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The Reason of Photons Angular Distribution at Electron-Positron Annihilation in a Positron-Emission Tomograph

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Abstract
It is shown that photon angular distribution of electron-positron annihilation is consequence of Doppler's effect in the reference frame of the electron and positron mass center. In the reference frame bound with electron the photon angular distribution is absent. But it is replaced by the Doppler's shift of photon frequencies. The received results can be used in work of a positron-emission tomograph.

Key words: Annihilation; Electron; Positron; Photon; Doppler's effect; Angular photon distribution; Positron-Emission Tomography (PET)

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INTRODUCTION

The analysis of angular distribution of flying out photons by energy $h\omega$(h - Planck's constant, ω - photon frequency) at annihilation of a positron e^0_{+1} and electron e^0_{-1} has great importance for high-grade use of the positron-emission tomographs (PET). PET is the advanced diagnostic device used for search tumors at the earliest stages of their occurrence.

Unfortunately the mechanism of annihilative process $e^0_{-1}+e^0_{+1}=2h\omega$ of the electron and positron is unknown. P. Dirac has been offered model of this process.

According to Dirac (Dirac, 1978; Heitler, 1956) the annihilation it is possible to present as transformation of the electron from a state of positive energy to the state with negative energy. According to the Dirac's theory vacuum holes the positron it is the hole in the field of vacuum. Interaction of the electron and positron i.e. them annihilation is a filling vacuum hole by the electron. Thus energy as two quantums of electromagnetic radiation is allocated.

1. ANGULAR AND POWER DISTRIBUTIONS OF ANNIHILATIVE RADIATIONS

Quantum-electrodynamical calculations of the annihilative process have been carried out enough for a long time. They were repeatedly checked and rechecked, including authors of the article. As a result of these calculations two formulas for the differential effective section of electromagnetic radiation quantums scattering in a solid angle dΩ have been found.

The first formula on time has been found by Heitler (Heitler, 1956). This formula looks like:

$$d\sigma=\frac{e^4}{4(4\pi)^2k^0p}\left\{\frac{\left(k^0\right)^2+p^2+p^2\sin^2\theta}{\left(k^0\right)^2-p^2\cos^2\theta}-\frac{2p^4\sin^4\theta}{\left(\left(k^0\right)^2-p^2\cos^2\theta\right)^2}\right\}d\Omega. \quad (1)$$

The formula is written in designations (Bogolubov, & Shirkov, 1976) where there is its detailed deduction. The so-called rational system of units which speed of light and Planck's constant are equal to unit c=h=1 is used. In this system of the units the energy, impulse and mass have the identical dimension.

In the Formula (1) e there is electron charge (or positron with an opposite sign), k^0 - the photon energy, p - the electron impulse, θ - the angle between impulses of the electron and one of the radiated photons. The formula (1) is found under condition of summation on all directions of photons polarization.

At the deduction (1) the reference frame connected to the center of mass interacting the electron and positron is used which the impulses of electron and positron are equal on the module and are opposite on the direction $\boldsymbol{p}_1=-\boldsymbol{p}_2=\boldsymbol{p}$. Impulses of photons also are equal on the module and opposite on the direction $\boldsymbol{k}_1=-\boldsymbol{k}_2$ (Heitler, 1956; Bogolubov, & Shirkov, 1976). We shall note that in this reference frame the condition of both photons supervision are identical.

The second formula which represents frequency or power distribution of the radiated quantums has been offered a little later by Feynman (2009):

$$\mathrm{d}\sigma = \frac{e^4 \omega_1^2}{64\pi^2 4m^2 |\boldsymbol{p}_+| (E_+ + m)} \left[\frac{\omega_1}{\omega_2} + \frac{\omega_2}{\omega_1} + 2 - 4(\boldsymbol{e}_1 \boldsymbol{e}_2)^2 \right] \mathrm{d}\Omega . \tag{2}$$

The Formula (2) is written down in designations (Feynman, 2009). As well as in the previous variant (1) the rational system of units is used.

In the Formula (2) $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ there are unit vectors of photons polarization radiated at annihilation, ω_1 and ω_2- the frequencies of the radiated photons, m - the mass of electron (or positron), $|\boldsymbol{p}_+|$ - the module of the positron impulse, E_+ - its energy.

The formula (2) is similar to Klein - Nishina formula for Compton effect (Feynman, 2009; Volobuev, & Tolstonogov, 2013). The main difference there are before the third and fourth addends in the square brackets the signs are changed on opposite.

