(1+\alpha)} + \frac{\sigma \exp\left[\langle \sigma - \sigma_\infty \rangle (\ln\varepsilon - \ln a)/b\right] - \sigma_\infty}{\xi_2}\frac{(\ln\varepsilon - \ln a)^\beta}{\Gamma(1+\beta)}. \quad (20)$$

In Equation (20), a is constant.

By Equation (20), can be deduced accelerate the strain rate load fractional order the mudstone creep damage constitutive model is

$$\sigma = \frac{E_0\xi_1\Gamma(1+\alpha)\xi_2\Gamma(1+\beta)\varepsilon + E_0\xi_1\Gamma(1+\alpha)(\ln\varepsilon - \ln a)^\beta\sigma_\infty}{\xi_1\Gamma(1+\alpha)\xi_2\Gamma(1+\beta) + E_0\xi_2\Gamma(1+\beta)(\ln\varepsilon - \ln a)^\beta + E_0\xi_1\Gamma(1+\alpha)(\ln\varepsilon - \ln a)^\beta \exp\left[\langle \sigma - \sigma_\infty \rangle (\ln\varepsilon - \ln a)/b\right]}. \quad (21)$$

When the outer load stress is greater than long-term strength of rock mass, that is $\sigma \geq \sigma_\infty$, rock mass will enter the stage of creep damage, that is $\exp[-\langle \sigma - \sigma_\infty \rangle t/b]$, the Equation (21) could be written as

$$\sigma = \frac{E_0\xi_1\Gamma(1+\alpha)\xi_2\Gamma(1+\beta)\varepsilon + E_0\xi_1\Gamma(1+\alpha)(\ln\varepsilon - \ln a)^\beta\sigma_\infty}{\xi_1\Gamma(1+\alpha)\xi_2\Gamma(1+\beta) + E_0\xi_2\Gamma(1+\beta)(\ln\varepsilon - \ln a)^\beta + E_0\xi_1\Gamma(1+\alpha)(\ln\varepsilon - \ln a)^\beta}. \quad (22)$$

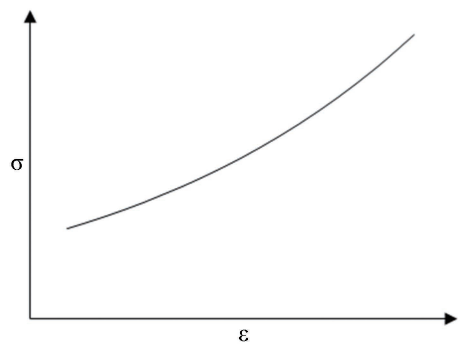

Figure 4
Rock Creep Stress-Strain Curve Diagram

According to the Equation (22), fitting the stress-strain curve diagram under the condition of rock creep, as shown in Figure 4. From the stress-strain curve diagram analysis with strain increase gradually, stress increases gradually, the strain rate also will increase. Namely that in the process of mudstone creep, the greater the strain, the greater the stress also.

When the β value is not at the same time, according to the Equation (22). Describe the stress-strain curve of the material as shown in Figure 5.

It can be seen from the Figure 5, with the increase of β value, stress increasing amplitude decreases. The smaller the value of β, the greater the dependent variable, the change of stress more obvious.

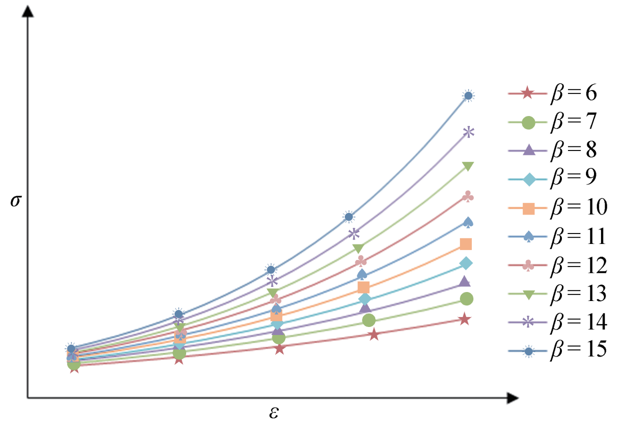

Figure 5
The Rock Creep Stress-Strain Curve Diagram Under Different β Values

3. THE FRACTIONAL ORDER MUDSTONE CREEP DAMAGE CONSTITUTIVE MODEL VALIDATION

Because the conventional triaxial test is strain rate loading test that Strain could be controlled, through the test verify the fractional order mudstone creep damage constitutive model. To verify the correctness and the rationality of the model in this paper, the author carries on the triaxial compression experiment, sample taken from the south area of Daqing oil field's oil and water wells casing damage zone. Through the experiment obtained rock mass mechanics parameters and stress-strain curve, compared to the experimental curves and fitting curve from Equation (18), as shown in Figure 6.

