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Research on a Model of Nonlinear Coupled-Vibration of Directional Well Drill String

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Abstract
Drill string vibrations are inevitable movement patterns during drilling process, the drill string vibrations form including longitudinal, lateral and torsional are recognized by most scholars. In the actual drilling process, the coupled vibration exhibits nonlinear features due to the impact of the eccentric rotation of the drill string and the influence of drilling fliud. Taking the whole drill string of directional well as the research object, a three-direction of longitudinal, transverse and torsional nonlinear coupled vibration model of drill string is established based on the nonlinear coupled deformations analysis in the longitudinal, transverse and torsional directions by using finite element analysis method. The nonlinear coupled vibration of drill string is studied by considering the effects of viscous forces of drilling fluid during the drilling process. The results show that the drill string resonance interval is relatively concentrated and the nonlinear coupled vibration frequency increase with the decrease of the length of the drill string.

Key words: Drill string; Nonlinear; Coupled vibration; Directional well; Resonance

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INTRODUCTION
Researching and analyzing the question of coupled nonlinear vibration of drill string are significant for drilling optimization design and reducing drilling costs. Because of the complex stress state of drill string in down hole, the drill string vibration is often manifested fully coupled vibration including longitudinal, lateral and torsional vibration. In the actual drilling, the coupled vibration exhibit nonlinear features. Therefore, it is necessary to establish a new model that closed to the actual field to study nonlinear coupled vibration of drill string in drilling process[1-3]. In this paper, a nonlinear coupled vibration model of drill string is established, which include three directions: Longitudinal vibration, transverse vibration and torsional vibration and study the effects of drill string eccentricity and drilling fluid viscous forces on the nonlinear coupled vibration of drill string during the drilling process.

1. NONLINEAR COUPLED VIBRATION MODEL OF ROTARY DRILL STRING

1.1 Basic Hypotheses of the Model
The drill string includes two parts: Drill pipe string and bottom hole assembly. The bottom hole assembly is mainly composed of drill collars, drill shock absorbers

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and other components. Abstracting and simplifying the drill string mechanics model by only considering simple system consisting of the drill pipe and drill collars, the model is shown in Figure 1, the 1, 2, 3, ..., n-1 segments are drill pipe segments based on the drill string form and n is the drill collar segment.

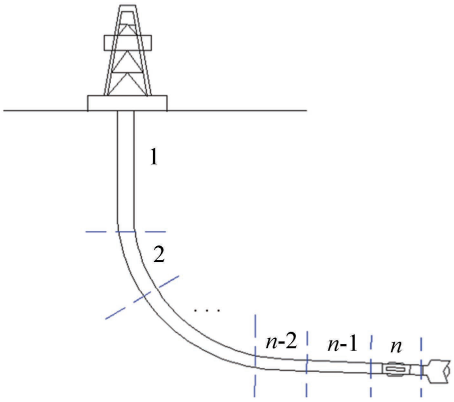

Figure1
Directional Well Drill String Model

According to the simplified model from Figure 1, the corresponding mathematical model is established, and the following hypothesis is made when establishing the dynamic model:

(a) The drill string and the borehole wall are rigid, and the cross section of the drill string is circular.

(b) The drill string is a elastomer with a tiny deformation, and the drill string axis is only slightly deviated from the borehole axis, while the friction and collision of the drill string joints between the borehole wall and the casing inner wall are neglected during the drilling process[4-7].

1.2 Establishing the Unit Stiffness Matrix and Mass Matrix

In order to establish the longitudinal, lateral and torsional coupled vibration equations of the drill string facilitativly, the global coordinate system $OXYZ$ is established as shown in Figure 2. The global coordinate system is the fixed Cartesian coordinate system and the wellhead coordinate is the coordinate origin O. Taking a space unit dx in the drill string model, at both ends of the unit the node numbers are i and j. The element coordinate system $ix'y'z'$ is established as shown in Figure 3. The positive direction of the drill string unit is from i to j, and the iy' axis, ix' axis and iz' axis constitute a right-handed orthogonal coordinate system. The origin of the coordinates, i and j, are the centroid of the drill string elements. Set point Cas the centroid of arbitrary drill string section, e as the eccentricity, ρ_0 (kg/m^3) as the drilling fluid density, d_i (m) as the inside diameter of the drill string, D_i (m) as the outer diameter, D_{0i}(m) as the wellbore diameter.