The major distinctive condition of the Formula (2) deduction is use of other reference frame in comparison with the Formula (1) deduction. The Formula (2) was deduced in the reference frame in which electron is at rest, and positron moves.

This reference frame as a whole is equivalent to the reference frame coupled with PET. Therefore we shall name this reference frame as laboratory. The electrons in the substance researched in PET basically are in the bound state. Positrons are result a β - positron radioactive decay of the shortly-lived radiopharmaceutical isotopes, for example C^{11}_{6}, O^{15}_{8}, N^{13}_{7}, F^{18}_{9}, P^{30}_{15}. Therefore electrons in laboratory reference frame it is possible to assume motionless (if to exclude chaotic thermal movement of molecules).

Both Formula (1) and Formula(2) were deduced with the help of Feynman standard diagram technique and diagrams of the second order of the perturbations theory. However results of the deductions essentially differ.

First, the Formula (1) assumes rather complex angular distribution of intensity I of annihilative radiation, since $\mathrm{d}\sigma \sim \mathrm{d}P \sim I\mathrm{d}\Omega$ where $\mathrm{d}P$ there is energy flux of radiation through the area $\mathrm{d}S$, intensity $I = \frac{\mathrm{d}P}{\mathrm{d}S}$. And this distribution is connected only to the electron impulse. The angle θ is present only at the complex with impulse p. In the Formula (2) the distinct form photons angular distribution in the obvious kind is absent.

Second, the Formula (2) assumes the opportunity of photons various energy from annihilation that is forbidden by the Formula (1) deduction owing to $\boldsymbol{k}_1=-\boldsymbol{k}_2$.

Therefore, first of all, there is a question what nature of angular distribution of the annihilative radiation intensity in (1)? Whether this distribution with annihilative process i.e. transformation "substance – energy" is connected or that is defined by other effects? Whether the given angular distribution of photons will be kept at transition to other reference frame connected, for example, to the PET?

2. THE REASONS OF ANGULAR AND POWER DISTRIBUTION OF THE ANNIHILATIVE RADIATIONS

For research of the angular dependence reason of differential effective section (1) we shall consider intermediate expression of the deduction which is not summarized yet on directions of the photons polarization (Bogolubov & Shirkov, 1976):

$$\mathrm{d}\sigma = \frac{1}{128\pi^2} \frac{e^4}{pk^0} \left\{ \begin{array}{l} \dfrac{(k^0)^4}{(pk_1)(pk_2)} - \dfrac{4(k^0)^4 (pe_1)^2 (pe_2)^2}{(pk_1)^2 (pk_2)^2} - \\ - \dfrac{4(k^0)^2 (e_1e_2)(pe_1)(pe_2)}{(pk_1)(pk_2)} - (e_1e_2)^2 \end{array} \right\} \mathrm{d}\Omega , \tag{3}$$

where k_1 and k_2 there are impulses of photons. Variables in square brackets: an impulse of electron, impulses of photons, unit vectors of photons polarization are written down as 4-vectors.

The formula (3) is simple for transforming to the kind:

$$\mathrm{d}\sigma = \frac{1}{128\pi^2} \frac{e^4}{pk^0} \left\{ \frac{(k^0)^4}{(pk_1)(pk_2)} - \left[2(k^0)^2 \frac{(pe_1)(pe_2)}{(pk_1)(pk_2)} + (e_1e_2) \right]^2 \right\} \mathrm{d}\Omega . \tag{4}$$

Let's transit in (4) to spatial vectors using a rule $(ab)=a^0b^0-\boldsymbol{ab}$ where $\boldsymbol{a}$ and $\boldsymbol{b}$ there are three-dimensional vectors which components change covariance, a^0 and b^0 - contravariance changing components of 4-vectors, in our case power components.

