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Pin on Disc Wear Volume Prediction Based on Grey System Theory

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Abstract

This paper is a study of pin on disc wear volume, with the MMW-1A vertical friction and wear testing machine as the testing equipment, under different lubrication conditions. In this paper, the pin wear volume GM(1,1) prediction model is built based on the grey system theory, GM(1,1) the model consists of a single variable in the first-order differential equation. The pin wear volume measured compare with GM(1,1) predicted wear volume, The comparison results showed that, the predicted values by the GM(1,1) are very close to the experiment measured values, and the precision of predicted results is quite high.

Key words: Grey system theory; Pin on disc; Wear volume; Prediction

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INTRODUCTION

Pin wear speed directly affects the effect of test when the friction and wear experiment is carried out by using the MMW-1A vertical friction and wear testing machine. Pin on disc friction pair is sliding friction, and the friction mainly occurs on slightly convex body contact surface of the pin and disc. Friction and wear is not material intrinsic characteristics, but has a strong system characteristics (Xie, 2010). Friction losses play a significant role for the performance (Bortoleto, 2013), Tradition methods such as the linear regression model need large number of samples (Huang, 2011).

Friction coefficient and wear rate are the main indicators that reflecting material tribological characteristics. Tribology workers have been engaged in how to accurately predict materials wear volume. Because of its complex factors, which affect the wear, measurable date is less and the poor information. And the grey system theory study uncertain systems, the characteristic of uncertain system is small sample and poor information that the partial information known, the partial information unknown (Xiao & Mao, 2013). Grey system theory is a new method which study few data and poor information uncertain question, it was first introduced in 1982 by Deng Julong (Liu, Yang, & Wu, 2014). The essence of grey system theory is weaken the randomness of the original random sequence by using generate information processing method, so the original data sequence is transformed into modeling new sequence (Chen & Qin, 2007). The principle of new information priority of small is emphasized in the grey model with rolling mechanism (Kumar & Jain, 2010; Akay & Atak, 2007; Kayacan, Ulutas, & Kaynak, 2010). As an emerging multiple attribute decision-making tool which requires a limited knowledge and understanding of an unascertainable system to solve problems, make good estimations or predictions, this theory soon caught the attention of scholars and practitioners form various discipline roots and scientific fields in the Chinese research communities (Yin, 2013). Grey system theory has gradually formed a complete discipline system after 30 years rapid development. At present, the grey system theory has been applied in agriculture, meteorology, economics, engineering, military, medical, social, and many other areas, and obtained many important achievements (Liu, Xu, & He, 2003).

1. THE GREY PREDICTION MODEL

1.1 The Establishment of The Grey Prediction Model

Grey system predict is based on the GM model to make quantitative prediction, it mainly accurately describe the system behavior, evolution, and further realize the quantitative prediction for its future changes through extract valuable information base on some known information (Liu, Yang, & Wu, 2014). The core of grey system theory is the GM(1,1) model. GM(1,1) model is suitable for the observed data with exponential distributing (Xu, Tan, Tu, & Qi, 2011).

GM(1,1) model is generated through accumulate the original data sequence, and it has less sample, simple operation, etc.. The process is as follows:

Definition: The original non-negative sequence is:

$$X^{(0)} = (x^{(0)}(1), x^{(0)}(2),...,x^{(0)}(n)). \tag{1}$$

$X^{(0)}$ is order accumulated and generate the $X^{(1)}$:

$$X^{(1)} = (x^{(1)}(1), \quad x^{(1)}(2),...,x^{(1)}(n)), \tag{2}$$

where $x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i)$, $k = 1,2,...,n$.

Definition: $Z^{(1)}$ is $X^{(1)}$ mean sequence generated by consecutive neighbors:

$$Z^{(1)} = (z^{(1)}(1), z^{(1)}(2),..., z^{(1)}(n)), \tag{3}$$

where $z^{(1)}(k) = \frac{1}{2}(x^{(1)}(k) + x^{(1)}(k\text{-}1))$, $k = 2,3,...,n$.

The equation $x^{(0)}(k) + az^{(1)}(k) = \mathrm{b}$ (4) is called GM(1,1) model, and $\hat{\boldsymbol{a}} = [a,b]^T$ is parameters sequence.