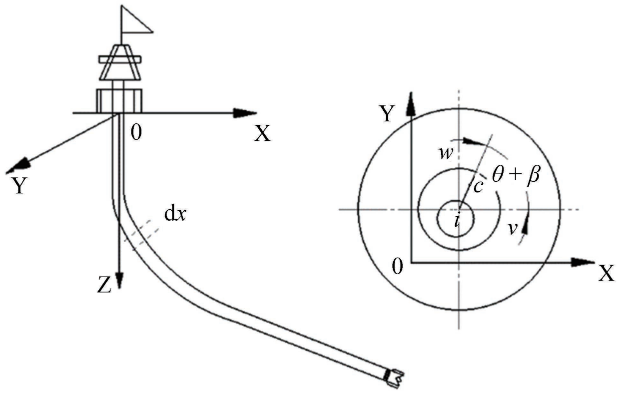

Figure 2
Nonlinear Coupled Vibration Model of Drill String in the Global Coordinate System

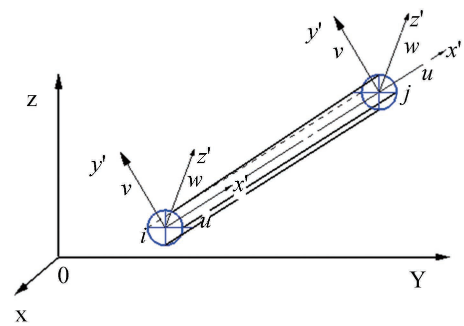

Figure 3
Nonlinear Coupled Vibration Model of Drilling String in the Local Coordinate System

The centroid velocity which the eccentricity of the drill string unit cross section is e is

$$V_c = \frac{\partial v_c}{\partial t} = \frac{\partial\left(v_c(x',t)\right)}{\partial t}\vec{i}\,' + \frac{\partial\left(w_c(x',t)\right)}{\partial t}\vec{j}\,' + \frac{\partial\left(u_c(x',t)\right)}{\partial t}\vec{k}\,'. \quad (1)$$

So the kinetic energy of the drill string unit can be expressed as

$$\begin{aligned} \mathrm{d}T &= \frac{1}{2}\int(\rho_i A V_c^{\,2})\mathrm{d}x' + \frac{1}{2}\int_0^L \rho_i \mathrm{I}\left(\frac{\partial \theta_x}{\partial t}\right)^2 \mathrm{d}x' \\ &= \frac{1}{2}\dot{\boldsymbol{\delta}}^T \mathbf{m}'_{ie}\dot{\boldsymbol{\delta}}. \end{aligned} \quad (2)$$

Where, ρ_i is the density of the drill string unit, kg/m^3; A is the cross section area of the drill string unit, m^2; I is the inertia moment of the cross section of the drill string unit, and $\mathbf{m}'_{ie}$ is the unit mass matrix of the drill string unit.

Where: $A = \pi\left(\frac{D_i^{\,2}}{4} - \frac{d_i^{\,2}}{4}\right)$, m^2.

The unit mass matrix of the nonlinear vibration of the drilling string is

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$$
\mathbf{m}'_{ie}=\begin{bmatrix}
\frac{\rho_1 AL}{3} & & & & & & \text{Symmetry} & & & & & \\
 & \frac{156\rho_1 AL}{420} & \frac{22\rho_1 AL^2}{420} & & & & & & & & & \\
 & \frac{22\rho_1 AL^2}{420} & \frac{4\rho_1 AL^3}{420} & & & & & & & & & \\
 & & & \frac{156\rho_1 AL}{420} & & & & & & & & \\
 & & & \frac{22\rho_1 AL^2}{420} & \frac{4\rho_1 AL^3}{420} & & & & & & & \\
 & & & & & \frac{JL+e^2\rho_1 AL}{3} & & & & & & \\
\frac{\rho_1 AL}{6} & & & & & & \frac{\rho_1 AL}{3} & & & & & \\
 & \frac{54\rho_1 AL}{420} & \frac{13\rho_1 AL^2}{420} & & & & & \frac{156\rho_1 AL}{420} & -\frac{22\rho_1 AL^2}{420} & & & \\
 & -\frac{13\rho_1 AL^2}{420} & -\frac{3\rho_1 AL^3}{420} & & & & & -\frac{22\rho_1 AL^2}{420} & \frac{4\rho_1 AL^3}{420} & & & \\
 & & & \frac{54\rho_1 AL}{420} & \frac{13\rho_1 AL^2}{420} & & & & & \frac{156\rho_1 AL}{420} & -\frac{22\rho_1 AL^2}{420} & \\
 & & & -\frac{13\rho_1 AL^2}{420} & -\frac{3\rho_1 AL^3}{420} & & & & & -\frac{22\rho_1 AL^2}{420} & \frac{4\rho_1 AL^3}{420} & \\
 & & & & & \frac{JL+e^2\rho_1 AL}{6} & & & & & & \frac{JL+e^2\rho_1 AL}{3}
\end{bmatrix} \qquad (3)
$$