Transiting to three-dimensional vectors, and also taking into account absence contravariance components at polarizing 4-vectors $e^0=0$ the expression (4) it is possible to present as:

$$d\sigma = \frac{1}{128\pi^2}\frac{e^4}{pk^0}\left\{\frac{\left(k^0\right)^4}{\left(k^0\right)^4-\left(pk_1\right)^2}-\left[2\left(k^0\right)^2\frac{\left(pe_1\right)\left(pe_2\right)}{\left(k^0\right)^4-\left(pk_1\right)^2}+\left(e_1e_2\right)\right]^2\right\}d\Omega$$

$$=\frac{1}{128\pi^2}\frac{e^4}{pk^0}\left\{\frac{1}{1-\left(\dfrac{pk_1}{\left(k^0\right)^2}\right)^2}-\left[\frac{2}{\left(k^0\right)^2}\frac{\left(pe_1\right)\left(pe_2\right)}{1-\left(\dfrac{pk_1}{\left(k^0\right)^2}\right)^2}+\left(e_1e_2\right)\right]^2\right\}d\Omega. \tag{5}$$

At the deduction (5) the condition of photons flying in strict opposite directions $\boldsymbol{k}_2=-\boldsymbol{k}_1$ also is used.

Taking into account $|\boldsymbol{k}_1|=k^0$, and also according to the energy conservation law $ck^0=mc^2$ (for clearly evident it is entered inside brackets the speed of light c=1) in the Formula (5) we shall replace $\dfrac{\boldsymbol{pk}_1}{\left(k^0\right)^2}=\dfrac{V}{c}\cos\theta$, where V - speed of electron. In result we shall receive:

$$d\sigma = \frac{1}{128\pi^2}\frac{e^4}{pk^0}\left\{\frac{1}{1-\left(\dfrac{V}{c}\cos\theta\right)^2}-\left[\frac{2}{1-\left(\dfrac{V}{c}\cos\theta\right)^2}\frac{\left(pe_1\right)\left(pe_2\right)}{\left(k^0\right)^2}+\left(e_1e_2\right)\right]^2\right\}d\Omega. \tag{6}$$

Let's transit in Formula (6) to the laboratory reference frame bound with electron. In this case $\boldsymbol{p}$=0, and V it is possible to examine as speed of a positron movement. The same there concerns and to value p in factor before brackets (p – positron impulse). In the given reference frame the Formula (6) becomes simpler:

$$d\sigma = \frac{1}{128\pi^2}\frac{e^4}{pk^0}\left\{\frac{1}{1-\left(\dfrac{V}{c}\cos\theta\right)^2}-\left(e_1e_2\right)^2\right\}d\Omega. \tag{7}$$

Our research is an auxiliary task.

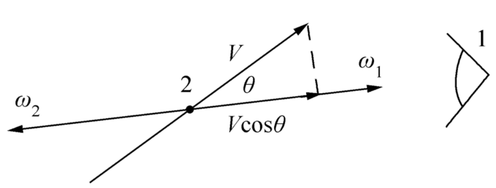

Figure 1
Supervision of the Photons Which Was Emitted by the Moving Particle

The observer 1 who is taking place in "motionless" (connected with the Earth) reference frame, Figure 1, examines some particle 2 moving with a speed V which in certain moment of time radiated two quantums opposite directed. At V=0 the quantum frequency is ω_0. The angle between speed of the particle and direction of one quantum propagation is equal θ. In the observer direction the particle has a component of speed $V\cos\theta$.

Due to Doppler's effect the quantum moving in the observer direction will have the increased frequency (Landay & Lifshits, 1967):

$$\omega_1=\omega_0\frac{\sqrt{1-\dfrac{V^2}{c^2}}}{1-\dfrac{V}{c}\cos\theta}. \tag{8}$$

For the quantum moving in an opposite direction so-called the "red displacement" of frequency will be observed:

$$\omega_2=\omega_0\frac{\sqrt{1-\dfrac{V^2}{c^2}}}{1+\dfrac{V}{c}\cos\theta}. \tag{9}$$

Using (8) and (9) we shall find size of the complex $\dfrac{\omega_1}{\omega_2}+\dfrac{\omega_2}{\omega_1}+2$ which is included into the Formula (2):

$$\frac{\omega_1}{\omega_2}+\frac{\omega_2}{\omega_1}+2=\frac{\left(\omega_1+\omega_2\right)^2}{\omega_1\omega_2}=\frac{4}{1-\left(\dfrac{V}{c}\cos\theta\right)^2}. \tag{10}$$

Let's note that distinction of frequencies of quantums in the examined task is determined by distinction in conditions of these quantums supervision: one quantum moves to the observer another leaves with him.