The least square method estimates the parameters

$$\hat{\boldsymbol{a}} = (\mathbf{B}^{\mathrm{T}}\mathbf{B})^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{Y}, \tag{5}$$

where

$$\mathbf{Y} = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}. \tag{6}$$

The whitenization equation of GM(1,1) model is:

$$\frac{\mathrm{d}x^{(1)}}{\mathrm{d}t} + ax^{(1)} = b, \tag{7}$$

make $x^{(1)}(0) = x^{(0)}(1)$.

so time response sequence is:

$$\hat{x}^{(1)}(k+1) = (x^{(0)}(1) - \frac{b}{a})e^{-ak} + \frac{b}{a}, k = 1,2,3,\cdots,n. \tag{8}$$

The predicted values can be obtain as follow:

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k), k = 1,2,3,\cdots,n \tag{9}$$

1.2 Test of the GM(1,1) Model

For the establishment of the GM(1,1) model whether to predict accurately, generally need to test the model, such as the residual error test and degree of grey incidence test, etc.. The smaller residual error showed that the predicted value is closer to actual value. According to the definition of the degree of correlation, the greater degree of correlation, geometry of the forecast data and actual data are closer, so the prediction model is better. The posteriori variance test mainly verifies the validity of the model.

1.2.1 Residual Error Test

Residual error: $q(k) = x^{(0)}(k) - \hat{x}^{(0)}(k)$, $k = 1,2,\cdots,n$. (10)

Relative error:

$$\varepsilon(k) = \frac{q(k)}{x^{(0)}(k)} = \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)}, k = 1,2,\cdots,n. \tag{11}$$

Average relative error:

$$\varepsilon = \frac{1}{n-1}\sum_{k=2}^{n}\left|\varepsilon(k)\right|, k = 2,3,\cdots,n. \tag{12}$$

1.2.2 Degree of Grey Incidence Test

Degree of grey incidence is to describe the geometry proximity of the predicted values and actual values. Generally, the greater the correlation, the geometry curve is closer.

Degree of grey incidence: $O = \frac{1+|s|+|\hat{s}|}{1+|s|+|\hat{s}|+|s-\hat{s}|}$. (13)

where

$$s = \left|\sum_{k=2}^{n}[x^{(0)}(k) - x^{(0)}(1)] + \frac{1}{2}[x^{(0)}(5) - x^{(0)}(1)]\right|, k = 2,3,\cdots,n \tag{14}$$

$$\hat{s} = \left|\sum_{k=2}^{n}[\hat{x}^{(0)}(k) - \hat{x}^{(0)}(1)] + \frac{1}{2}[\hat{x}^{(0)}(5) - \hat{x}^{(0)}(1)]\right|, k = 2,3,\cdots,n \tag{15}$$

1.2.3 Posteriori Variance Test

$\overline{x}$ is average value of the original sequence

$$\overline{x} = \frac{1}{n}\sum_{k=1}^{n} x^{(0)}(k). \tag{16}$$

The mean square error of original sequence is

$$S_1 = \sqrt{\frac{1}{n}\sum_{k=1}^{n}(x^{(0)}(k) - \overline{x})^2}. \tag{17}$$

The average value of the residual error is

$$\overline{q} = \frac{1}{n}\sum_{k=1}^{n'} q(k), \quad n' < n. \tag{18}$$

The mean square error of the residual error is

$$S_2 = \sqrt{\frac{1}{n'}\sum_{k=1}^{n'}(q(k) - \overline{q})^2}, \quad n' < n. \tag{19}$$

According to the definition, the posteriori variance ratio C and the little error of frequency P are:

$$C=\frac{S_2}{S_1}, \tag{20}$$

$$P=P\{|q(k)-\bar{q}|<0.6745S_1\}. \tag{21}$$

The prediction model accuracy can be divided into four levels according to the size of the posteriori variance ratio and little error of frequency as shown in Table 1.

Table 1
Grades of Forecast Precision

	Posteriori variance ratio C	Little error of frequency P
Level 1	0.35	0.95
Level 2	0.50	0.80
Level 3	0.65	0.70
Level 4	0.80	0.60

2. PIN ON DISC WEAR VOLUME GREY PREDICTION MODEL

2.1 The Establishment of Pin on Disc Wear volume Grey Prediction Model

The experiment using MMW-1A vertical friction and wear testing machine, loading range is 2-1,000N, speed range is 1~2,000r/min, with continuous speed regulation system. The pin on disc is 45 steel, sample size is Φ 4.8×12.7mm, inner diameter Φ 16mm× outsider diameterΦ 32mm×10mm.