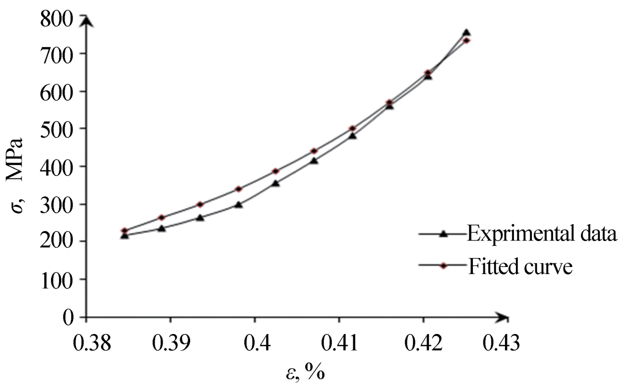

Figure 6
Stress-Strain Curves

According to the Figure 6, accelerate strain rate loading fractional order mudstone creep damage constitutive model's result agree well with experimental data fitting curve, results indicate that the correctness of the constitutive model is established in this paper. With the increase of strain rate, all the value of β is 10.13, so the fractional order β of the same kind of mudstone don't change with confining pressure, and be able to reflect the hardness of mudstone.

CONCLUSION

(a) According to classical theory of solid and fluid mechanics, based on the theory of fractional order calculus, introducing damage variable, establish accelerated strain rate loading fractional order mudstone creep damage constitutive model. The model can better reflect the mudstone of stress-strain relationship in the accelerated strain rate loading conditions.

(b) By accelerated strain rate loading fractional order mudstone creep damage constitutive model, the relationship between strain and time is exponential function in the accelerated creep stage, strain increases, the stress increases and the strain rate also increase. With the increase of β value, stress increasing amplitude decreases. The smaller the value of β, the greater the dependent variable, the change of stress more obvious.

(c) Conventional triaxial test of mudstone (accelerated strain rate loading) is able to verify the accelerate strain rate loading fractional order mudstone creep damage constitutive model. The model's result agrees well with experimental data fitting curve. The fractional orders β of the same kind of mudstone don't change with confining pressure, and be able to reflect the hardness of mudstone.

In this paper the innovation lies in the accelerated strain rate loading stage, the results of constitutive model showed that there appears to be an exponential function between strain and time. The mudstone creep constitutive model considering the accelerated strain rate loading conditions was established based on the theory of fractional order can describe the relation between stress and strain.

REFERENCES

[1] Song, Y. J., & Lei, S. Y. (2013). Mechanical model of rock nonlinear creep damage based on fractional calculus. *Chinese Journal of Underground Space and Engineering*, (9), 91-95.

[2] Zhang, Z. T., Wang, H., & Tao, Z. Y. (1996). The study of rock creep property. *Journal of Yangtze River Scientific Research Institute*, *S1*, 2-6.

[3] Deng, R. G., Zhou, D. P., & Zhang, Z. Y. (2001). A new rheological model for rocks. *Chinese Journal of Rock Mechanics and Engineering*, *20*(6), 780-784.

[4] Chen, Y. J., Pan, C. L., & Cao, P. (2003). A new mechanical model for soft rock rheology. *Rock and Soil Mechanics*, *24*(2), 209-214.

[5] Cao, S. G., Bian, J., & Li, P. (2002). Rheologic constitutive relationship of rocks and a modifical model. *Chinese Journal of Rock Mechanics and Engineering*, *21*(5), 632-634.

[6] Wang, L. G., He, F., & Liu, X. F. (2004). Non-linear creep model and stability analysis of rock. *Chinese Journal of Rock Mechanics and Engineering*, *23*(10), 1640-1642.

[7] Yin, D. S., Ren, J. J., & He, C. L. (2009). Stress-stain relation of soft soil based on fractional calculus operator's theory. *Chinese Journal of Rock Mechanics and Engineering*, *28*, 2973-2979.

[8] Liu, B. G., & Cui, S. D. (2010). Experimental study of creep damage of mudstone. *Chinese Journal of Rock Mechanics and Engineering*, *10*(29), 2127-2133.