In the formula (7) the considered auxiliary task is actually realized. Thus the moving particle is meant as a positron, and the observer is on "motionless" electron. Therefore substituting (10) in (7) we shall find:

$$d\sigma = \frac{1}{128\pi^2}\frac{e^4}{pk^0}\left\{\frac{1}{4}\left(\frac{\omega_1}{\omega_2}+\frac{\omega_2}{\omega_1}+2\right)-\left(e_1e_2\right)^2\right\}d\Omega$$

$$=\frac{1}{128\pi^2}\frac{e^4}{4pk^0}\left\{\frac{\omega_1}{\omega_2}+\frac{\omega_2}{\omega_1}+2-4\left(e_1e_2\right)^2\right\}d\Omega. \tag{11}$$

Let's note that at use of the formula (10) we have actually refused the condition $\boldsymbol{k}_2=-\boldsymbol{k}_1$.

If in factor before brackets in the Formula (2) to use $k^0=E_+=m=\omega_1$ the formula (2) and Formula (11) become identical.

We shall note one important point arising at transition from the reference frame of the electron and positron mass center to the reference frame bound for electron or laboratory reference frame. If to divide the Formula (9) on the Formula (8) the result which differs from the result received in monographies. For example, (Itzukson & Zuber, 1984; Bjorken & Drell,1978) turns out.

At division (9) on (8) and accepting c=1 we receive:

$$\frac{\omega_2}{\omega_1}=\frac{k_2}{k_1}=\frac{1-V\cos\theta}{1+V\cos\theta} \tag{12}$$

In (Itzukson & Zuber, 1984; Bjorken & Drell, 1978) the following ratio is offered:

$$\frac{k_2}{k_1} = \frac{E_+ - p_+ \cos\theta}{m} \, . \qquad (13)$$

Taking into account $E_+=m$ and $p_+=mV$ we find:

$$\frac{k_2}{k_1} = 1 - V\cos\theta \, . \qquad (14)$$

The Formula (14) differs from the Formula (12) a little. It is connected by that the Formula (14) is received within the framework of the first approximation of the perturbation theory. Therefore it is essentially inexact. The Formula (12) follows from exact formulas of Doppler's effect. Thus remaining only within the framework of the first approximation of the perturbation theory it is impossible to establish equivalence of Formula (1) and Formula (2).

In summary we shall summarize the Formula (11) on photons with polarization. Coming back to polarise 4-vectors with the account $e^0=0$ also using $\sum_{e_1,e_2}(e_1e_2)^2 = 2$ (Bogolubov & Shirkov, 1976), we shall find:

$$\begin{aligned} d\sigma &= \frac{1}{128\pi^2}\frac{e^4}{4pk^0}\left\{\left|\frac{\omega_1}{\omega_2} + \frac{\omega_2}{\omega_1} + 2 - 4(e_1e_2)^2\right|\right\}d\Omega \\ &= \frac{1}{128\pi^2}\frac{e^4}{pk^0}\left\{\left|1 - \frac{(\omega_2-\omega_1)^2}{4\omega_1\omega_2}\right|\right\}d\Omega \, . \end{aligned} \qquad (15)$$

The module used owing to the standard use in the module of a compound matrix element of the finding of the process differential effective section (Heitler, 1956).

3. APPLICATION OF THE ANNIHILATIVE RADIATIONS IN A POSITRON-EMISSION TOMOGRAPH

Taking into account that in a positron-emission tomography the positrons speed are insignificant, and also taking into account Formula (8) and Formula (9) it is possible to write down:

$$\omega_1\omega_2 = \omega_0^2 \frac{1-\dfrac{V^2}{c^2}}{1-\left(\dfrac{V}{c}\right)^2\cos^2\theta} \approx \omega_0^2 \, . \qquad (16)$$

Substituting (16) in (15), we shall find:

$$d\sigma \approx \frac{1}{128\pi^2}\frac{e^4}{pk^0}\left\{\left|1 - \frac{(\Delta\omega)^2}{4\omega_0^2}\right|\right\}d\Omega \, . \qquad (17)$$

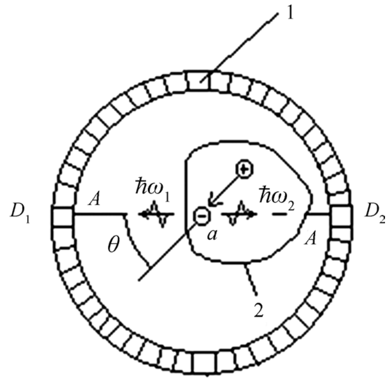

Figure 2
The Basic Scheme of Photons Registration in the Positron-Emission Tomography