In the MMW-1A vertical friction and wear testing machine, using the pin on disc friction pair under base oil lubrication. The speed of the pin is set to 60r/min, loading is 80N, and every wear time is 5 min. Five groups experiment was carried out. The pin wear volume is measured in the Table 2.

Table 2
The Pin Wear Volume

Wear time (min)	5	10	15	20	25
Pin wear volume (mg)	1.23	3.41	4.89	7.34	9.29

The original sequence form Table 1 is

$X^{(0)}$ = (1.23, 3.41, 4.89, 7.34, 9.29).

The one-time AGO sequence according to Equation (2) is

$X^{(1)}$ = (1.23, 4.64, 9.53, 16.87, 26.16).

According to Equation (3) , $Z^{(1)}$is $Z^{(1)}(k)$ = (2.935, 7.085, 13.2, 21.515).

B and **Y** matrix sequence can be obtained follow Equation (6)

$$\mathbf{B}=\begin{bmatrix}-2.935 & 1\\ -7.085 & 1\\ -13.2 & 1\\ -21.515 & 1\end{bmatrix} \quad \mathbf{Y}=\begin{bmatrix}3.41\\ 4.89\\ 7.34\\ 9.29\end{bmatrix}.$$

According to Equation (5), $\hat{\boldsymbol{a}}=(\mathbf{B}^{T}\mathbf{B})^{-1}\mathbf{B}^{T}\mathbf{Y}$ = [-0.32, 2.6537]^{T}.

So a = -0.32, b = 2.6537 .

The whitenization equation of GM(1,1) model is $\frac{dx^{(1)}}{dt}-0.32x^{(1)}=2.6537$.

So time response sequence is:

$$\hat{x}^{(1)}(k)=(x^{(0)}(1)-\frac{b}{a})e^{-ak}+\frac{b}{a}=9.52e^{0.32k}-8.29.$$

The model $X^{(1)}$ can be obtained according to above Equation.

$\hat{X}^{(1)}$ = (1.23, 4.8213, 9.767, 16.5779, 25.9573).

Hence, According to Equation (9) the forecast value is $\hat{X}^{(0)}$ = (1.23, 3.5913, 4.9457, 6.8109, 9.3794).

The pin wear actual values and predicted values are shown in Table 3.

Table 3
The Pin Wear Actual Value and Predicted Value

Wear time (min)	Wear volume (mg)	Predicted value (mg)	Residual error ($q(k)$)	Relative error ($\varepsilon(k)$)
5	1.23	1.23	0	0%
10	3.41	3.5913	-0.1813	5.32%
15	4.89	4.9457	-0.0557	1.14%
20	7.34	6.8109	0.5291	7.21%
25	9.29	9.3794	-0.0894	0.96%

2.2 Test of Pin on Disc Wear Volume Grey Prediction Model

2.2.1 Residual Error Test of GM(1,1)

The residual error results are shown in Table 3.

$$\text{Average relative error: }\varepsilon=\frac{1}{5-1}\sum_{k=2}^{5}|\varepsilon(k)|=3.7\%.$$

Therefore, the accuracy of the GM(1,1) prediction model is 96.3%

2.2.2 Degree of Grey Incidence Test of GM(1,1)

$$s=\left|\sum_{k=2}^{4}[x^{(0)}(k)-x^{(0)}(1)]+\frac{1}{2}[x^{(0)}(5)-x^{(0)}(1)]\right|=15.98$$

$$\hat{s}=\left|\sum_{k=2}^{4}[\hat{x}^{(0)}(k)-\hat{x}^{(0)}(1)]+\frac{1}{2}[\hat{x}^{(0)}(5)-\hat{x}^{(0)}(1)]\right|=15.73$$

$$|s-\hat{s}|=|15.98-15.7326|=0.25.$$

So the degree of grey incidence is

$$O=\frac{1+|s|+|\hat{s}|}{1+|s|+|\hat{s}|+|s-\hat{s}|}=0.99.$$

According to above calculation, the degree of grey incidence of pin wear actual volume and predicted volume is 0.99. Therefore, the geometry curve of the pin actual wear volume is very close to the predicted volume, and the trends of the geometry curve are also very close. The geometry curve of the pin actual wear volume and forecast volume are shown in Figure 1.