In the formula: The moment of inertia $J = \rho LI$, sectional polar moment of inertia $I = \pi \frac{\left(D_i^4 - d_i^4\right)}{64}$.

The elastic potential energy of the coupled vibration of the drill string unit is

$$
\begin{aligned}
\mathrm{d}\Lambda &= \boldsymbol{\delta}_U^T \int_0^L \mathbf{N}'^{T}_U EA\mathbf{N}'_U \mathrm{d}x' \boldsymbol{\delta}_U + \frac{1}{2}\int_0^L \boldsymbol{\delta}_V^{\,T} \mathbf{N}''^{T}_V EA\mathbf{N}''_w \mathrm{d}x' \boldsymbol{\delta}_w \\
&+ \boldsymbol{\delta}_V^{\,T} \int_0^L \mathbf{N}''^{T}_V EI\mathbf{N}''_V \mathrm{d}x' \boldsymbol{\delta}_V + \boldsymbol{\delta}_w^{\,T} \int_0^L \mathbf{N}''^{T}_w EI\mathbf{N}''_w \mathrm{d}x' \boldsymbol{\delta}_w + \boldsymbol{\delta}_\theta^{\,T} \int_0^L \mathbf{N}'^{T}_\theta GJ\mathbf{N}'_\theta \mathrm{d}x' \boldsymbol{\delta}_\theta \\
&\cdot 2\sqrt{\rho_0 \mu V_0^{\,3} L}\left(1-\left(\frac{A_1}{A_2}\right)^{\frac{3}{2}}\right)\int_0^L \left(\boldsymbol{\delta}_w^{\,T} \mathbf{N}'^{T}_w \mathrm{d}x' \mathbf{N}'_w \boldsymbol{\delta}_w + \boldsymbol{\delta}_v^{\,T} \mathbf{N}'^{T}_v \mathrm{d}x' \mathbf{N}'_v \boldsymbol{\delta}_v + \boldsymbol{\delta}_u^{\,T} \mathbf{N}'^{T}_u \mathrm{d}x' \mathbf{N}'_u \boldsymbol{\delta}_u\right) \\
&= \boldsymbol{\delta}^{\mathrm{T}} \mathbf{k}'_{ie} \boldsymbol{\delta}.
\end{aligned} \qquad (4)
$$

The unit stiffness matrix of the coupled vibration of the drill string unit is

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$$
\mathbf{k}'_{ie}=\begin{bmatrix}
\frac{EA}{L} & & & & & & & \text{Symmetry} & & & & \\
 & \frac{12EI}{(1+\varphi)L^3} & \frac{6EI}{(1+\varphi)L^2} & & & & & & & & & \\
 & \frac{6EI}{(1+\varphi)L^2} & \frac{(4+\varphi)4EI}{(1+\varphi)L}+\frac{6EA}{L^3} & \frac{3EA}{L^2} & & & & & & & & \\
 & & \frac{3EA}{L^2} & \frac{12EI}{(1+\varphi)L^3}+\frac{2EA}{L} & \frac{6EI}{(1+\varphi)L^2} & & & & & & & \\
 & & & \frac{6EI}{(1+\varphi)L^2} & \frac{(4+\varphi)4EI}{(1+\varphi)L} & & & & & & & \\
 & & & & & \frac{GJ}{L} & & & & & & \\
-\frac{EA}{L} & & & & & & \frac{EA}{L} & & & & & \\
 & -\frac{12EI}{(1+\varphi)L^3} & -\frac{6EI}{(1+\varphi)L^2} & & & & & \frac{12EI}{(1+\varphi)L^3} & -\frac{6EI}{(1+\varphi)L^2} & & & \\
 & \frac{6EI}{(1+\varphi)L^2} & \frac{(2-\varphi)2EI}{(1+\varphi)L}-\frac{6EA}{L^3} & -\frac{3EA}{L^2} & & & & -\frac{6EI}{(1+\varphi)L^2} & \frac{(4+\varphi)4EI}{(1+\varphi)L}+\frac{6EA}{L^3} & -\frac{3EA}{L^2} & & \\
 & & \frac{3EA}{L^2} & \frac{EA}{L}-\frac{12EI}{(1+\varphi)L^3} & -\frac{6EI}{(1+\varphi)L^2} & & & & -\frac{3EA}{L^2} & \frac{2EA}{L}+\frac{12EI}{(1+\varphi)L^3} & -\frac{6EI}{(1+\varphi)L^2} & \\
 & & & \frac{6EI}{(1+\varphi)L^2} & \frac{(2-\varphi)2EI}{(1+\varphi)L} & & & & & -\frac{6EI}{(1+\varphi)L^2} & \frac{(4+\varphi)4EI}{(1+\varphi)L} & \\
 & & & & & -\frac{GJ}{L} & & & & & & \frac{GJ}{L}
\end{bmatrix}
$$