Let's find the frequencies difference of radiated photons, i.e. size $\Delta\omega=\omega_1-\omega_2$, using the Formula (8) and Formula (9):

$$\Delta\omega = \omega_0\frac{\sqrt{1-\dfrac{V^2}{c^2}}}{1-\dfrac{V}{c}\cos\theta} - \omega_0\frac{\sqrt{1-\dfrac{V^2}{c^2}}}{1+\dfrac{V}{c}\cos\theta} = \omega_0\frac{2\dfrac{V}{c}\cos\theta\sqrt{1-\dfrac{V^2}{c^2}}}{1-\left(\dfrac{V}{c}\cos\theta\right)^2} \, . \qquad (18)$$

If the angle $\theta=0$, i.e. a positron moves on the line connecting detectors γ - radiation D_1 and D_2, the difference of the photons frequencies will be greatest and the Formula (18) will be transformed to the kind:

$$\Delta\omega_{\max} = \omega_0\frac{2\dfrac{V}{c}}{\sqrt{1-\dfrac{V^2}{c^2}}} \, . \qquad (19)$$

Taking into account $V<c$, we shall find:

$$\frac{\Delta\omega_{\max}}{2\omega_0} = \frac{V}{c} \qquad (20)$$

The size ω_0 can be found from the approached equality $h\omega_0 \approx mc^2$. In this case:

$$\Delta\omega_{\max} = 2\frac{V}{\lambda\!\!\!^{-}} \qquad (21)$$

where $\lambda\!\!\!^{-} = \dfrac{\hbar}{mc} = 3{,}86159 \cdot 10^{-13}$ m there is Compton's length of an electron wave (Javorsky & Detlavs, 1990).

In Figure 2 the basic scheme of the photons registration in the positron-emission tomography (Volobuev, 2011) is shown.

The researched object 2 is placed in the ring of detectors 1. At the annihilation of a positron and electron, taking place in a point *a*, the two quantums energies $\hbar\omega_1$ and $\hbar\omega_2$ in opposite directions radiated (Planck's constant h=1 used for clearing). If the quantums flying on line *A-A*, are registered by detectors D_1 and D_2 simultaneously the point of quantums emission is in the middle between detectors D_1 and D_2. Detectors in a ring 1 from the point of Doppler's effect view in the reference frame bound with electrons play a role of motionless observers.

By number of the quantums which are radiated in different directions process is spherical symmetric. Therefore the density of detectors in a ring 1 should be uniform. However the quantum frequencies and consequently also their energy depending on a direction on the detector (observer) due to the Doppler's effect can be different size $\Delta\omega=\omega_2-\omega_1$.

Measuring the frequencies or energies difference of the quantums which radiated opposite directions also using the maximal value of this difference during measurement $\Delta\omega_{\max}$ it is possible to find the speeds of positrons movement under the Formula (21). Taking into account that speed of positron is proportional to the density of a tissue $\rho\sim V$ through which it moves we receive the necessary information on density of a tissue in the tumor. This additional information can be received during diagnostics of an organism with help of the positron-emission tomograph.

CONCLUSION

By results of the carried out analysis we can draw the following conclusions.

Formulas Heitler (1) and Feynman (2) it is adequate in different reference frames describe scattering photons of annihilation of electron and positron.

In the laboratory reference frame bound with electron the angular distribution of number photons is absent however due to distinction in conditions of quantums supervision owing to Doppler's effect there is a distinction in frequencies of the radiated quantums.

To transition in the reference frame bound to the center of mass of electron and positron the distinction in frequencies of the radiated quantum is reduced in angular distribution of photons which also is consequence of Doppler's effect.

Investigating angular distribution of electromagnetic radiation intensity at annihilation of a positron and electron in the reference frame of their mass center our research not annihilation, and other physical phenomenon – Doppler's effect which accompanies with annihilation radiation. Hence the first not disappearing amendment of the perturbation theory received on the basis of a holes Dirac's hypothesis does not result in confirmation or denying of this hypothesis even if experiments confirm angular distribution of the annihilative radiation intensity.

Measuring the frequencies or energy difference of the quantum which have flung out opposite directions it is possible to find the speeds of positrons movement, see Formula (21). Taking into account that speed of a positron is proportional to density of substance through which it moves it is possible to receive the information on density of researched substance.

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