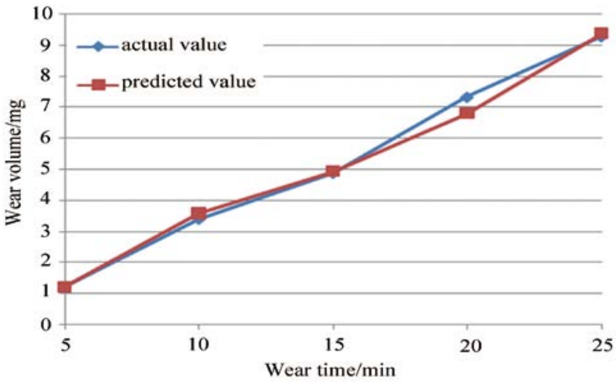

Figure 1
Curve of Pin on Disc Grey Prediction Model

2.2.3 Posteriori Variance Test of GM(1,1)

Using the posteriori variance test the GM(1,1) prediction results and evaluate the precision of the prediction model. According to the calculation, the posteriori variance ratio *C* is 0.0123 and the little error of frequency *P* is 1. So the posteriori variance ratio *C* is less than 0.35, and the little error of frequency *P* is greater than 0.95. Hence, the precision of the GM(1,1) prediction model is level 1.

CONCLUSION

Based on the grey system theory, this paper has established pin on disc wear volume GM(1,1) prediction model, and compared the actual measured values with predicted values. The study test the established GM(1,1) model by using residual error test, degree of grey incidence test and posteriori variance test. Test results shown that the accuracy of the prediction model is 96.3%, the degree of grey correlation is 0.99. The posteriori variance ratio *C* is 0.0123 less than 0.35, and the little error of frequency *P* is 1 greater than 0.95. The analysis results shown that pin actual measured wear volume and predicted volume is very consistent. Therefore, the GM(1,1) prediction model can truly reflect the pin on disc actual wear volume and accurately predict the pin on disc wear volume in the next time. So we can use the GM(1,1) prediction model to predict the pin on disc wear volume is 12.92 mg when the wear time is 30 min. Using the GM(1,1) prediction model to predict the pin on disc wear volume is very meaningful to our friction and wear experiment.

REFERENCES

Akay, D., & Atak, M. (2007). Grey prediction with rolling mechanism for electricity demand forecast of Turkey. *Energy, 32*(9), 1670-1675.

Bortoleto, E .M., Rovani, A. C., Seriacopi, V., Profito, F. J., Zachariadis, D. C., Machado, I. F., Sinatora, A., & Souza R. M. (2013). Experimental and numerical analysis of dry contact in the pin on disc test. *Wear, 301*(1-2), 19-26.

Chen, Q. B., & Qin, S. R. (2007). Application of prediction to machinery test and diagnosis. *China Measurement Technology, 33*(5), 10-13, 52.

Huang, J.-C. (2011). Application of grey system theory in telecare. *Computers in Biology and Medicine, 41*(5), 302-306.

Kayacan, E., Ulutas, B., & Kaynak, O. (2010). Grey system theory-based models in time series prediction. *Expert Systems With Applications, 37*(2), 1784-1789.

Kumar, U., & Jain, V. K. (2010). Time series models (Grey-Markov, Grey Model with rolling mechanism and singular spectrum analysis) to forecast energy consumption in India. *Energy, 35*(4),1709-1716.

Liu, Q. P., Xu, X. C., & He, Y. C. (2003). The grey forecast on the tribological behaviors of the film. *Lubrication Engineering, 40*(7), 53-55, 58.

Liu, S. F., Yang, Y. J., & Wu, L. F. (2014). *Grey system theory and applications* (7th ed.). Beijing, China: Science Press.

Xiao, X. P., & Mao, S. H. (2013). *Grey forecast and decision method.* Beijing, China: Science Press.

Xie, Y. B. (2010). Research and modeling theory of tribology system. *Journal of Tribology, 30*(1), 1-8.

Xu, J., Tan, T., Tu, M., & Qi, L. (2011). Improvement of grey models by least squares. *Expert Systems With Applications, 38*(11), 13961-13966.

Yin, M.-S. (2013). Fifteen years of grey system theory research: A historical review and bibliometric analysis. *Expert Systems With Applications, 40*(7), 2767-2775.