$$
+2\sqrt{\rho_0\mu V_0^3 L}\left(1-\left(\frac{A_1}{A_2}\right)^{\frac{3}{2}}\right)\begin{bmatrix}
\frac{1}{L} & & & & & & \frac{1}{L} & & & & & 0\\
 & \frac{6\cos^2(\beta+\theta)}{5L} & \frac{\cos^2(\beta+\theta)}{10} & & & & & \frac{6\cos^2(\beta+\theta)}{5L} & \frac{\cos^2(\beta+\theta)}{10} & & & \\
 & \frac{\cos^2(\beta+\theta)}{10} & \frac{2L\cos^2(\beta+\theta)}{15} & & & & & -\frac{\cos^2(\beta+\theta)}{10} & -\frac{L\cos^2(\beta+\theta)}{30} & & & \\
 & & & \frac{6\sin^2(\beta+\theta)}{5L} & \frac{\sin^2(\beta+\theta)}{10} & & & & & \frac{6\sin^2(\beta+\theta)}{5L} & \frac{\sin^2(\beta+\theta)}{10} & \\
 & & & \frac{\sin^2(\beta+\theta)}{10} & \frac{2L\sin^2(\beta+\theta)}{15} & & & & & -\frac{\sin^2(\beta+\theta)}{10} & -\frac{L\sin^2(\beta+\theta)}{30} & \\
 & & & & & 0 & & & & & & 0\\
-\frac{1}{L} & & & & & & \frac{1}{L} & & & & & \\
 & -\frac{6\cos^2(\beta+\theta)}{5L} & -\frac{\cos^2(\beta+\theta)}{10} & & & & & \frac{6\cos^2(\beta+\theta)}{5L} & -\frac{\cos^2(\beta+\theta)}{10} & & & \\
 & \frac{\cos^2(\beta+\theta)}{10} & -\frac{L\cos^2(\beta+\theta)}{30} & & & & & -\frac{\cos^2(\beta+\theta)}{10} & \frac{2L\cos^2(\beta+\theta)}{15} & & & \\
 & & & -\frac{6\sin^2(\beta+\theta)}{5L} & -\frac{\sin^2(\beta+\theta)}{10} & & & & & \frac{6\sin^2(\beta+\theta)}{5L} & -\frac{\sin^2(\beta+\theta)}{10} & \\
 & & & \frac{\sin^2(\beta+\theta)}{10} & -\frac{L\sin^2(\beta+\theta)}{30} & & & & & -\frac{\sin^2(\beta+\theta)}{10} & \frac{2L\sin^2(\beta+\theta)}{15} & \\
0 & & & & & 0 & & & & & & 0
\end{bmatrix} \tag{5}
$$

In the formula: $\phi=\frac{12kEI}{GAL^2}$, is the influence factor of the shear deformation on the deflection of the drill string, and k is the shape factor of the drill string section.

2. MATEIX SOLUTION

2.1 Coordinate Transformation

The coordinate transformation matrix is get from the node displacement vector is changed through the coordinate transformation.

$$
[T_1]=\begin{bmatrix}\cos\alpha\cos\varphi & \cos\alpha\sin\varphi & -\sin\alpha\\ -\sin\varphi & \cos\varphi & 0\\ \sin\alpha\cos\varphi & \sin\alpha\sin\varphi & \cos\alpha\end{bmatrix}. \tag{6}
$$

Where α is the angle of inclination of the drill string unit, °; φ is the azimuth angle of the drill string unit, °.

From the coordinate transformation matrix, it can be known that in the global coordinates, the discrete mass matrix and stiffness matrix of the drill string unit by using the finite element method are

$$[m]=[T]^{-1}[m_{ie}'][T], \tag{7}$$

$$[k]=[\mathrm{T}]^{-1}[k_{ie}'][T]i. \tag{8}$$

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2.2 The Superposition Solution of the Global Stiffness Matrix and Mass Matrix in Global Coordinate System

The unit matrix of the global coordinate system which is getting from the Formulas (7) and (8) is partitioned into blocks. Then the block matrix of the stiffness matrix and the mass matrix of the drill string unit *i* is obtained. According to the idea of finite element analysis, combining the unit block matrix and the total unit mass matrix and stiffness matrix of the global drill string analysis are obtained.

$$M=\begin{bmatrix} m_{11}^{1} & m_{12}^{1} & & & & \\ m_{21}^{1} & m_{22}^{1}+m_{11}^{2} & m_{12}^{2} & & & \\ & m_{12}^{2} & m_{22}^{2}+m_{11}^{3} & & & \\ & & & \ddots & & \\ & & & & m_{22}^{n-1}+m_{11}^{n} & m_{12}^{n} \\ & & & & m_{21}^{n} & m_{22}^{n} \end{bmatrix}. \quad (9)$$

$$K=\begin{bmatrix} k_{11}^{1} & k_{12}^{1} & & & & \\ k_{21}^{1} & k_{22}^{1}+k_{11}^{2} & k_{12}^{2} & & & \\ & k_{12}^{2} & k_{22}^{2}+k_{11}^{3} & & & \\ & & & \ddots & & \\ & & & & k_{22}^{n-1}+k_{11}^{n} & k_{12}^{n} \\ & & & & k_{21}^{n} & k_{22}^{n} \end{bmatrix}. \quad (10)$$

After obtaining the total mass matrix and stiffness matrix of the drill string, the coupled vibration equation of the drilling string can be established in the drilling process.

$$[M]\ddot{\boldsymbol{\delta}}+[C]\dot{\boldsymbol{\delta}}+[K]\boldsymbol{\delta}=[\vec{F}]. \quad (11)$$

Where: $[C]$ is the damping matrix that drill fluid acts on the drill string unit; $[\vec{F}]$ is the external load matrix[8].

In order to study the structural performance of the drilling string system, the free vibration equation of drill string is solved to get the following equation.

$$[M]\ddot{\boldsymbol{\delta}}+[K]\boldsymbol{\delta}=0. \quad (12)$$

The motion spectrum equation of the drill string system can be obtained as Formula (13) from the Formula (12).

$$[\omega^2] = [K][M]^{-1}. \quad (13)$$

The natural frequency matrix of the coupled vibration of the drill string without damping can be obtained by putting the matrix *M* and *K* into the Formula (13).

$$[\omega^2]=\begin{bmatrix} \omega_1^2 & & & & & \\ & \omega_2^2 & & & & \\ & & \omega_3^2 & & & \\ & & & \cdot & & \\ & & & & \omega_{n-1}^2 & \\ & & & & & \omega_n^2 \end{bmatrix}. \quad (14)$$

The data on the main diagonal is the square of the natural vibration frequency of the drill string corresponding to each order, and the natural frequency of each order vibration is get. In drilling progress, the vibration frequency caused by outside interference on the bit should avoid to close the natural frequency ω of the coupled vibration of the drill string system.

3. APPLICATION OF NONLINEAR COUPLED VIBRATION OF DRILLING STRING

Taking a directional well in Jilin oilfield as an example, the design datas of the well are:

(a) The Physical Parameters of the Drill String

Steel density 7,850 kg/m^3, elastic modulus 2.06 × 1,011 N/m^2, shear modulus 784.53 × 105 KPa, drilling fluid density 1,200 kg/m^3, drilling fluid viscosity 35 mPa·s, drilling pump displacement 75 L/s.

(b) The Parameters of Well Deviation

Table 1
Well Deviation Parameters of a Well in Jilin Oilfield

Well depth (m)	Deviation angle (°)	Azimuth angle (°)
0	0	0
50	0.48	169.22
100	0.73	114.86
200	1.4	166.92
…	…	…
1,567.09	84.57	202.37
1,701.8	90.62	196.56
1,740.62	84.95	196.43
…	…	…
2,293.95	87.35	196.48
2,303.92	87.01	197.2
2,310.81	86.59	197.63
2,330	86.59	197.63

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(c) The Design Parameters of Casing String and the Wellbore Structure

In first spudding, the size of the drill bit is 444.5 mm, the length of the hole section drilled by the fist spudding bit is 202 m, the size of casing is 339.7 mm and the length of the casing is 200 m. In second spudding, the size of the drill bit is 311.9 mm, the length of the hole section drilled by the fist spudding bit is 1,008 m, the size of casing is 244.5 mm and the length of the casing is 1,000 m. In third spudding, the size of the drill bit is 215.9 mm, the length of the hole section drilled by the fist spudding bit is 2,330 m, the size of casing is 139.7 mm and the length of the casing is 2,000 m.

Table 2
Drilling Tool Parameters of a Well in Jilin Oilfield

Name	Length (m)	Outer diameter (mm)	Inside diameter (mm)	Joint outer diameter (mm)	Joint inner diameter (mm)	roots
Drill bit	0.3	215.9	95	101.6	95	1
Stud	7.14	172	71.44	172	71.44	1
Non-magnetic Drill collar	7.14	165.1	71.44	165.1	71.44	1
Centralizer	1.5	205	71.44	205	71.44	1
HWDP	274	127	76.2	165.1	76.2	30
Drill stem	2,040	127	101.6	161.9	101.6	222

(d) Drilling Tool Parameters

In order to analyze the effect of nonlinear coupled on the vibration of drill string during the drilling progress, the linear coupled and nonlinear coupled vibration frequency of drill string can be calculated and compared by using the Delphi programming software, as shown in Figure 5.

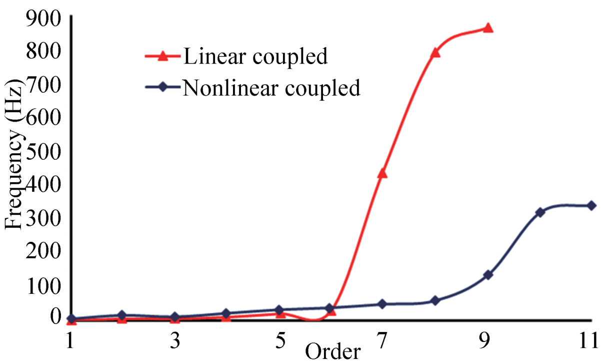

Figure 5
Comparison Between the Linear Coupled and Nonlinear Coupled Vibration of the Drill String of a Well in Jilin Oilfield

From Figure 5, it shown that the vibration frequency of the drill string is relatively concentrated after considering the coupled nonlinear vibration of drill string. The low order frequency is small and high frequency is big in linear coupled vibration of the drill string, which is enough to avoid resonance speed.

It can be analyzed that the vibration state of the monolithic drill string along with the well depth variation through the influence of the drill string length on the vibration frequency of the drill string. On this basis, the influence of the drill string length on the natural frequency of drill string is analyzed combining the above-mentioned drilling data of a well in Jilin Oilfield. The results are shown in Table 3.

Table 3
Vibration Frequency of Drill String With the Change of the Drill String Length of a Well in Jilin Oilfield

Drill pipe length (m)	1 order	2 order	3 order	4 order	5 order	6 order
1,130	4	5	5	8	35	436
800	4	5	7	11	41	512
400	4	4	5	22	43	705
200	4	4	5	32	44	828
100	4	5	5	33	89	867
50	4	5	7	34	178	877

From Table 3, it shows that with the increase of the drill string length, the each order natural vibration frequency of drill string is reduced, but the frequency of the low order vibration of drill string caused by the change of the drill string length is not obvious.

CONCLUSION

(a) In this paper, the nonlinear coupled vibration model of rotating drill string is derived by using the finite element method based on the longitudinal, lateral and torsional nonlinear deformation of drill string and considering the viscous force of the drill fluid by using the boundary layer theory and the rule of nonlinear coupled vibration of the rotating drill string is obtained. It can be gained that the vibration frequency of drill string is relatively concentrated after considering the nonlinear coupled vibration of the drill string.

(b) With the increase of the drill string length, the natural vibration frequency of drill string is reduced, but the

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frequency of the low order vibration of drill string caused by the change of the drill string length is not obvious.

(c) The coupled vibration frequency of drill string is related to the material performance of the drill string, and it can be avoided by optimizing the drilling string.

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